LOGARITHMS

Lesson 7

Learning Outcomes and Assessment Standards

Learning outcome 1: Number and number relationships
Assessment Standard 12.1.2

- Demonstrates an understanding of the definition of a logarithm and any laws needed to solve real life problems (eg growth and decay)

Overview

In this lesson you will:

- learn what is meant by a logarithm
- change identities from log form to exponential form and visa versa
- use the laws of logarithms to simplify expressions and solve basic equations.

Lesson

$3^2 = 9$

This is an expression written in exponential form. The inverse of this exponential expression is the logarithmic expression.

We can write this same expression in logarithmic form: $2 = \log_3 9$.

So: If $y = \log_b x$ then $x = b^y$ where $b > 0$; $b \neq 1$ and as a result $x > 0$. We say that the *log* is the *power* that a certain base must be raised to obtain a certain answer.

Change from exponential form to log form.

1. $2^4 = 16$ $\quad 4 = \log_2 16$
2. $3^{-1} = \frac{1}{3}$ $\quad -1 = \log_3 \frac{1}{3}$
3. $25^{\frac{1}{2}} = 5$ $\quad \frac{1}{2} = \log_{25} 5$
4. $10^2 = 100$ $\quad 2 = \log_{10} 100$

When we don't specify the base we assume we are using base 10.

Change from log form to exponential form:

a) $4 = \log_3 81$ $\quad 3^4 = 81$
b) $-1 = \log_3 \frac{1}{3}$ $\quad 3^{-1} = \frac{1}{3}$
c) $6 = \log_2 64$ $\quad 2^6 = 64$
d) $3 = \log 1\,000$ $\quad 10^3 = 1\,000$

Some conclusions

$3^0 = 1$ $\quad 10^0 = 1$

$0 = \log_3 1$ $\quad 0 = \log_{10} 1$

In general

$a^0 = 1$ also $\quad 2^1 = 2$

$0 = \log_a 1$ $\quad 1 = \log_2 2$

$\left(\frac{1}{3}\right)^1 = \frac{1}{3}$

$\frac{1}{3}1 = \log_a$

$10^1 = 10$

$1 = \log 10$

In general

$a^1 = a$

$1 = \log_a a$

Remember: If the base is not specified then it is base 10. $\log 2 = \log_{10} 2$

Log laws

1. $\log_a (xy) = \log_a x + \log_a y$ (If we multiply numbers, we add their logs)
2. $\log_a \frac{x}{y} = \log_a x - \log_a y$ (If we divide numbers, we subtract their logs)

Examples

1. Simplify $\log_6 2 + \log_6 3$

   $= \log_6 (2 \times 3)$

   $= \log_6 6$

   $= 1$

2. Simplify $\log_2 4 + \log_2 3 - \log_2 6 - \log_2 2$

   $= \log_2 \frac{4 \times 3}{6 \times 2}$

   $= \log_2 1$

   $= 0$

The power law

$\log_a x^y = y \log_a x$ (the power of the base multiplies with the log)

Examples

1. $\log_3 27$

   $= \log_3 3^3$ (prime the number)

   $= 3 \log_3 3$ (power comes down)

   $= 3$ ($\log_a a = 1$)

2. $\log_2 3$

   $= \log_2 2^{-2}$ (prime the number)

   $= -2 \log_2 2$ (power comes down)

   $= -2$ ($\log_a a = 1$)

3. Simplify $2 \log_{12} 3 + 4 \log_{12} 2$

   $\log_{12} 3^2 + \log_{12} 2^4$ (power goes back)

   $= \log_{12} (3^2 \times 2^4)$ (law 1)

   $= \log_{12} 144$

   $= \log_{12} 12^2$ (forming $\log_a a^k = k$)

   $= 2 \log_{12} 12$

   $= 2$

4. Simplify $2 \log 5 + 2 \log 2$

   $= \log 5^2 + \log 2^2$

   $= \log (25 \times 4)$

   $= \log 100$

$= \log 10^2$
$= 2 \log 10$
$= 2$

Be careful not to abuse the laws – obey them

$\log_2 32 - \log_2 8$
$= \log_2 \frac{32}{8}$
$= \log_2 4$
$= \log_2 2^2$
$= 2$

But $\log 32 \div \log 8 = \frac{\log 32}{\log 8} = \frac{\log 2^5}{\log 2^3} = \frac{5\log 2}{3\log 2} = \frac{5}{3}$

Also $\log(6 + 4)$

$\neq \log 6 + \log 4$

$\log 6 + \log 4$

$= \log 24$

One last one

If $\log (x + y) = \log x + \log y$

express x in terms of y

$\log(x + y) = \log (xy)$

$x + y = xy$

$y = xy - x$

$x = x(y - 1)$

$x = \frac{y}{y - 1} \quad x > 0 \quad y > 0 \quad y \neq 1$

Activity 1

Express as the log of a single number and simplify if possible

1. $\log_5 30 - \log_5 6$
2. $3 \log_2 8 - \log_2 16$
3. $2 \log 5 + 3 \log 2 - \log 2$
4. $\log_3 \frac{1}{9} - \log_3 \frac{1}{2}$
5. $\frac{1}{2}\log 25 - \log 2 + \frac{1}{3}\log 64$

Activity 2

1. $\log_5 125 + \log_5 625$
2. $\log_5 125 \div \log_5 625$
3. $\log_3 81 + \log_3 9$
4. $\log_3 81 \div \log_3 \frac{1}{9}$
5. $\frac{\log 64 + \log 2}{\log 32 - \log 2}$
6. $\log_9(\log_2 8)$
7. $(\log_2 8)(\log_2 16)$
8. $\log_4 \frac{1}{2} + \log_4 \frac{1}{6} - \log_4 \frac{1}{18}$
9. $\log_3 27 + \log_2 \frac{1}{8} + \log_7 49$
10. $\log_3 \sqrt[4]{72} + \log_3 \sqrt{3}$
11. $\frac{3}{5}\log_6 32 + \frac{2}{3}\log_6 72 + \frac{1}{2}\log_6 25 - \log_6 10$