education

Department:
Education
REPUBLIC OF SOUTH AFRICA

NATIONAL
SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

PREPARATORY EXAMINATION 2008

MARKS: 150

TIME: 3 hours

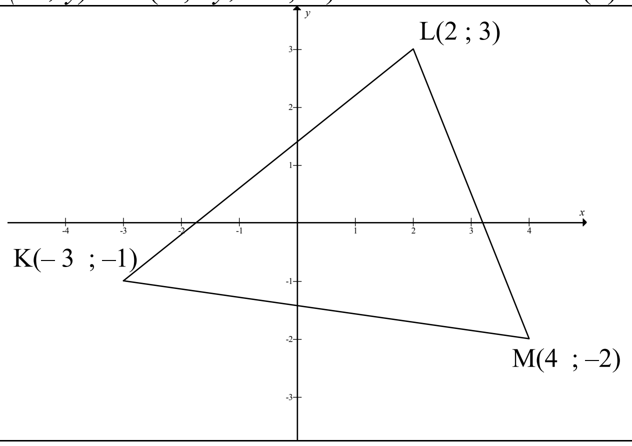
MARKING LEGEND	
SYMBOL	EXPLANATION OF SYMBOL
√ A	Accuracy
√ M	Method
√ CA	Continuous Accuracy *(follow-up)*
√ B/D	Break down
√ S	Statement
√ R	Reason
√ S/R	Statement and Reason

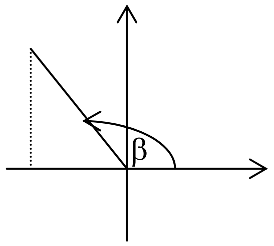
This memorandum consists of 12 pages.

QUESTION 1		[25]
	SOLUTION	**EXPLANATION**
1.1.1	$BC = \sqrt{(x_C - x_B)^2 + (y_C - y_B)^2}$ √ M $= \sqrt{(0-2)^2 + (-6-4)^2}$ √A $= \sqrt{104}$ $= 2\sqrt{26}$ √ CA (3)	M – for distance formula in any format (or if implied) A- for correct substitution into distance formula. CA – simplification of answer in **surd form.** **ANSWER ONLY in simplified surd form : Full marks.**
1.1.2	$(x_M; y_M) = (\frac{x_B + x_C}{2} ; \frac{y_B + y_C}{2})$ $= (1 ; -1)$ √A √A M(1; – 1) (2)	2A – 1 for x value of M 1 for y value of M **ANSWER ONLY: Full marks. Answer need not be in coordinate form.** **Accept: x = 1 ; y = – 1**
1.1.3	$m_{BC} = \frac{y_C - y_B}{x_C - x_B}$ √M $= \frac{-6-4}{0-2}$ $= 5$ √ CA (2)	M- Use of gradient formula (or implied) CA – for gradient value **ANSWER ONLY: Full marks.**
1.1.4	$m_{BC} = \tan\theta$ √M $\therefore \tan\theta = 5$ $\therefore \theta = 78{,}69^\circ$ √CA (2)	M- inclination is related to gradient of straight line. CA- value of θ. **No penalty for rounding off.** **Accept 79^0 and $78{,}7^0$.** **ANSWER ONLY: Full marks.**
1.2	AM ⊥ BC - given $\therefore m_{AM} = -\frac{1}{5}$ √CA *(product of gradients = – 1)* $y = mx + c$ √M $(-1) = \frac{-1}{5}(1) + c$ √CA √CA $\therefore c = \frac{-4}{5}$ √CA **OR** **Required equation is:** $y = \frac{-1}{5}x - \frac{4}{5}$ √CA (5)	CA – for value of gradient of AM M – formula of straight line CA – substitution of (1 ;– 1) in straight line equation CA – substitution of gradient into straight line equation. CA – value of c OR for equation in standard form.

1.3	$y = \frac{-1}{5}x - \frac{4}{5}$ ....(1) $y = -x + 8$ ....(2) (2) – (1) : $0 = -\frac{4}{5}x + \frac{44}{5}$ √CA $\frac{4}{5}x = \frac{44}{5}$ √CA $x = 11$√CA Sub $x = 11$ in (2): $y = -(11) + 8$ √CA $y = -3$ √CA ∴ A (11 ; – 3) (5)	CA- Eliminating y from both equations CA – rewriting with $\frac{4}{5}x$ on LHS and constant value on RHS. CA – Solving for x CA – Substituting x into either (1) OR (2) CA – Solving for y
1.4	A'(22 ; –6) √CA B'(4 ; 8) √CA C'(0 ; – 12) √CA (3)	3 CA- 1 for each points coordinates
1.5	$\frac{\text{Area}\,\Delta\,\text{A'B'C'}}{\text{Area}\,\Delta\,\text{ABC}} = \frac{\frac{1}{2}(B'C')(A'M')}{\frac{1}{2}(BC)(AM)}$ √M $= \frac{(2\text{BC})(2\text{AM})}{(\text{BC})(\text{AM})}$ √M $= \frac{4}{1}$ √CA (3) OR $\frac{\text{Area}\,\Delta\,\text{A'B'C'}}{\text{Area}\,\Delta\,\text{ABC}} = \frac{k^2}{1} = \frac{4}{1}$	M- Area formula M- substitution B'C' = 2BC and A'M' = 2AM CA – value of ratio. **ANSWER ONLY: Full marks**

QUESTION 2		[25]
	SOLUTION	**EXPLANATION**
2.1	$y = \frac{1}{3}x + \frac{11}{3}$ √M $m = \frac{1}{3}$ √CA $m_{TU} = \frac{y_U - y_T}{x_U - x_T}$ $= \frac{3}{k+6}$ √A Thus $m_{TU} = \frac{1}{3}$ √M *(TU is parallel to given line)* $\frac{1}{3} = \frac{3}{k+6}$ √M $k + 6 = 9$ $k = 3$ √CA (5)	M- equation of straight line in standard form CA – value of gradient of given line A- value of gradient of straight line TU M- Using the fact that parallel lines have equal gradients M – equating the two gradients CA – value of *k*
2.2.1	$x^2 + y^2 - 6x + 2y + t = 0$ $(x - 3)^2 + (y + 1)^2 = 10 - t$ √A √A √A M (3 ; – 1) √CA (4)	3A: 2A for LHS – binomial 1A for RHS CA – coordinates of M
2.2.2	$r = \sqrt{10-t}$ √CA (1)	CA – value of radius in terms of *t*
2.2.3	$(AM)^2 = (MB)^2 + (AB)^2$ √M (Theorem of Pythagoras) $(\sqrt{35})^2 = (\sqrt{10-t})^2 + (3\sqrt{2})^2$ √CA $35 = 10 - t + 18$ √A $t = -7$ √CA (4)	M – use of Pythagoras CA – substitution A – simplification CA – value of *t*

QUESTION 3		**[18]**
	SOLUTION	**EXPLANATION**
3.1.1	C'(–2 ; –4) √A√A (2)	2A – 1 for x value 1 for y value
3.1.2	$A'B' = 2\sqrt{29}$ √A (1)	A – for value of $A'B'$
3.1.3	In a translation the shape and size of the original figure is retained, hence the lengths of the sides of the triangle will remain invariant under a translation. √A√A (2)	2A – look for terms shape and size remaining the same /invariant
3.1.4 (a)	**Stage 1: (rotated 90^o in a clockwise direction about the origin)** $(x ; y) \rightarrow (y ; -x)$ √M **Stage 2: (enlargement by factor of 0,5 through origin)** $(x ; y) \rightarrow (0{,}5x ; 0{,}5y)$ √M **Combining stages 1 and 2:** $(x ; y) \rightarrow (0{,}5y ; -0{,}5x)$ √CA √CA (4)	M - application of transformation rule M - Application of second transformation rule 2CA – 1 for each ordinate
3.1.4 (b)		√CA √CA √CA 1 for each coordinate.
3.2	A (1 ; –2) ; $A'(x'; y')$ $x' = x\cos\alpha - y\sin\alpha$ √M $= (1)\cos(60^o) - (-2)\sin(60^o)$ √A $= \frac{1}{2} + 2\left(\frac{\sqrt{3}}{2}\right)$ $= \sqrt{3} + \frac{1}{2}$ √CA $y' = y\cos\alpha + x\sin\alpha$ √M $= (-2)\cos(60^o) + 1\sin(60^o)$ √A $= -1 + \left(\frac{\sqrt{3}}{2}\right)$ √CA $A'\left(\sqrt{3} + \frac{1}{2} ; -1 + \frac{\sqrt{3}}{2}\right)$ (6) Or $A'(2{,}23;-0{,}13)$	M – formula A- substitution correctly into formula CA – answer M – formula A- substitution correctly into formula CA – answer

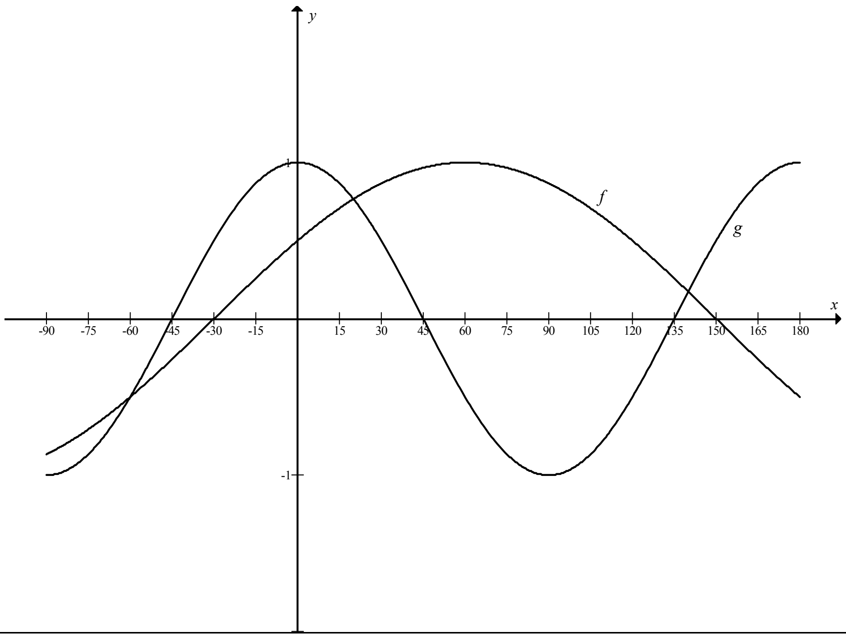
QUESTION 4		[17]
	SOLUTION	**EXPLANATION**
4.1	 √M sin β = a Let unknown side be x: Using Pythagoras: x $= -\sqrt{1-a^2}$ √M √M tan β = $\frac{-a}{\sqrt{1-a^2}}$ √CA (4)	M – for angle in correct quadrant in Cartesian plane M – using of Pythagoras to calculate unknown side M – selecting the x-value in the correct quadrant CA – value of tan β
4.2	$\frac{\sin 15^o + 2\cos(-135^o)}{\sin 300^o}$ $= \frac{\sin(45^o - 30^o) + 2\cos 45^o}{-\sin 60^o}$ √M √M √M $= \frac{\sin 45^o \cos 30^o - \cos 45^o \sin 30^o + 2(\frac{\sqrt{2}}{2})}{-\frac{\sqrt{3}}{2}}$ √M√A √A $= \frac{(\frac{\sqrt{2}}{2})(\frac{\sqrt{3}}{2}) - (\frac{\sqrt{2}}{2})(\frac{1}{2}) + \sqrt{2}}{\frac{-\sqrt{3}}{2}}$ $= \frac{3\sqrt{2} + \sqrt{6}}{-2\sqrt{3}}$ √CA Or $= \frac{\sqrt{2}(3+\sqrt{3})}{-2\sqrt{3}}$ (7)	3M : 1M for rewriting sin 15º as sin (45 – 30) 1M for reduction formula for cos(-135) 1M for reduction formula for sin (360-60) M – for expansion of sin (45 – 30) A – for value of cos 45 A – for value of sin 60 CA – for simplification and solution. **Answer only : max 1 /7**
4.3	$\frac{\sin(-\theta).\sin(180^o - \theta) + \cos(90^o + \theta)}{-\sin(360^o - \theta) - \tan 315^0}$ $= \frac{-\sin\theta . \sin\theta - \sin\theta}{+\sin\theta + \tan 45^o}$ √A√A√A√A√A $= \frac{-\sin\theta(\sin\theta + 1)}{\sin\theta + 1}$ $= -\sin\theta$ √CA **(6)**	5A: 1A for - sin θ 1A for sin θ 1A for - sin θ 1 A for + sin θ 1 A for + tan 45º 1CA for simplification

QUESTION 5		[8]
5.1	Undefined for $\cos A = \sin A$ $\tan A = 1$ $A = 45^o + k.180^o$ √M for $k \in Z$ OR $\cos 2A = 0$ $2A = 90^o + k.180^o$ $A = 45^o + k.90^o$ √M $A = 45^o + k.90^o$ for $k \in Z$ is the overall answer (2)	M –value of A M- value of A
5.2	$LHS = \frac{1}{\cos 2A} + \frac{\sin 2A}{\cos 2A}$ $= \frac{1+\sin 2A}{\cos 2A}$ √M $= \frac{\sin^2 A + \cos^2 A + 2\sin A.\cos A}{\cos^2 A - \sin^2 A}$ √M√M√M $= \frac{(\sin A + \cos A)^2}{(\cos A - \sin A).(\cos A + \sin A)}$ √M√M $= \frac{\sin A + \cos A}{\cos A - \sin A}$ $\therefore \frac{1}{\cos 2A} + \frac{\sin 2A}{\cos 2A} = \frac{\sin A + \cos A}{\sin A - \cos A}$ (6)	M- common denominator M – for writing 1 = $\sin^2$ A + $\cos^2$ A M – for writing sin2A = 2 sinAcosA M – for writing cos 2A as $\cos^2$ A – $\sin^2$ A M- factorizing numerator M- factorizing denominator
QUESTION 6		**[7]**
6.1	$1 - 2\sin^2 x$ √A	A – for value of cos 2*x*
6.2	$\cos 2x + \sin x = 0$ $1 - 2\sin^2 x + \sin x = 0$ √M $2\sin^2 x - \sin x - 1 = 0$ $(2\sin x + 1).(\sin x - 1) = 0$ $\sin x = -\frac{1}{2}$ √ A OR $\sin x = 1$ √A $x = 210^o + k.360^o$ √CA OR $x = 90^o + k.360^o$ √ CA where $k \in Z$ $x = 330^o + k.360^o$ √CA (6)	M – substitution for value of cos 2x A for value of sin x = - 0,5 A for sin x =1 3CA : 1 each for angles

QUESTION 7 [20]

7.1

	$f(x)$	$g(x)$
x-intercepts	$(-30^o ; 0)$ $(150^o ; 0)$ √ M	$(-45^0 ; 0)$ $(45^0 ; 0)$ $(135^0 ; 0)$ √ M
Y – intercepts	$(0^o; 0,5)$ √ M	$(0^o; 1)$ √ M
Turning point	$(60^o ; 1)$ √ M	$(0^o; 1)$ $(-90^o; 1)$ $(90^o; -1)$ $(180^o; 1)$ √ M
Shape	√ M	√M



ALL intercepts and ALL turning points must be shown

7.2

$\sin(x+30^o)=\cos 2x$

$\sin(x+30^o)=\sin(90^o-2x)$ √M

$x+30^o=90^o-2x+k.360^o$ √ M AND

$x+30^o=180^o-(90^o-2x)+k.360^o$ √M√M

$x=20^o+k.120^o$ √ M AND

$x=-60^o+k.360^o$ √M

For the given interval : $x=-60^o;20^o$ √CA√CA

(8)

M – cos 2x = sin (90° –2*x*)
M – equating both sides
2M – use of reduction formula and addition of k.360^0
M –solution of x
M –solution of x (negative angle)
2CA – 1 CA for each value within the given interval

7.3

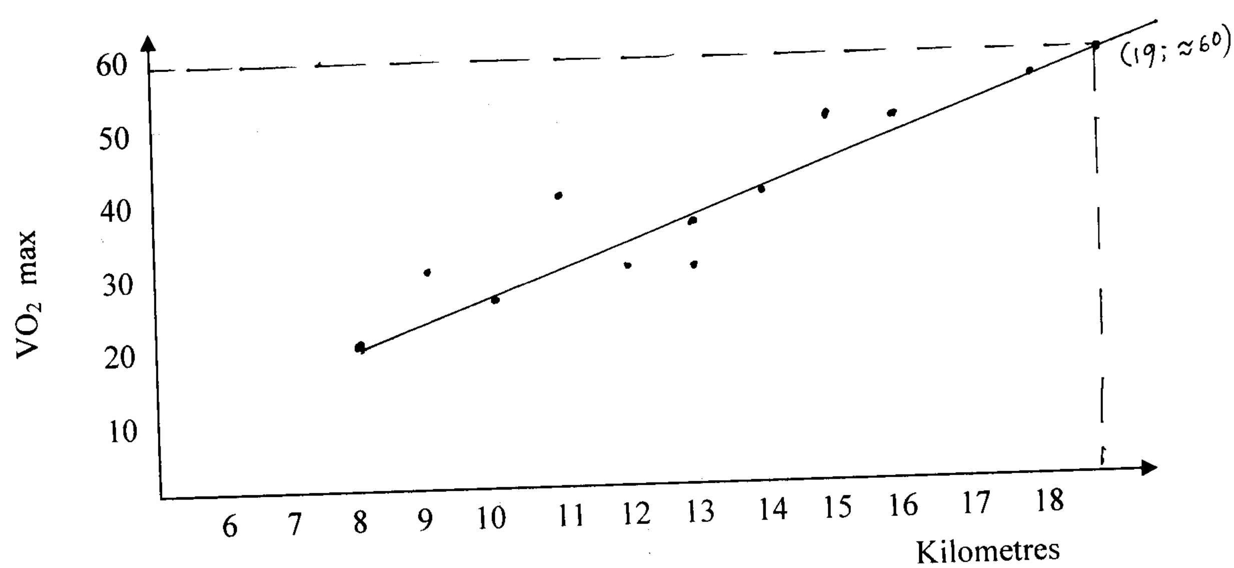
$x\in[20^o;140^o]$ √CA√CA

7.4

$x\in[-90^o;-45^o]$ √ CA $\cup$ $x\in[90^o;135^o]$ √CA

QUESTION 8		**[12]**
8.1	$\tan y = \frac{h}{OL}$ √M $OL = \frac{h}{\tan y}$ √A	
8.2	In ΔFLE: $\frac{\sin 2x}{h} = \frac{\sin(90^o - x)}{2}$ √ Mfor sine rule and *2x* angle $2\ \sin 2x = h\cos x$ √M for co-function $h = \frac{2\ \sin 2x}{\cos x}$ √ $h = \frac{4\sin x.\cos x}{\cos x}$ √M for expanding *sin2x* $h = 4\sin x$ √A $OL = \frac{4\ \sin x}{\tan y}$ √A	
8.3	$Area = \frac{1}{2} h\ .EL.\sin(180° - 106° - (90° - 53°))$ √M $Area = 0,5 \times 3,2 \times 2 \times \sin 37^o$ √ √ A $Area = 1,93\ units^2$ √A	

QUESTION 9		[11]
9.1.1	See graph below √M√M√M	3M –for plotting points
9.1.2	See graph below √CA√CA	2CA for drawing of graph.
9.1.3	See graph below √CA, i.e approximately 60 units of VO_2 max	1CA for estimated value
9.1.4	It would appear that the more kilometers the athletes covered the more VO_2 max they used. √CA√CA	2CA for interpretation from graph



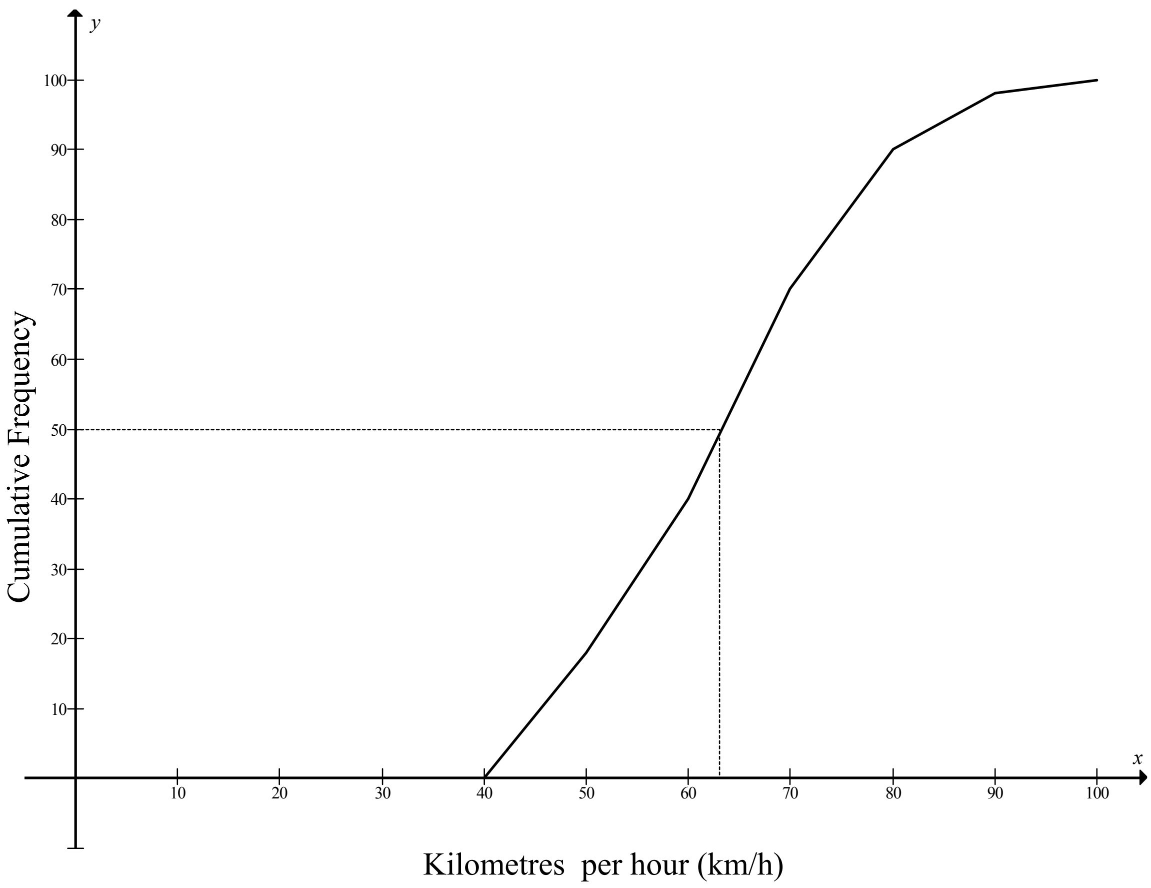
9.2.	1 | 2 5 5 7 8 8 9 2 | 0 1 2 3 3 | 3 4 4 8 8 8	
9.2.1	Median=21 √	√
9.2.2	Upper quartile=34 √	√
9.2.3	Lower quartile=17,5 √	√

QUESTION 10.1

Speed intervals (in km/h)	Number of cars (frequency)	Cumulative frequency
$40 \leq speed < 50$	18	**18**
$50 \leq speed < 60$	22	**40**
$60 \leq speed < 70$	30	**70**
$70 \leq speed < 80$	20	**90**
$80 \leq speed < 90$	8	**98**
$90 \leq speed < 100$	2	**100**
	Total : 100	

√A√A for values

QUESTION 10.2



10.3	63 km/h √A (1)	
10.4	The majority of drivers are exceeding the speed limit. √CA √CA (2)	2CA for conclusion from ogive curve

QUESTION 11		[8]
11.1	The lowest and the highest values (marks) √M	
11.2	The mark 64 represents the upper quartile,i.e: 0,75 X 31 = 23,25 √M The **23rd learner** obtained a mark of 64 √CA	
11.3	Yes,the learner is correct. Since the "gap" between the median and the lower quartile is not the same as the "gap" between the median and the upper quartile. √M√M	
11.4	Interval is (26,3 ; 64,7) √M 16 of the 31 results lie within 1 standard deviation of the mean , hence 15 will lie outside one standard deviation of the mean. √CA√CA	