education

Department:
Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

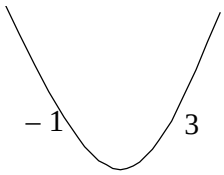
FEBRUARY/MARCH 2009

MEMORANDUM

MARKS: 150

This memorandum consists of 15 pages.

QUESTION 1

1.1.1	$3x+\frac{1}{x}=4$ $3x^2-4x+1=0$ $(3x-1)(x-1)=0$ $x=\frac{1}{3}$ or $x=1$	P standard form P factors PP answers (4)
1.1.2	$5x(x-3)=2$ $5x^2-15x-2=0$ $x=\frac{-(-15)\pm\sqrt{(-15)^2-4(5)(-2)}}{2(5)}$ $x=\frac{15\pm\sqrt{265}}{10}$ $x=3,13$ or $x=-0,13$	P standard form P substitution into formula P 265 PP answers (5)
1.1.3	$x^2-2x>3$ $x^2-2x-3>0$ $(x-3)(x+1)>0$ + 0 − 0 + −1 3 OR  $x<-1$ or $x>3$	P standard form P critical values PP answers (4)
1.2	$x-3y=1$ $x=1+3y$ $(1+3y)^2-2(1+3y)y+9y^2=17$ $1+6y+9y^2-2y-6y^2+9y^2-17=0$ $12y^2+4y-16=0$ $3y^2+y-4=0$ $(3y+4)(y-1)=0$ $y=-\frac{4}{3}$ or $y=1$ $x=-3$ or $x=4$	P $x=1+3y$ P substitution P simplification P factors P answers P Panswers (7

	OR $x-3y=1$ $y=\frac{x-1}{3}$ $x^2-2x\left(\frac{x-1}{3}\right)+9\left(\frac{x-1}{3}\right)^2=17$ $x^2-\frac{2x^2-2x}{3}+9\left(\frac{x^2-2x+1}{9}\right)=17$ $x^2-\frac{2x^2-2x}{3}+x^2-2x+1=17$ $3x^2-2x^2+2x+3x^2-6x+3=51$ $4x^2-4x-48=0$ $x^2-x-12=0$ $(x+3)(x-4)=0$ $x=-3$ or $x=4$ $y=-\frac{4}{3}$ or $y=1$	
1.3	let 1234567893 = n Then $1234567893\times1234567894-1234567895\times1234567892$ $=n(n+1)-(n+2)(n-1)$ $=n^2+n-n^2-n+2$ $=2$ **OR** Full marks for $(3\times4)-(5\times2)=2$	P method P simplification P answer (3) **[23]**

QUESTION 2

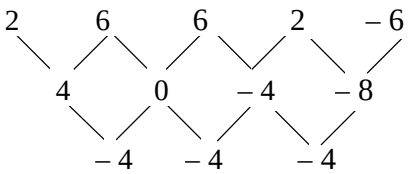
2.1.1	$S_2 = \frac{1}{1\times 2} + \frac{1}{2\times 3}$ $= \frac{1}{2} + \frac{1}{6}$ $= \frac{2}{3}$	P answer (1)
2.1.2	$S_3 = \frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4}$ $= \frac{2}{3} + \frac{1}{12}$ $= \frac{9}{12}$ $= \frac{3}{4}$	P answer (1)
2.1.3	$S_4 = \frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} + \frac{1}{4\times 5}$ $= \frac{3}{4} + \frac{1}{20}$ $= \frac{16}{20}$ $= \frac{4}{5}$	P answer (1)
2.2	If n represents the number of terms then the sum will be n divided by $n + 1$. **OR** $S_n = \frac{n}{n+1}$	P P answer (2)
2.3	$S_n = \frac{2008}{2009}$	P answer (1) **[6]**

QUESTION 3

3.1	$(2p-3)-(1-p)=(p+5)-(2p-3)$ $2p-3-1+p=p+5-2p+3$ $3p-4=-p+8$ $4p=12$ $p=3$	P equating differences P simplification P answer (3)
3.2.1	First term = $1-p=1-3=-2$	P – 2 (1)
3.2.2	-2 ; 3 ; 8 Common difference = 5 **OR** $3p-4=3(3)-4=5$	P answer (1)
3.3	After the first term -2, all the other terms end in either a 3 or an 8. Perfect squares never end in a 3 or an 8.	PP explanation (2) **[7]**

QUESTION 4

4.1	6 ; 6 ; 2 ; – 6 ; – 18 ;… First difference: 0, – 4, – 8, – 12, … Therefore the next term is: – 18 – 16 = – 34	P – 34 (1)
4.2	6 6 2 – 6 0 – 4 – 8 – 4 – 4 $2a=-4$ $a=-2$ $T_n=-2n^2+bn+c$ $6=-2+b+c$ $8=b+c$ ...(i) $6=-8+2b+c$ $14=2b+c$ …(ii) (ii) – (i): $6=b$ $\therefore c=2$ $T_n=-2n^2+6n+2$ **OR**	ü $a=-2$ ü solving simultaneously ü $b=6$ ü $c=2$ ü general term (5)

	 $2a = -4$ $a = -2$ $T_0 = c = 2$ $T_1 = -2 + b + 2 = 6$ $b = 6$ $T_n = -2n^2 + 6n + 2$ **OR** $T_2 - T_1 = 0$ $T_3 - T_2 = -4$ $T_4 - T_3 = -8$ M $T_n - T_{n-1} = -4(n-2) = 8 - 4n$ Adding both sides $T_n - 6 = -4[1 + 2 + 3 + ...(n-2)]$ $T_n = -4\left[\frac{(n-2)(n-1)}{2}\right] + 6$ $T_n = -2n^2 + 6n + 2$	ü $a = -2$ P $c = 2$ P substitution P $b = 6$ P general term (5) Pobtaining differences P adding both sides P substituting $T_1 = 6$ P expression for sum of terms Psimplification (5)
4.3	$-6838 = -2n^2 + 6n + 2$ $-2n^2 + 6n + 6840 = 0$ $n^2 - 3n - 3420 = 0$ $n = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(-3420)}}{2}$ $n = \frac{3 \pm 117}{2}$ n = 60 or n = - 57 not possible $\therefore T_{60} = -6838$	P standard form P substitution P values of n P answer (4) **[10]**

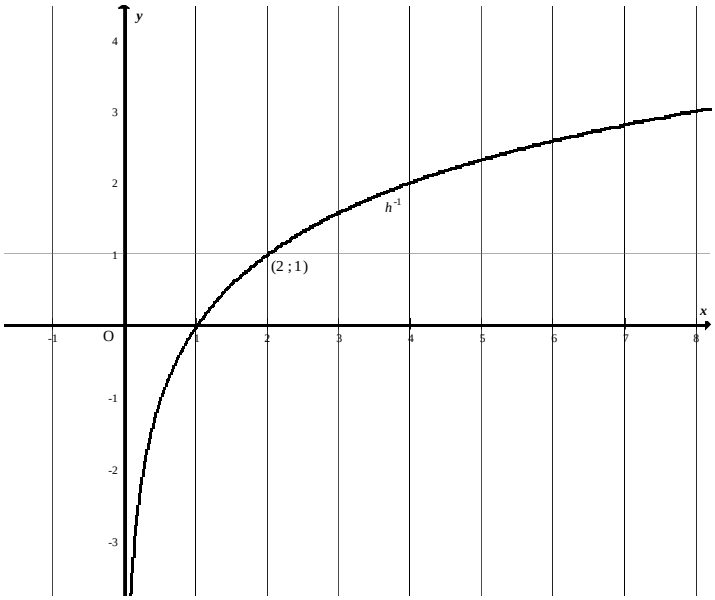
QUESTION 5

5.1	Area of unshaded square = $1-\frac{1}{16}$ $= \frac{15}{16}$	P1 P $\frac{1}{16}$ P answer (2)
5.2	Sum of the unshaded areas of the first seven squares $=(1-1)+\left(1-\frac{1}{4}\right)+\left(1-\frac{1}{4^2}\right)+\ldots+\left(1-\frac{1}{4^6}\right)$ $=7-\left(1+\frac{1}{4}+\frac{1}{4^2}+\ldots+\frac{1}{4^6}\right)$ $=7-\left(\frac{1\left(1-\left(\frac{1}{4}\right)^7\right)}{1-\frac{1}{4}}\right)$ $=7-1,333251953\ldots$ $=5,666748047\ldots$ $=5,67$ **OR** Sum of the unshaded areas of the first seven squares $=(1-1)+\left(1-\frac{1}{4}\right)+\left(1-\frac{1}{4^2}\right)+\ldots+\left(1-\frac{1}{4^6}\right)$ $=6-\left(\frac{1}{4}+\frac{1}{4^2}+\ldots+\frac{1}{4^6}\right)$ $=6-\frac{\frac{1}{4}\left(1-\left(\frac{1}{4}\right)^6\right)}{1-\frac{1}{4}}$ $=6-0,332519531\ldots$ $=5,666748047\ldots$ $=5,67$	Pexpression for sum of unshaded areas P simplification P answer (5) **[7]**

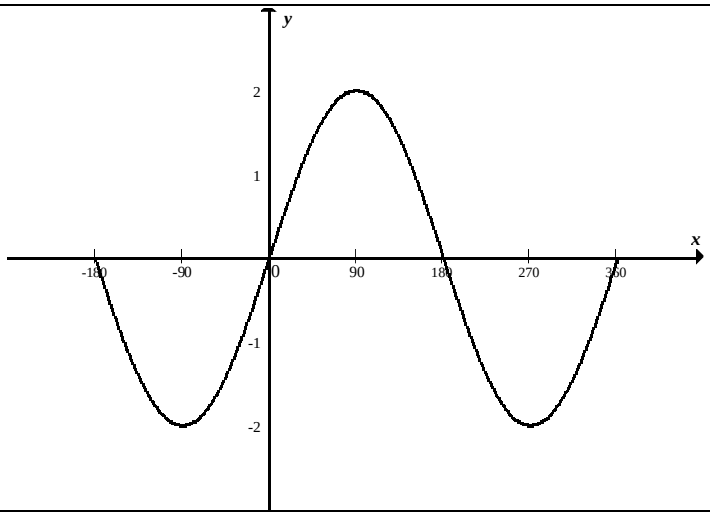
QUESTION 6

6.1	$y = \frac{a}{x-1} + 2$ $0 = \frac{a}{0-1} + 2$ $0 = -a + 2$ $a = 2$ $y = \frac{2}{x-1} + 2$ **OR** $(x-1)(y-2) = k$ But g passes through the origin $\therefore (-1)(-2) = k = 2$ $\therefore (x-1)(y-2) = 2$	P $b = 1$ P $c = 2$ P substitution of $(0\ ;\ 0)$ P $a = 2$ (4) P equation P through origin P substitution P equation (4)
6.2	$g(x) = (x-1)^2 + q$ $0 = (2{,}5-1)^2 + q$ $q = -\frac{9}{4}$ Turning point $\left(1\ ;\ -\frac{9}{4}\right)$	P $p = 1$ P substitution of $(2{,}5\ ;\ 0)$ P $q = -\frac{9}{4}$ P answer (4)
6.3	$y = 2$ $x = 2$	P answer P answer (2)
6.4	$h(x) = -(x-1)^2 + \frac{9}{4}$	P answer (1) **[11]**

QUESTION 7

7.1	Any base raised to the power 0 is 1 which means the *y*-intercept of the graph $h(x) = a^x$ will be (0 ; 1) therefore Q(0 ; 1) **OR** $h(0) = a^0 = 1$ $\therefore$ Q(0 ; 1)	P *y*-intercept P any base raised to power 0 is 1 (2)
7.2	$a^{-1} = \frac{1}{2}$ $\frac{1}{a} = \frac{1}{2}$ $a = 2$	P substitution P answer (2)
7.3	$2^y = x$ $y = \log_2 x$	P interchanging *x* and *y* P answer (2)
7.4		P point (0,5 ; – 1) or any other valid point P point (1 ; 0) P shape (3)
7.5	$x > 0{,}5$	P reading off from graph P answer (2)
7.6	$\therefore 2 + x\log 3 = x\log 2$ $\therefore x = \frac{2}{\log \frac{2}{3}} = -11.36$ **OR** $\left(\frac{2}{3}\right)^x = 100$ $\therefore x \log \left(\frac{2}{3}\right) = 2$ $\therefore x = \frac{2}{\log \frac{2}{3}} = -11.36$	P equating P logs both sides P answer (3) **[14]**

QUESTION 8

8.1		P shape P amplitude (2)
8.2	$y \in [-4\,;\,4]$	PP answer (2)
8.3	$720°$	PP answer (2)
8.4	Shift f $90°$ to the left to get $2\cos x$. $\therefore \theta = 90° + n.360°$	PP answer Accept $90°$ (2) **[8]**

QUESTION 9

9.1	$2000\left(1+\frac{i}{12}\right)^{18}=2860$ $\left(1+\frac{i}{12}\right)^{18}=1,43$ $1+\frac{i}{12}=1,020069541...$ $\frac{i}{12}=0,020069541...$ $i=0,24083...$ $i=24,08\%$	P substitution P $\left(1+\frac{i}{12}\right)^{18}=1,43$ P $i=0,24083\ldots$ P answer (4)
9.2	$F_v=\frac{100\left[\left(1+\frac{0,08}{12}\right)^{12}-1\right]}{\frac{0,08}{12}}$ $=R1\,244,99$ The accumulated amount is less than R1 300 required to buy the bike. Farouk will not be able to buy the bike on 1 January 2009.	P formula P substitution PP answer P conclusion (5) **[9]**

QUESTION 10

10.1	Loan = $125000-\frac{15}{100}\times125000$ Loan = R 106 250 OR Loan = $0,85\times125\,000$ Loan = R 106 250	P answer (1)
10.2	$106250=x\left[\frac{1-\left(1+\frac{0,125}{12}\right)^{-6\times12}}{\frac{0,125}{12}}\right]$ $1106,770833=x\left(1-\left(1+\frac{0,125}{12}\right)^{-6\times12}\right)$ $x=$ R 2104,94	P substitution P – 72 P $\left(\frac{0,125}{12}\right)$ P simplification P answer (5) **[6]**

QUESTION 11

11.1	$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h\to 0} \frac{(x+h)^2 - 2(x+h) - x^2 + 2x}{h}$ $= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - 2x - 2h - x^2 + 2x}{h}$ $= \lim_{h\to 0} \frac{2xh + h^2 - 2h}{h}$ $= \lim_{h\to 0} \frac{h(2x + h - 2)}{h}$ $= \lim_{h\to 0}(2x - 2 + h)$ $= 2x - 2$	ü method ü substitution ü simplification ü factorising ü answer (5)
11.2.1	$D_x[(x^3 - 3)^2]$ $= D_x\left[x^6 - 6x^3 + 9\right]$ $= 6x^5 - 18x^2$	P simplification PP answer (3)
11.2.2	$y = \frac{4}{\sqrt{x}} - \frac{x^3}{9}$ $y = 4x^{\frac{-1}{2}} - \frac{1}{9}x^3$ $\frac{dy}{dx} = -2x^{\frac{-3}{2}} - \frac{x^2}{3}$	P power form PP answer (3) **[11]**

QUESTION 12

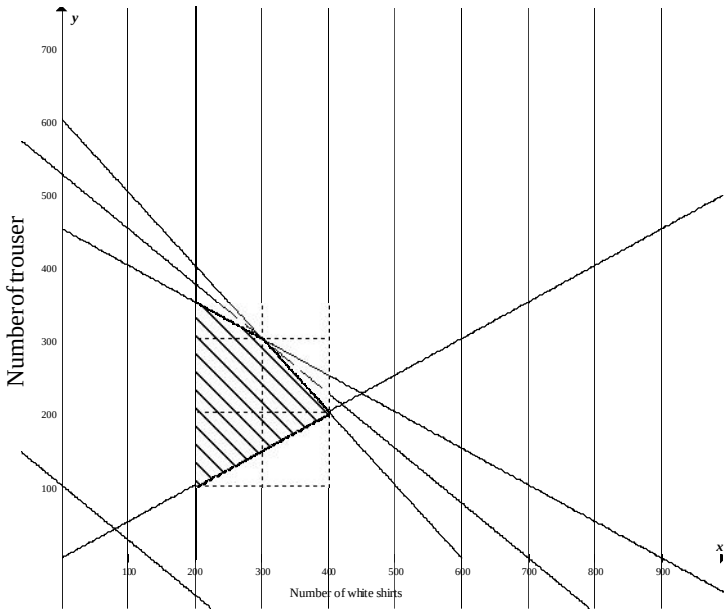
12.1	$h'(x) = -3x^2 + 2ax + b$ $h'(-1) = -3(-1)^2 + 2a(-1) + b$ $0 = -3 - 2a + b$ $2a - b = -3$ K (i) $h'(2) = -3(2)^2 + 2a(2) + b$ $0 = -12 + 4a + b$ $4a + b = 12$ K (ii) (ii) + (i): $6a = 9$ $a = \frac{3}{2}$ $\therefore 2\left(\frac{3}{2}\right) - b = -3$ $b = 6$ **OR** $h(-1) = -(-1)^3 + a(-1)^2 + b(-1) = \frac{-7}{2}$ $\therefore a - b = \frac{-9}{2}$ $2a - 2b = -9$ L (i) $h(2) = -(2)^3 + a(2)^2 + b(2) = 10$ $4a + 2b = 18$ L (ii) (i) + (ii): $6a = 9$ $a = \frac{3}{2}$ $\left(\frac{3}{2}\right) - b = \frac{-9}{2}$ $b = 6$	P $h'(x)$ P substitution of $x = -1$ P $h'(x) = 0$ P simplification P substitution P solving simultaneously (6) P substitution of $x = -1$ P $h(-1) = \frac{-7}{2}$ P simplification P substitution of $x = 2$ and $h(2) = 10$ P simplification P solving simultaneously (6)
12.2	Average Gradient $= \frac{10 - (-3,5)}{2 - (-1)}$ $= \frac{13,5}{3}$ $= \frac{9}{2}$	P substitution P answer (2)

12.3	$h'(x) = -3x^2 + 3x + 6$ $h'(-2) = -3(-2)^2 + 3(-2) + 6$ $h'(-2) = -12$ Point of contact $(-2\ ;\ 2)$ $y - 2 = -12(x + 2)$ $y = -12x - 22$	P $h'(x)$ P substitution P gradient P point P answer (5)
12.4	$h'(x) = -3x^2 + 3x + 6$ $h''(x) = -6x + 3$ $-6x + 3 = 0$ $x = \frac{1}{2}$ **OR** $x = \frac{-1+2}{2}$ $x = \frac{1}{2}$	ü second derivative ü = 0 ü answer (3)
12.5	$p > 3{,}5$ or $p < -10$	PP answer (2) **[19]**

QUESTION 13

13.1	$AB^2 = (a^2 - 0)^2 + (a - 3)^2$ $AB^2 = a^4 + a^2 - 6a + 9$	P substitution P simplification (2)
13.2	$\frac{d}{da} AB^2 = 4a^3 + 2a - 6$ $0 = 4a^3 + 2a - 6$ $0 = 2a^3 + a - 3$ $0 = (a-1)(2a^2 + 2a + 3)$ $a = 1$ There is no real solution for $2a^2 + 2a + 3 = 0$	P $\frac{d}{da} AB^2$ P $\frac{d}{da} AB^2 = 0$ P simplification P factorisation P answer (5) **[7]**

QUESTION 14

14.1	$x \geq 200$ $x + y \leq 600$ $50x + 100y \leq 45000$	P answer P answer P answer (3)
14.2 and 14.3		P 600 P450 P 200 P 600 P 900 (5) P feasible region (1)
14.4	P = 30x + 40y	PP answer (2)
14.5	Maximum at (300 ; 300)	P search line P answer (2) **[13]**

TOTAL: 150