ANSWERS AND SOLUTIONS

Lesson 1

Solutions to Worksheet 1

1. $$\frac{(x+2)(x-3)}{(3-x)6}$$
$$\frac{x+2}{-6}$$

2. $$\frac{9-12x+4x^2}{2x^2-x-3} \times \frac{4+x-3x^2}{6x^2-17x+12}$$
$$= \frac{(3-2x)^2}{(2x-3)^2(x+1)} \times \frac{(4-3x)(1+x)}{(3x-4)}$$
$$= \frac{\cancel{(3-2x)^2}}{\cancel{(2x-3)^2}\cancel{(x+1)}} \times \frac{^{(-1)}\cancel{(4-3x)}\cancel{(1+x)}}{\cancel{(3x-4)}}$$
$$= -1$$

3. $$\frac{(x-1)-2(1-x)^2}{(2x-3)(x+2)}$$
$$= \frac{(x-1)[1-2(x-1)]}{(2x-3)(x+2)}$$
$$= \frac{(x-1)\cancel{(3-2x)}^{(-1)}}{\cancel{(2x-3)}(x+2)}$$
$$= \frac{-x+1}{x+2}$$

4. $$\frac{(p-5)(p+3)}{6(p-2)} \times \frac{2-p}{5-p}$$
$$= \frac{p+3}{6}$$

5. $$\frac{y(x-3)+4(x-3)}{(3-x)} \times \frac{2y}{y(4+y)}$$
$$= \frac{^{(-1)}\cancel{(x-3)}\cancel{(y+4)}}{\cancel{(3-x)}} \times \frac{2y}{y\cancel{(4+y)}}$$
$$= -2$$

Solutions to Worksheet 2

1. $$\frac{1}{y-2} - \frac{3}{y-3} - \frac{4}{(y-3)^2}$$
$$= \frac{(y-3)^2 - 3(y-2)(y-3) - 4(y-2)}{(y-2)(y-3)^2}$$
$$= \frac{y^2-6y+9-3(y^2-5y+6)-4y-8}{(y-2)(y-3)^2}$$
$$= \frac{-2y^2+5y-17}{(y-2)(y-3)^2}$$

2. $$\frac{3x}{x+2} + \frac{2}{2x-5} + \frac{4x}{2}$$
$$= \frac{6x(2x-5)+4(x+2)+4x(x+2)(2x-5)}{2(x+2)(2x-5)}$$
$$= \frac{12x^2-30x+4x+8+4x(2x^2-x-10)}{2(x+2)(2x-5)}$$
$$= \frac{8x^3+8x^2-66x+8}{2(x+2)(2x-5)}$$

3. $$\frac{-6}{(x-3)(x+3)} - \frac{3}{(x+3)^2} + \frac{1}{x-3}$$
$$= \frac{-6(x+3)-3(x-3)+(x+3)^2}{(x-3)(x+3)^2}$$
$$= \frac{-6x-18-3x+9+x^2+6x+9}{(x-3)(x+3)^2}$$
$$= \frac{x^2-3x}{(x-3)(x+3)^2}$$

4. $$\frac{3y+2x}{y+x} - \frac{3}{(3y+2x)(y+x)}$$
$$= \frac{(3y+2x)^2-3}{(y+x)(3y+2x)}$$
$$= \frac{9y^2+12xy+4x^2-3}{(y+x)(3y+x)}$$

5. $$x + \frac{1}{x-1} - \frac{2}{x-2}$$
$$= \frac{x(x-1)(x-2)+(x-2)-2(x-1)}{(x-1)(x-2)}$$
$$= \frac{x^3-3x^2+2x+x-2-2x+2}{(x-1)(x-2)}$$
$$= \frac{x^3-3x^2+x}{(x-1)(x-2)}$$

6. $$\frac{2}{(m-1)^2} - \frac{3}{m-1}$$
$$= \frac{2-3(m-1)}{(m-1)^2}$$
$$= \frac{2-3m+3}{(m-1)^2}$$
$$= \frac{5-3m}{(m-1)^2}$$

Solutions to Worksheet 3

1) $\frac{2}{2x-3}+\frac{4}{x}$

$=\frac{2x+8-12}{x(2x-3)}$

$=\frac{10x-12}{x(2x-3)}$

2. $\frac{2}{2x-3}=\frac{4}{x}$; $2x-3 \neq 0 \Rightarrow x \neq \frac{3}{2}$ and $x \neq 0$

$\therefore \frac{2}{2x-3} \times x(2x-3) = \frac{4}{x} \times x(2x-3)$

$\therefore 2x = 4(2x-3)$

$\therefore 2x = 8x - 12$

$\therefore -6x = -12$

$\therefore x = 2$

3. $2+\frac{5}{x-1}$

$\therefore \frac{2(x-1)+5}{x-1}$

$\therefore \frac{2x-2+5}{x-1}$

$\therefore \frac{2x+3}{x-1}$

4. $2=\frac{5}{x-1}$; $x-1 \neq 0$; $x \neq 1$

$\therefore 2(x-1) = 5$

$\therefore 2x - 2 = 5$

$\therefore 2x = 7$

$\therefore x = \frac{7}{2}$

5. $\frac{3}{2x}+\frac{4}{3x}-\frac{1}{x+3}$

$=\frac{9(x+3)+8(x+3)-6x}{6x(x+3)}$

$=\frac{9x+27\ 8x+24-6x}{6x(x+3)}$

$=\frac{11x+51}{6x(x+3)}$

6. $\frac{3}{2x}+\frac{4}{3x}=\frac{1}{x+3}$

Restrictions on x: $2x \neq 0$ $\quad 3x \neq 0$ $\quad x+3 \neq 0$
$x \neq 0$ $\quad x \neq 0$ $\quad \therefore x \neq -3$

$\therefore \frac{3}{2x} \times 6x(x+3) + \frac{4}{3x} \times 6x(x+3)$

$= \frac{1}{x+3} \times 6x(x+3)$

$\therefore 9(x+3) + 8(x+3) = 6x$

$\therefore 9x + 27 + 8x + 24 = 6x$

$\therefore 11x = -51$

$\therefore x = -\frac{51}{11}$

7. $\frac{2}{x+1}+\frac{3}{1-x}$

$\therefore \frac{2(x-1)-3(x+1)}{(x+1)(x-1)}$

$\therefore \frac{2x-2-3x-3}{(x+1)(x-1)}$

$\therefore \frac{-x-5}{(x+1)(x-1)}$

8. $\frac{2}{x+1}+\frac{3}{1-x}=0$

Restrictions on x: $x+1 \neq 0$; $\quad x \neq -1$
and $\quad x-1 \neq 0$; $x \neq 1$

$\therefore \frac{2}{x+1}-\frac{3}{x-1}=0$

$\therefore \frac{2}{x+1}=\frac{3}{x-1}$

$\therefore 2(x-1) = 3(x+1)$

$\therefore 2x - 2 = 3x + 3$

$\therefore -x = 5$

$\therefore x = -5$

Lesson 2

Solutions to Activity 1

1. $(4)^2 = 16$
2. $\left(\frac{7}{2}\right)^2 = \frac{49}{4}$
3. $(-3)2 = 9$
4. $\left(-\frac{5}{2}\right)^2 = \frac{25}{4}$
5. $\left(\frac{1}{4} \times 2\right)^2 = \frac{1}{4}$
6. $\left(-\frac{2}{5} \times 2\right)^2 = \frac{16}{25}$

Solutions to Activity 2

1. a) $x^2 + 3x - 7$

$= x^2 + 3x + \frac{9}{4} - \frac{9}{4} - 7$

$= \left(x + \frac{3}{2}\right)^2 - \frac{9}{4} - \frac{28}{4}$

$= \left(x + \frac{3}{2}\right)^2 - \frac{37}{4}$

b) $-\left[x^2 + 5x + \left(\frac{5}{2}\right)^2 - \frac{25}{4} + 1\right]$

$= -\left(x + \frac{5}{2}\right)^2 + \frac{25}{4} - \frac{4}{4}$

$= -\left(x + \frac{5}{2}\right)^2 + \frac{21}{4}$

Max. value of $\frac{21}{4}$ at $x = -\frac{5}{2}$

c) $3x^2 + 2x + 7$

$= 3\left(x^2 + \frac{2}{3}x + \frac{1}{9} - \frac{1}{9} + \frac{7}{3}\right)$

$= 3\left[\left(x + \frac{1}{3}\right)^2 - \frac{1}{9} + \frac{21}{9}\right]$

$= 3\left[\left(x + \frac{1}{3}\right)^2 + \frac{20}{9}\right]$

$= 3\left(x + \frac{1}{3}\right)^2 + \frac{20}{3}$

Min value of $\frac{20}{3}$ when $x = -\frac{1}{3}$

d) $\left[x^2 + \frac{4}{5} + \left(\frac{2}{5}\right)^2 - \frac{4}{25} + \frac{3}{5}\right]$

$= 5\left(x + \frac{2}{5}\right)^2 - \frac{4}{5} + 3$

$= 5\left(x + \frac{2}{5}\right)^2 + \frac{11}{5}$

Min. value $\frac{11}{5}$ at $x = -\frac{2}{5}$

e) $x^2 + px + \left(\frac{p}{2}\right)^2 - \frac{p^2}{4} = 3$

$= \left(x + \frac{p}{2}\right)^2 + 3 - \frac{p^2}{4}$

$= \left(x + \frac{p}{2}\right)^2 + \frac{12 - p^2}{4}$

Min. value of $\frac{12 - p^2}{4}$ at $x = \frac{-p}{2}$

f) $px^2 + qx + r$

$= p\left(x^2 + \frac{q}{p}x + \frac{q^2}{4p^2} - \frac{q^2}{4p^2} + \frac{r}{p}\right)$

$= p\left[\left(x + \frac{q}{2p}\right)^2 - \frac{q^2}{4p^2} + \frac{4pr}{4p^2}\right]$

$= p\left(x + \frac{q}{2p}\right)^2 + \frac{4pr - q^2}{4p}$

If $p < 0$; max value is $\frac{4pr - q^2}{4p}$ when $x = -\frac{q}{2p}$

If $p > 0$; min value $\frac{4pr - q^2}{4p}$ when $x = -\frac{q}{2p}$

2. a) $x^2 - x + \left(-\frac{1}{2}\right)^2 - \frac{1}{4} + 5$

$= \left(x - \frac{1}{2}\right)^2 + \frac{19}{4}$

Min. value is $+\frac{19}{4}$

$\therefore$ the expression is always positive.

b) $2[x^2 - \frac{3}{2}x + \left(\frac{-3}{4}\right)^2 - \frac{9}{16} + 4]$

$= 2\left(x - \frac{3}{4}\right)^2 - \frac{9}{8} + 8$

$= 2\left(x - \frac{3}{4}\right)^2 + \frac{55}{8}$

Min. value is $\frac{55}{8}$

$\therefore$ the expression is always positive.

c) $10[x^2 + \frac{1}{2}x + \left(\frac{1}{4}\right)^2 - \frac{1}{16} + \frac{1}{5}$

$= 10\left(x + \frac{1}{4}\right)^2 - \frac{5}{8} + 2$

$= 10\left(x + \frac{1}{4}\right)^2 + \frac{11}{8}$

$= 10\left(x + \frac{1}{4}\right)^2 \geq 0$

$\therefore\ 10\left(x + \frac{1}{4}\right)^2 + \frac{11}{8} \geq \frac{11}{8} > 0$

3. a) $-[x^2 - 3x + 5]$

$= -[x^2 - 3x + \left(\frac{-3}{2}\right)^2 - \frac{9}{4} + 5]$

$= -\left(x - \frac{3}{2}\right)^2 + \frac{9}{4} - 5$

$= -\left(x - \frac{3}{2}\right)^2 - \frac{11}{4}$

Max. value of $-\frac{11}{4}$

$\therefore$ the expression is always negative.

b) $-4(x^2 - \frac{1}{2}x + \left(-\frac{1}{4}\right)^2 - \frac{1}{16} + \frac{1}{4})$

$= -4\left(x - \frac{1}{4}\right)^2 + \frac{1}{4} - 1$

$= -4\left(x - \frac{1}{4}\right)^2 - \frac{3}{4}$

$\left(x - \frac{1}{4}\right)^2 \geq 0$

$\therefore -4\left(x - \frac{1}{4}\right)^2 \leq 0$

$\therefore -4\left(x - \frac{1}{4}\right)^2 - \frac{3}{4} \leq -\frac{3}{4} < 0$

4. i) $A = x(40 - x)$

$A = -x^2 + 40x$

ii) $A = -(x^2 - 40x + (-20)^2 - 400)$

$a = -(x - 20)^2 + 400$

Max. is 400 when $x = 20$

Max. area is 400 m^2

iii) dimension $x = 20$

$\therefore$ L = 20 B = 20

Lesson 3

Activity 1

1. $\left[\frac{3^3a^4}{2^3a}\right]^{\frac{2}{3}}$

$= \left[\left(\frac{3a}{2}\right)^3\right]^{\frac{2}{3}}$

$= \frac{9a^2}{4}$

2. $\left[\frac{2^4y^6}{3^2x^4}\right]^{-\frac{1}{2}}$

$= \left[\frac{3^2x^4}{2^4y^6}\right]^{\frac{1}{2}}$

$= \frac{3x^2}{4y^3}$

3. $\frac{(2.5)^{n+3}.5^{n-1}}{(5^2.2)^{n+2}}$

$= \frac{2^{n+3}.5^{n+3}.5^{n-1}}{5^{2n+4}.2^{n+2}}$

$= 2^{n+3-n-2}.5^{2n+2-2n-4}$

$= 2.5^{-2}$

$= \frac{2}{25}$

4. $\frac{(3^2)^{2k+1}.3^{-k}.(2^2)^{2-k}}{3^{5k-2}.(2.3)^{3-2k}.2}$

$= \frac{3^{4k+2}.3^{-k}.2^{4-2k}}{3^{5k-2}.2^{3-2k}.3^{3-2k}.2}$

$= 3^{4k+2-k-5k+2-3+2k}.2^{4-2k-3+2k-1}$

$= 3^1.2^0$

$= 3$

5. $\frac{3^{2n-1}.3^{-8n}.3^{3n+3}}{3^{-3n-1}}$

$= 3^{2n-1-8n+3n+3+3n+1}$

$= 3^3$

$= 27$

6. $\frac{3^x(2^13^1)^{2x-1}\cdot 2^{4x+1}}{(2^2\cdot 3)^{3x}}$

$= \frac{3^x\cdot 2^{2x-1}\cdot 3^{2x-1}\cdot 2^{4x+1}}{2^{6x}\cdot 3^{3x}}$

$= 3^{x+2x-1-3x}\cdot 2^{6x-6x} = 3^{-1}\cdot 2^0 = \frac{1}{3}$

Activity 2

1. $\frac{3^x + 3^x\cdot 3}{3^x - 3^x\cdot 3^2}$

$= \frac{3^x(1+3)}{3^x(1-9)}$

$= -\frac{4}{8}$

$= -\frac{1}{2}$

2. $\frac{2\cdot 2^x\cdot 2^1 + 8\cdot 2^x\cdot 2^{-3}}{4\cdot 2^x\cdot 2^{-1} - 16\cdot 2^x\cdot 2^{-4}}$

$= \frac{\cancel{2^x}\left(8 + 8 \times \frac{1}{8}\right)}{\cancel{2^x}\left(2 - 16 \times \frac{1}{16}\right)}$

$= \frac{9}{1}$

$= 9$

3. $\frac{6\cdot 3^x\cdot 3^{-1} - 2\cdot 3^x\cdot 3^1}{5\cdot 3^x + 3^x\cdot 3^2}$

$= \frac{3^x(2-6)}{3^x(5+9)}$

$= -\frac{4}{14} = -\frac{2}{7}$

4. $\frac{5^x + 5^x \cdot 5^{-2}}{2 \cdot 5^x \cdot 5^{-1} - 3 \cdot 5^x \cdot 5^{-2}}$

$= \frac{5^x\left(1 - \frac{1}{25}\right)}{5^x\left(\frac{2}{5} - \frac{3}{25}\right)}$

$= \frac{24}{25} \div \frac{10-3}{25}$

$= \frac{24}{25} \times \frac{25}{7}$

$= \frac{24}{7}$

5. $(a + b) - \left(a - 2a^{\frac{1}{2}}b^{\frac{1}{2}} + b\right)$

$= a + b - a + 2a^{\frac{1}{2}}b^{\frac{1}{2}} - b$

$= 2a^{\frac{1}{2}}b^{\frac{1}{2}}$

Lesson 4

Activity 1

1. $x = 3$ or $x = -7$

2. $x = 5$ or $x = 8$

3. $x = \frac{3}{2}$ or $x = \frac{4}{3}$

4. $x = 5$ (note $x \neq -1$)

5. Restr: $x \neq 0$; $x \neq 2$

LCD: $2x(x - 2)$

$\therefore\ 60x - x(x - 2) = 30(x - 2).2$

$\therefore\ 60x - x^2 + 2x = 60x - 120$

$\therefore\ x^2 - 2x - 120 = 0$

$\therefore\ (x + 10)(x - 12) = 0$

$\therefore\ x = -10$ or $x = 12$

6. $\frac{4x}{3(x+4)} - \frac{1}{2} = \frac{1}{2(x-1)}$ Restr: $x \neq -4$; $x = 1$

LCD: $6(x - 1)(x + 4)$

$\therefore\ 2.4x(x - 1) - 3(x - 1)(x + 4) = 3(x + 4)$

$\therefore\ 8x^2 - 8x - 3(x^2 + 3x - 4) = 3x + 12$

$\therefore\ 5x^2 - 17x + 12 = 3x + 12$

$\therefore\ 5x^2 - 20x = 0$

$\therefore\ x^2 - 4x = 0$

$x(x - 4) = 0$

$x = 0$ or $x = 4$

7. Restr: $x \neq 6$; $x \neq -2$

LCD: $8(x + 2)(x - 6)$

$21(x + 2) - 5(x - 6) + 8(x + 2)(x - 6) = 0$

$\therefore\ 21x + 42 - 5x + 30 + 8(x^2 - 4x - 12) = 0$

$\therefore\ 16x + 72 + 8x^2 - 32x - 96 = 0$

$\therefore\ 8x^2 - 16x - 24 = 0$

$\therefore\ x^2 - 2x - 3 = 0$

$(x + 1)(x - 3) = 0$

$x = -1$ or $x = 3$

8. LCD: $3x(x + 1)$

Restr: $x \neq 0$; $x \neq -1$

$\frac{x+1}{x} - \frac{5x}{3(x+1)} = \frac{2}{3}$

$\therefore\ 3(x + 1)^2 - 5x^2 = 2x(x + 1)$

$\therefore\ 3(x^2 + 2x + 1) - 5x^2 = 2x^2 + 2x$

$\therefore\ 3x^2 + 6x + 3 - 5x^2 = 2x^2 + 2x$

$\therefore\ 4x^2 - 4x - 3 = 0$

$(2x + 1)(x - 3) = 0$

$x = -\frac{1}{2}$ or $x = 3$

9. $x = \frac{3}{5}$ or $x = 3$

10. a) $x = 0$

b) $x = 0$ or $x = -\frac{3}{2}$

c) $x = \pm\sqrt{3}$

d) $x = 0$ or $x = -\frac{3}{2}$ or $x = \pm\sqrt{3}$

11. $\dfrac{2+\frac{1}{x}}{2} = \dfrac{3-\frac{1}{x}}{3+\frac{1}{x}}$

$\dfrac{2x+1}{2x} = \dfrac{3x-1}{3x+1}$

Restr: $x \neq 0 \quad x \neq -\frac{1}{3}$

LCD: $2x(3x + 1)$

$\Rightarrow$ $(2x + 1)(3x + 1) = 2x(3x - 1)$

$6x^2 + 5x + 1 = 6x^2 - 2x$

$\therefore\ 7x = -1$

$x = -\frac{1}{7}$

Activity 2

1. $2x + 1 + 2x - 1 = 4x$ which is an even number

$= 2(2x)$

2. $x^2 - (x - 1)^2$ which is an odd number

$= x^2 - (x^2 - 2x + 1)$

$= x^2 - x^2 + 2x - 1$

$= 2x - 1$

3. a) $7^2 - 4^2$ this is a multiple of three

$= 49 - 16$

$= 33$

$= 3(11)$

b) $100 - 36$ this is a multiple of 4

$= 64$

$= 4(16)$

c) $144 - 49$ this is a multiple of 5

$= 5(19)$

d) The square of any number minus the square of another number is a multiple of the difference between the two numbers.

e) Let the one number be n and the other number be m with $n > m$.

So: $n^2 - m^2 = (n - m)(n + m)$

Thus with $(n - m)$ being a factor of $n^2 - m^2$, it means that $n^2 - m^2$ is a multiple of $(n - m)$.

this means that the answer is divisible by $(n - m)$

Question 4 is for you to do on your own.

Lesson 5

Solutions to Activity 1

1. (a) $x = 5$

b) $\sqrt{11 - x} = x + 1$

Restr: $11 - x \geq 0$ and $x + 1 \geq 0$

$x \leq 1$ $\quad x \geq -1$

$\therefore -1 \leq x \leq 1$

Now $11 - x = x^2 + 2x + 1$

$x^2 + 3x - 10 = 0$

$(x + 5)(x - 2) = 0$

$\therefore x = -5$ or $x = 2$

But $-1 \leq x \leq 1$

$\therefore$ No solution

c) $\sqrt{5x - 1} = 2x - 1$

Restr: $5x - 1 \geq 0$ and $2x - 1 \geq 0$

$x \geq \frac{1}{5}$ and $x \geq \frac{1}{2}$

$\therefore x \geq \frac{1}{2}$

Now: $5x - 1 = 4x^2 - 4x + 1$

$\therefore 4x^2 - 9x + 2 = 0$

$(4x - 1)(x - 2) = 0$

$\therefore x = \frac{1}{4}$ or $x = 2$

d) $\sqrt{x - 2} = 4 - x$

Restr: $x - 2 \geq 0$ or $4 - x \geq 0$

$x \geq 2$ or $x \leq 4$

$\therefore 2 \leq x \leq 4$

$x - 2 = x^2 - 8x + 16$

$\therefore x^2 - 9x + 18 = 0$

$(x - 3)(x - 6) = 0$

$x = 3$ or $x = 6$

e) $\sqrt{10 - 3x} = x - 2$

Restr: $10 - 3x \geq 0$ or $x - 2 \geq 0$

$3x \leq 10$ $\quad x \geq 2$

$x \leq \frac{10}{3}$

$\therefore x \leq x \leq \frac{10}{3}$

$\therefore 10 - 3x = x^2 - 4x + 4$

$\therefore x^2 - x - 6 = 0$

$(x + 2)(x - 3) = 0$

$\therefore x = -2$ or $x = 3$

f) $(x - 3)\sqrt{x - 2} - 5(x - 3) = 0$

$\therefore (x - 3)\left(\sqrt{x - 2} - 5\right) = 0$ Restr: $x - 2 \geq 0$

$\therefore x = 3$ or $\sqrt{x - 2} = 5$ $\quad \therefore x \geq 2$

$x - 2 = 25$

$x = 27$

2. (b) : $x = 2$ only (c) $x \leq -2$; So $x = -14$

3. $x = 41$

4. $x = \frac{9}{11}$ or $x = -\frac{3}{2}$

Solution to Activity 2

2. $n \geq 0 \quad m \geq 0$

3. $c \leq 0 \qquad a \leq 0$

Lesson 6

1. $x^2 - 2x + (1)^2 - 1 - 7 = 0$

$(x - 1)^2 = 8$

$x - 1 = \pm\sqrt{8}$

$x = 1 \pm 2\sqrt{2}$

$4x^2 - 8x + 4 = 28 + 4$

$(2x - 2)^2 = 32$

$2x - 2 = \pm 4\sqrt{2}$

$x - 1 = \pm 2\sqrt{2}$

$x = 1 \pm 2\sqrt{2}$

2. $x^2 - 4x + 7 = 0$

$\therefore x^2 - 4x + 4 + 3 = 0$

$\therefore (x - 2)^2 = -3$

x is non $\mathbb{R}$.

3. $p^2 - 3p + \left(-\frac{3}{2}\right)^2 - \frac{9}{4} + 1 = 0$

$\left(p - \frac{3}{2}\right)^2 = \frac{9}{4} - 1$

$p - \frac{3}{2} = \pm\sqrt{\frac{9-4}{4}}$

$p = \frac{+3}{2} \pm \frac{\sqrt{5}}{2}$

$p = \frac{+3 \pm \sqrt{5}}{2}$

$4p^2 - 12p + 9 = -4 + 9$

$(2p - 3)^2 = 5$

$2p - 3 = \pm\sqrt{5}$

$p = \frac{3 \pm \sqrt{5}}{2}$

4. $2x^2 - 3x - 1 = 0$

$4a = 8 \quad 16x^2 - 24x + 9 = 8 + 9$

$b^2 = 9 \quad (4x - 3)^2 = 17$

$\therefore 4x - 3 = \pm\sqrt{17}$

$x = \frac{3 \pm \sqrt{17}}{4}$

5. $x^2 - 2x + 1 = 24 + 1$

$(x - 1)^2 = 25$

$\therefore x - 1 = \pm 5$

$x = 1 \pm 5$

$\therefore x = 6$ or $x = -4$

6. $5x^2 - 3x - 1 = 0$

$4a = 20 \qquad 100x^2 - 60x + 9 = +20 + 9$

$b^2 = 9 \qquad \therefore (10x - 3)^2 = 29$

$\therefore 10x - 3 = \pm\sqrt{29}$

$\therefore x = \frac{3 \pm \sqrt{29}}{10}$

7. $-2x^2 - x + 5 = 0$

$2x^2 + x - 5 = 0$

$x^2 + \frac{1}{2}x \qquad -\frac{5}{2} = 0$

$x^2 + \frac{1}{2}x + \left(\frac{1}{4}\right)^2 - \frac{1}{16} - \frac{5}{2} = 0$

$\left(x + \frac{1}{4}\right)^2 = \frac{1}{16} + \frac{5}{2}$

$x + \frac{1}{4} = \pm\sqrt{\frac{1 + 40}{16}}$

$x = -\frac{1}{4} \pm \frac{\sqrt{41}}{4}$

$= \frac{-1 \pm \sqrt{41}}{4}$

$4a = -8$

$b^2 = 1$

$16x^2 + 8x + 1 = 40 + 1$

$(4x + 1)^2 = 41$

$x = \frac{-1 \pm \sqrt{41}}{4}$

8. $3x^2 - 6x - 2 = 0$

$4a = 12 \qquad 36x^2 - 72x + 36 = 24 + 36$

$b^2 = 36 \qquad (6x - 6)^2 = 60$

$\therefore 6x - 6 = \pm\sqrt{60}$

$\therefore x = \frac{6 \pm \sqrt{60}}{6}$

9. $9x^2 - 6mx + m^2 - 3m = 0$

$9\left[x^2 - \frac{6}{9}mx + \frac{m^2 - 3m}{9}\right] = 0$

$\therefore x^2 - \frac{2}{3}mx + \frac{1}{9}m^2 = \frac{3m - m^2}{9} + \frac{m^2}{9}$

$\therefore \left(x - \frac{m}{3}\right)^2 = \frac{3m}{9}$

$\therefore x - \frac{m}{3} = \pm\sqrt{\frac{m}{3}}$

$\therefore x = \frac{m}{3} \pm \sqrt{\frac{m}{3}}$

$\therefore x = \frac{m}{3} \pm \sqrt{\frac{m}{3}}$: $m \geq 0$

Lesson 7

1. $9x(x - 1) = -2 \Rightarrow 9x^2 - 9x + 2 = 0$

$a = 9, b = -9, c = 2$

$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-9) \pm \sqrt{(-9)^2 - 4(9)(2)}}{2(9)}$

$= \frac{9 \pm \sqrt{81 - 72}}{18}$

$= \frac{9 \pm \sqrt{9}}{18} = \frac{9 \pm 3}{18}$

$\therefore x = \frac{9 + 3}{18} = \frac{12}{18} = \frac{2}{3}$

or

$x = \frac{9 - 3}{18} = \frac{6}{18} = \frac{1}{3}$

2. $a = 2 \quad b = 4 \quad c = 3$

$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-4 \pm \sqrt{16 - 4(6)}}{4}$

$= \frac{-4 \pm \sqrt{-8}}{4}$

3. $a = 2 \quad b = 1 \quad c = -2$

$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-1 \pm \sqrt{1 - 4(-4)}}{4}$

$= \frac{-1 \pm \sqrt{17}}{4}$

$\therefore x = \frac{-1 + \sqrt{17}}{4}$

or

$x = \frac{-1 - \sqrt{17}}{4}$

4. $\therefore\ 3x^2 + 4x - 84 = 0$

$a = 3 \quad b = 4 \quad c = -84$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-4 \pm \sqrt{16 - 4(3)(-84)}}{2(3)}$$
$$= \frac{-4 \pm \sqrt{16 + 1\,008}}{6}$$
$$= \frac{-4 \pm \sqrt{1\,024}}{6}$$
$$= \frac{-4 \pm 32}{6}$$
$$= \frac{-2 \pm 16}{3}$$

$x = \frac{-2 + 16}{3}$ or $x = \frac{-2 - 16}{3}$

$x = \frac{14}{3}$ or $x = -\frac{18}{3}$

$x = \frac{14}{3}$ or $x = -3$

5. $24x^2b^2 + 2bx - 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-2b \pm \sqrt{4b^2 - 4(24b^2)(-1)}}{2(24b^2)}$$
$$= \frac{-2b \pm \sqrt{102b^2}}{48b^2}$$
$$= \frac{-2b \pm b\sqrt{102}}{48b^2}$$
$$= \frac{-2 \pm \sqrt{102}}{48b}$$

6. $36x^4 - 25x^2 + 4 = 0$

$a = 36 \quad b = -25 \quad c = 4$

$$x^2 = \frac{25 \pm \sqrt{625 - 4(36)(4)}}{72}$$
$$= \frac{25 \pm \sqrt{49}}{72}$$
$$= \frac{25 \pm 7}{72}$$

$\therefore\ x^2 = \frac{32}{72}$ or $x^2 = \frac{18}{72}$

$\therefore\ x = \pm\sqrt{\frac{16 \times 2}{36 \times 2}}$ $\quad x^2 = 4$

$x = \pm\,\frac{4}{9}$ or $x = \pm 2$

7. $x^2(x + m) - 2x(x + m) + (x + m) = 0$

$(x + m)(x^2 - 2x + 1) = 0$

$\therefore\ x = -m$ or $(x - 1)^2 = 0$

$x = 1$

8. $x^2 - 6x + 2{,}6 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{6 \pm \sqrt{36 - 4(2{,}6)}}{2}$$
$$= \frac{6 \pm \sqrt{25{,}6}}{2}$$

$\therefore\ x = 5{,}53$ or $x = 0{,}47$

9. $(x + 3)(2 - x) = 7x$

$\therefore\ -(x + 3)(x - 2) = +7x$

$-x^2 - x + 6 - 7x = 0$

$\therefore\ x^2 + 8x - 6 = 0$

$$x = \frac{-8 \pm \sqrt{64 - 4(-6)}}{2}$$
$$= \frac{-8 \pm \sqrt{88}}{2}$$

$\therefore\ x = 0{,}69$ or $x = -8{,}69$

Lesson 8

Activity 1

1. Let $k = 3x^2 - x : k \neq 0$

$\therefore k + \frac{6}{k+2} = 5$

$\therefore k^2 + 2k + 6 = 5k + 10$

$\therefore k^2 - 3k - 4 = 0$

$(k+1)(k-4) = 0$

$\therefore 3x^2 - x + 1 = 0$ or $3x^2 - x - 4 = 0$

$\therefore x = \frac{1 \pm \sqrt{1-3}}{6}$ $\quad (3x-4)(x+1)$

$x = \frac{1 \pm \sqrt{-2}}{6}$ $\quad x = \frac{4}{3}$ or $x = -1$

$\therefore x$ is non $\mathbb{R}$.

2. Let $k = x^2 - 5x$;

$k^2 - 2k - 24 = 0$

$(k+4)(k-6) = 0$

$x^2 - 5x + 4 = 0$ or $x^2 - 5x - 6 = 0$

$(x-1)(x-4) = 0 \quad (x-6)(x+1) = 0$

$x = 1$ or $x = 4 \therefore x = 6$ or $x = -1$

3. $\sqrt{3x-1} + 1 = \frac{6}{\sqrt{3x-1}}$

If $k = \sqrt{3x-1}$; $k > 0$

$k + 1 = \frac{6}{k}$

$\therefore k^2 + k - 6 = 0$

$(k+3)(k-2) = 0$

$\therefore \sqrt{3x-1} + 3 = 0$ or $\sqrt{3x-1} - 2 = 0$

$\therefore \sqrt{3x-1} = -3 \quad 3x - 1 = 4$

impossible $\quad 3x = 5$

$x = \frac{5}{3}$

4. Let $k = (x^2 - 3x)$

$k^2 - 5k + 4 = 0$

$(k-4)(k-1) = 0$

so $x^2 - 3x - 4 = 0$ or $x^2 - 3x - 1 = 0$

$(x-4)(x+1) = 0 \quad x = \frac{3 \pm \sqrt{9 - 4(-1)}}{2}$

$x = 4$ or $x = -1 \quad = \frac{3 \pm \sqrt{13}}{2}$

$\therefore x = 3{,}30$ or $x = -0{,}30$

5. Let $k = 3x^2 + x : k \neq 0$

So $4(k+1) - \frac{10(k-1)}{k} = 7$

$\therefore 4k(k+1) - 10(k-1) = 7k$

$\therefore 4k^2 + 4k - 10k + 10 - 7k = 0$

$\therefore 4k^2 - 13k + 10 = 0$

$(4k-5)(k-2) = 0$

$\therefore 4(3x^2 + x) - 5 = 0 \quad 3x^2 + x - 2 = 0$

$\therefore 12x^2 + 4x - 5 = 0 \quad (3x-2)(x+1) = 0$

$\therefore (2x-1)(6x+5) = 0 \quad x = \frac{2}{3}$ or $x = -1$

$x = \frac{1}{2}$ or $x = -\frac{5}{6}$

6. $2x - 3 - \frac{3}{2x-1} = 0$

Let $k = 2x - 1 : k \neq 0$

$\therefore k - 2 - \frac{3}{k} = 0$

$\therefore k^2 - 2k - 3 = 0$

$(k+1)(k-3) = 0$

$2x - 1 + 1 = 0 \quad 2x - 1 - 3 = 0$

$2x = 0 \quad 2x = 4$

$x = 0 \quad x = 2$

7. $x^2 + 6x - 2 = \frac{35}{x^2 + 6x}$

Let $k = x^2 + 6x \neq 0$

$\therefore\ k - 2 = \frac{35}{k}$

$\therefore\ k^2 - 2k - 35 = 0$

$(k + 5)(k - 7) = 0$

$x^2 + 6x + 5 = 0$ or $x^2 + 6x - 7 = 0$

$(x + 1)(x + 5) = 0$ $(x - 1)(x + 7) = 0$

$x = -1$ or $x = -5$ or $x = 1$ or $x = -7$

8. $(2x^2 - 3x)^2 - 2(2x^2 - 3x) - 3 = 0$

Let $2x^2 - 3x = k$

$k^2 - 2k - 3 = 0$

$(k - 3)(k + 1) = 0$

$2x^2 - 3x = 3$ or $2x^2 - 3x = -1$

$2x^2 - 3x - 3 = 0$ or $2x^2 - 3x + 1 = 0$

$x = \frac{3 \pm \sqrt{9 - 4(2)(-3)}}{4}$ or $(2x - 1)(x - 1) = 0$

$= \frac{3 \pm \sqrt{33}}{4}$ or $x = \frac{1}{2}$ or $x = 1$

9. let $\sqrt{2x - 1} = k;\ k > 0$

$\frac{k}{2} - \frac{4}{k} - 1 = 0$

lcd $2k$

$k2 - 8 - 2k = 0$

$k2 - 2k - 8 = 0$

$(k - 4)(k + 2) = 0$

$k = 4$ or $k = -2$

$\sqrt{2x - 1} = 4$ or n.a

$2x - 1 = 16$

$2x = 17$

$x = \frac{17}{2}$

10. $x^2 - \frac{5}{x^2 + x} = 4 - x$

$\therefore\ x^2 + x - \frac{5}{x^2 + x} = 4$

Let $k = x^2 + x \neq 0$

$\therefore\ k - \frac{5}{k} = 4$

$\therefore\ k^2 - 4k - 5 = 0$

$(k - 5)(k + 1) = 0$

$x^2 + x - 5 = 0$

$x = \frac{-1 \pm \sqrt{1 + 20}}{2} = \frac{-1 \pm \sqrt{21}}{2}$

$\therefore\ x = 1{,}79$ or $x = -2{,}79$

or $x^2 + x + 1 = 0$

$x = \frac{-1 \pm \sqrt{1 - 4}}{2a}$ non $\mathbb{R}$

Activity 2

Solution no 1:

⇒ Let the digits be x and y.

x – the tens digit

y – the units digit

$\therefore\ 7(x + y) = 10x + y$ … (1)

$xy - x = \frac{12x}{y}$ … (2)

From (1): $7x + 7y = 10x + y$

$\therefore\ -3x = -6y$

$x = 2y$ … (3)

(3) → (2): $2y.y - 2y = \frac{12.2y}{y}$

$\therefore\ 2y^2 - 2y - 24 = 0$

$\therefore\ y^2 - y - 12 = 0$

$\therefore\ (y - 4)(y + 3) = 0$

$\therefore\ y = 4$ or $y = -3$

If $y = 4$, then $x = 2(4)$ or if $y = -3$, then $x = 2(-3)$

$x = 8$ $x = -6$

Solution to no 2

⇒ Let the one integer be x and the other y.

Then the consecutive integer (y) will be $y = x +$
1

Thus: $xy = 90$

$x(x + 1) = 90$

$\therefore\ x^2 + x - 90 = 0$

$\therefore\ (x + 10)(x - 9) = 0$

$\therefore\ x = -10$ or $x = 9$

Thus the integers are –10 and –9 or 9 and 10.

1. $3^{2x} - 12 \cdot 3^x + 27 = 0$

$(3^x = k) \Rightarrow \quad k^2 - 12k + 27 = 0$

$(k - 9)(k - 3) = 0$

$3^x = 9 \quad 3^x = 3$

$x = 2 \qquad x = 1$

2. $3^x \cdot 3 - 10 \cdot 3^x \cdot 3^{-1} + 3 = 0$

$(3^x = k) \Rightarrow 3k - \frac{10}{3}k + 3 = 0$

$\Rightarrow 9k - 10k + 9 = 0$

$k = 9$

$x = 2$

3. Let $2^{\frac{x}{2}} = k$

$\left(k - \frac{4}{k}\right)\left(k - \frac{8}{k}\right) = 0$

$\Rightarrow (k^2 - 4)(k^2 - 8) = 0$

$k^2 = 4 \quad k^2 = 8$

$\left(2^{\frac{x}{2}}\right)^2 = 2^2 \qquad \left(2^{\frac{x}{2}}\right)^2 = 2^3$

$x = 2 \quad x = 3$

4. Let $2^{\frac{2x}{3}} = k$

$\Rightarrow 4k - 33 + \frac{8}{k} = 0$

$\Rightarrow 4k^2 - 33k + 8 = 0$

$\Rightarrow (4k - 1)(k - 8) = 0$

$k = \frac{1}{4} \qquad k = 8$

$2^{\frac{2x}{3}} = 2^{-2} \qquad 2^{\frac{2x}{3}} = 2^3$

$\Rightarrow \frac{2x}{3} = -2 \qquad \frac{2x}{3} = 3$

$\Rightarrow x = -3 \qquad x = 4\frac{1}{2}$

5. $4 \cdot 2^{2x} \cdot 4 - 17 \cdot 2^x + 1 = 0$

Let $2^x = k$

$16k^2 - 17k + 1 = 0$

$(16k - 1)(k - 1) = 0$

$k = \frac{1}{16} \; k = 1$

$2^x = 2(-4) \qquad 2^x = 2^0$

$x = -4 \; x = 0$

6. Let $2^x = k$

$8k^2 + 10k - 3 = 0$

$(4k - 1)(2k + 3) = 0$

$k = \frac{1}{4} \qquad k = -\frac{3}{2}$

$2^x = 2^{-2} \qquad$ invalid

$x = -2$

Lesson 9

Activity 1

1. $y = 9 - x \rightarrow x^2 + x(9 - x) + (9 - x)^2 = 61$

$\therefore x^2 + 9x - x^2 + 81 - 18x + x^2 - 61 = 0$

$\therefore x^2 - 9x + 20 = 0$

$(x - 4)(x - 5) = 0$

$x = 4 \quad$ or $\quad x = 5$

$y = 9 - 4 = 5 \quad y = 9 - 5 = 4$

$\therefore (4; 5) \quad$ or $\quad (5; 4)$

2. $x - 2y = 3$

$x = 2y + 3 \rightarrow 4(2y + 3)^2 - 5y(2y + 3) = 3 - 6y$

$\therefore 4(4y^2 + 12y + 9) - 10y^2 - 15y = 3 - 6y$

$\therefore 16y^2 + 48y + 36 - 10y^2 - 9y - 3 = 0$

$\therefore 6y^2 + 39y + 33 = 0$

$\therefore 2y^2 + 13y + 11 = 0$

$(2y + 11)(y + 1) = 0$

$\therefore y = -\frac{1}{2} \quad$ or $\quad y = -1$

$x = 2\left(-\frac{11}{2}\right) + 3 \quad$ or $\quad x = 2(-1) + 3$

$= -11 + 3 \qquad x = 1$

$x = -8$

$\left(-8; -\frac{1}{2}\right)$ or $(1; -1)$

3. $(x-1)^2 + (y-2)^2 = 5 \qquad y = -2x - 1$

$\therefore (x-1)^2 + (-2x-1-2)^2 = 5$

$\therefore x^2 - 2x + 1 + 4x^2 + 12x + 9 - 5 = 0$

$\therefore 5x^2 + 10x + 5 = 0$

$\therefore x^2 + 2x + 1 = 0$

$(x+1)^2 = 0$

$\therefore x = -1$

$y = 2 - 1 = 1$

$(-1; 1)$

4. $(x-2)^2 + (y-3)^2 = 0$

Since $a^2 + b^2 > 0 \quad a; b \in \mathbb{R}$, this equation will equal zero if and only if $x = 2$ and $y = 3$

5. $(x^2 + 2)(y - 3) = 0$

$a.b = 0$ if and only if one of a or b is equal to zero.

So $x^2 + 2 = 0$ or $\qquad y - 3 = 0$

$x^2 = -2 \qquad y = 3$ only

rom $\mathbb{R}$

6. $2x - 3y = 1$

$2x = 3y + 1$

$x = \frac{3y+1}{2} \rightarrow x(x-6) - y = (1-y)(1+y)$

$\therefore \left(\frac{3y+1}{2}\right)\left(\frac{3y+1}{2} - 6\right) - y = 1 - y^2$

$\left(\frac{3y+1}{2}\right)^2 - 6\left(\frac{3y+1}{2}\right) - y = 1 - y^2$

$\therefore \frac{9y^2 + 6y + 1}{4} - 3(3y+1) - y = 1 - y^2$

$\therefore 9y^2 + 6y + 1 - 12(3y+1) - 4y = 4 - 4y^2$

$\therefore 9y^2 + 6y + 1 - 36y - 12 - 4y - 4 + 4y^2 = 0$

$\therefore 13y^2 - 34y - 15 = 0$

$\therefore (13y + 5)(y - 3) = 0$

$y = -\frac{5}{13}$ or $y = 3$

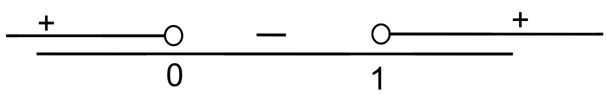
so $x = \frac{3\left(\frac{-5}{13}\right) + 1}{2}$ or $x = \frac{3(3) + 1}{2}$

$x = -\frac{1}{13} \qquad x = 5$

Lesson 10

Activity 1

1. $x(x-1) > 0$



$x < 0$ or $x > 1$

2. $x \in (-2; 3]$

3. $x \leq -2$ or $x \geq 4$

4. $x < 0$ or $x > 4$

5. $0 \leq x \leq 3$

Activity 2

1. $[-3; 0]$ or $[4; \infty)$

2. $(-\infty; -4]$ or $(5; \infty)$

3. $[-2; 0)$ or $(1; 3]$

4. $(-6; -1)$ or $[0; 2]$

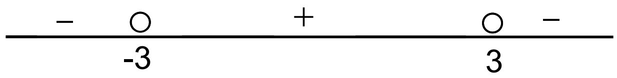
5. $(-\infty ; -1)$ or $(1 ; 4]$ 6. $(-1 ; 0)$ or $(2 ; \infty)$

Activity 3

1. $9 - k^2 \geq 0$

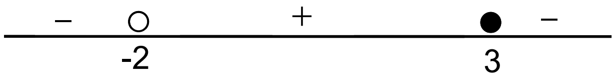
$(3 - k)(3 + k) \geq 0$

$k \in [-3 ; 3]$



2. $\frac{3 - p}{2 + p} \geq 0$

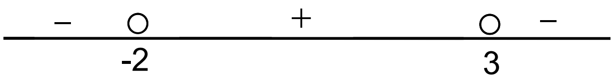
$p \in (-2 ; 3]$



3. $6 + m - m^2 < 0$ or $m - 1 = 0$

$(3 - m)(2 + m) < 0$

$m < -2$ or $m > 3$ or $m = 1$



4. $\frac{-(t-1)^2}{2t} \geq 0 \quad t = 1$

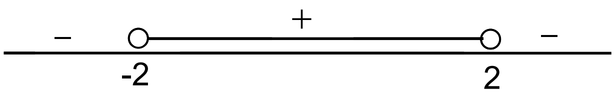
$2t < 0$

$t < 0$ or $t = 1$

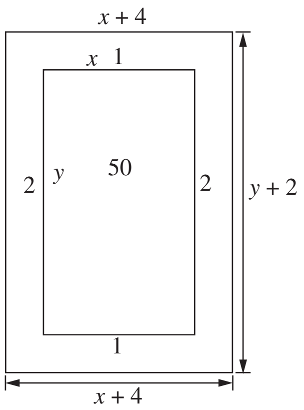
5. $4 - p^2 \geq 0$ but $p \neq 1$

$(2 - p)(2 + p) \geq 0$

$p \in [-2 : 2]$ but $p \neq 1$



Lesson 11



1.

$xy = 50$ … (1) (area)

$2(x + 4) + 2(y + 2) = 42$ (perimeter)

$x + 4 + y + 2 = 21$

$x + y = 15$

$\therefore\ y = 15 - x$ … (2)

$\therefore\ x(15 - x) = 50$

$-x^2 + 15x - 50 = 0$

$\therefore\ x^2 - 15x + 50 = 0$

$(x - 5)(x - 10) = 0$

$x = 5$ or $x = 10$

Then: $y = 15 - 5 = 10$ or $y = 15 - 10 = 5$

$\therefore$ Dimensions of card: L = 5 + 4 = 9

B = 10 + 2 = 12

or L = 10 + 4 = 14

B = 5 + 2 = 7

So (9 × 12) or (7 × 14)

2. Let the numbers be x and y.

$x - y = 3$ $(x > y)$

$xy = 40$ (Product)

So $x = y + 3 \rightarrow (y + 3)y = 40$

$\therefore\ y^2 + 3y - 40 = 0$

$\therefore\ (y - 5)(y + 8) = 0$

$\therefore\ y = 5$ or $y = -8$

$x = 8$ n.a.

The two numbers are 8 and 5.

3. Let the speed of the old locomotive be x km/h.

	S	t	D
Old	x	$\frac{100}{x}$	100
New	$x + 10$	$\frac{100}{x + 10}$	100

$T_{slow} = T_{fast} + \frac{1}{2}$ hour

$\therefore\ \frac{100}{x} = \frac{100}{x + 10} + \frac{1}{2}$

$\therefore\ 200(x + 10) = 200x + x(x + 10)$

$\therefore\ 200x + 2\ 000 = 200x + x^2 + 10x$

$\therefore\ x^2 + 10x - 2\ 000 = 0$

$(x + 50)(x - 40) = 0$

$x = 40$ km/h

4. Let the apprentice take x days.

Then the electrician takes $(x - 9)$ days.

$\frac{1}{x} + \frac{1}{x-9} = \frac{1}{20}$ Restriction: $x - 9 > 0$

$20(x - 9) + 20x = x^2 - 9x$ $\therefore x > 9$

$\therefore 20x - 180 + 20x = x^2 - 9x$

$\therefore x^2 - 49x + 180 = 0$

$\therefore (x - 45)(x - 4) = 0$

$\therefore x = 45$ or $x = 4$

n.a.

The apprentice takes 45 days and the electrician takes 36 days.

5. Let the stream flow be x km/h (x > 0)

	S	D	t
Up	$30 - x$	12	$\frac{12}{30-x}$
Down	$30 + x$	12	$\frac{12}{30+x}$

$T_{up} + T_{down} = 1$ hour

$\therefore \frac{12}{30-x} + \frac{12}{30+x} = 1$

$\therefore 12(30 + x) + 12(30 - x) = 900 - x^2$

$\therefore 360 + 12x + 360 - 12x = 900 - x^2$

$\therefore x^2 = 180$

$\therefore x = \pm\sqrt{180}$

$\therefore x = \sqrt{36 \times 5} = 6\sqrt{5}$ $(x > 0)$

$\therefore$ The stream flowed at $6\sqrt{5}$ km/h.

6.

	D (km)	S (km/h)	t (min)
Clear	$1{,}5 + x$	S_1	$\frac{5x}{2}$
Rainy	$1{,}5 + x$	52	$\frac{15x}{4}$

20 km/h $= \frac{20}{60}$ km/min

$S_1 = S_2 + \frac{1}{3}$

$\frac{1{,}5 + x}{\frac{5x}{2}} = \frac{1{,}5 + x}{\frac{15x}{4}} + \frac{1}{3}$

$\therefore \frac{3 + 2x}{5x} = \frac{6 + 4x}{15x} + \frac{1}{3}$

$\therefore 3(3 + 2x) = 6 + 4x + 5x$

$\therefore 9 + 6x = 6 + 9x$

$\therefore 3x = 3$

$x = 1$ km

The train is 1 km long.

7. Let the denominator be x and numerator be y:

$x = 1 + y^2$ … (1) $x > 0$ or $x < 0$

$y > 0$ $y < 0$

$\frac{y+1}{x-3} = \frac{1}{4}$ … (2)

$\therefore 4y + 4 = x + 3$

$= 1 + y^2 + 3$

$\therefore y^2 + 4 = 4y + 4$

$\therefore y^2 - 4y = 0$

$y(y - 4) = 0$

$\therefore y = 0$ or $y = 4$

n.a.

Then $x = 1 + 16 = 17$

So the fraction is $\frac{4}{17}$.

8. a) Each person pays $R\frac{120}{x}$

b) $R\left(\frac{120}{x-4}\right)$ each

c) $\frac{120}{x} + 5 = \frac{120}{x-4}$

$\therefore 120(x-4) + 5x(x-4) = 120x$

$\therefore 120x - 480 + 5x^2 - 20x - 120x = 0$

$\therefore 5x^2 - 20x - 480 = 0$

$\therefore x^2 - 4x - 96 = 0$

$\therefore (x-12)(x+8) = 0$

$\therefore x = 12$ or $x = -8$

n.a.

There were 12 people in the original group.

9. Assume he bought x sheep: $(x > 0)$

He paid $\frac{960}{x}$ rand per sheep.

He had $(x - 4)$ to sell.

So: $\left(\frac{960}{x} + 8\right)$ is the selling price per sheep.

$\therefore \left(\frac{960}{x-4}\right) = \frac{960}{x} + 8$

So $960x = 960(x-4) + 8x(x-4)$

$\therefore 960x = 960x - 3\ 840 + 8x^2 - 32x$

$\therefore 8x^2 - 32x - 3\ 840 = 0$

$\therefore x^2 - 4x - 480 = 0$

$\therefore (x+20)(x-24) = 0$

$\therefore x = 24$

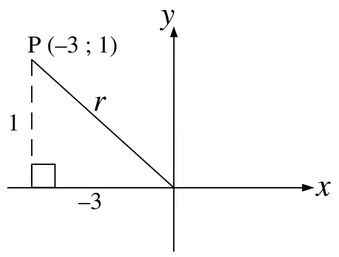
$\therefore$ He bought 24 sheep.

Lessons 12–13

1. (a) 2nd (b) 2nd (c) 3rd

(d) 3rd (e) 1st (f) 3rd

2.



$x = -3 \quad y = 1 \quad r^2 = 10$

$r = \sqrt{10}$

(a) $\frac{1}{\sqrt{10}} - \frac{3}{\sqrt{10}} = -\frac{2}{\sqrt{10}}$

(b) $1 + \left(\frac{1}{\sqrt{10}}\right)^2 = 1 + \frac{1}{10} = \frac{11}{10}$

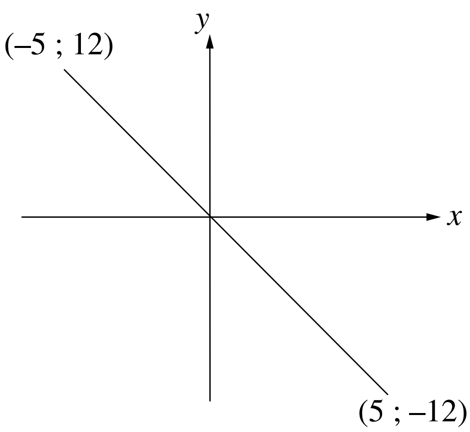
(c) $\sin^2\theta + \cos^2\theta$

$= \left(\frac{1}{\sqrt{10}}\right)^2 + \left(\frac{-3}{\sqrt{10}}\right)^2 = 1$

3. $\tan\theta = -\frac{5}{12}$

$r^2 = 25 + 144$

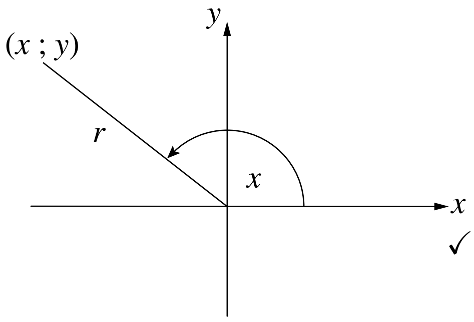
$r = 13$



a) $\sin\theta = \pm\frac{12}{13}$

b) $\cos\theta = \pm\frac{5}{13}$

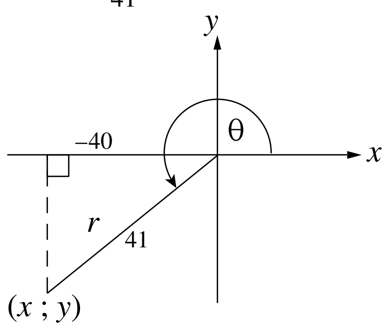
4. $\cos x = \frac{-24}{25}$



$x = -24 \quad y = +7 \quad r = +25$

$$\frac{2\left(\frac{-7}{24}\right)}{\frac{7}{25}} = \frac{-14}{24} \times \frac{25}{7} = \frac{-25}{12}$$

5. $\cos \theta = \frac{-40}{41}$

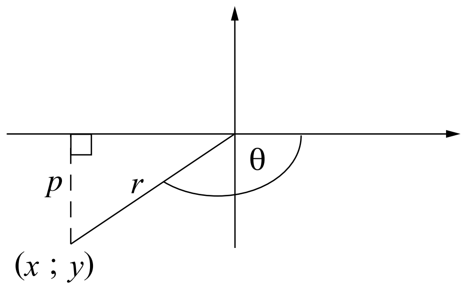


$x = -40 \quad y = +9 \quad r = +41$

(a) $1 + \left(-\frac{9}{40}\right)^2 = 1 + \frac{81}{1600}$
$= \frac{1681}{1600}$

(b) $2\left(\frac{18}{41}\right) - \left(\frac{-40}{41}\right) = \frac{76}{41}$

6. $\sin \theta = \frac{\text{p}}{1}$



$y = -\text{p} \quad r = 1 \quad x = -\sqrt{1 - \text{p}^2}$

$\cos \theta = -\sqrt{1 - \text{p}^2}$

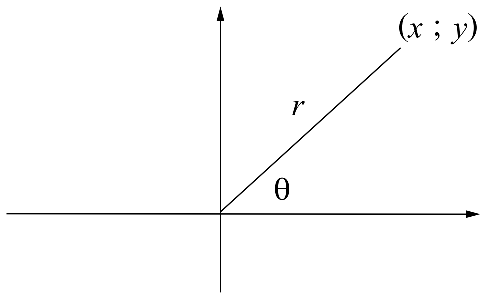
7. $\tan \theta = \frac{m}{1}$

$\sin^2 \theta - \cos^2 \theta$

$= \left(\frac{m}{\sqrt{m^2 + 1}}\right)^2 - \left(\frac{1}{\sqrt{m^2 + 1}}\right)^2$

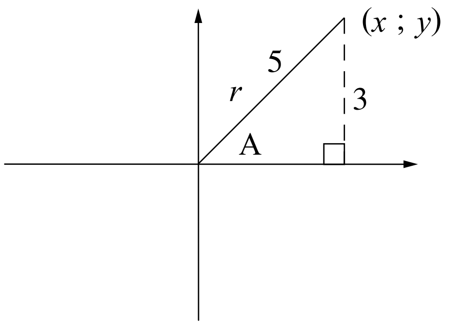
$= \frac{m^2}{m^2 + 1} - \frac{1}{m^2 + 1}$

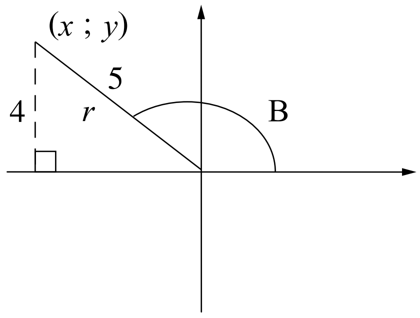
$= \frac{m^2 - 1}{m^2 + 1}$



$y = m \quad x = 1 \quad r^2 = m^2 + 1$
$r = \sqrt{m^2 + 1}$

8.





$x = +4$
$y = +3$
$r = +5$

$x = -3$
$y = +4$
$r = +5$

$$\frac{\left(\frac{4}{5}\right) + \left(\frac{-3}{5}\right)}{\left(\frac{3}{4}\right) - \left(\frac{-4}{3}\right)} = \frac{48 - 36}{45 + 80} = \frac{12}{125}$$

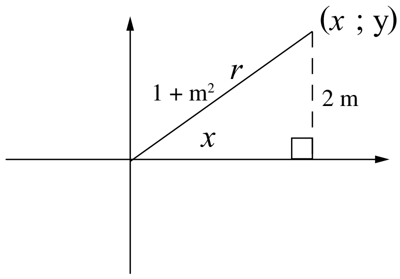
9. $\sin x = \dfrac{2m}{1 + m^2}$

$\dfrac{1 + m^2}{1 - m^2} + \dfrac{2m}{1 - m^2}$

$= \dfrac{1 + 2m + m^2}{(1 - m^2)}$

$= \dfrac{(1 + m)^2}{(1 - m)(1 + m)}$

$= \dfrac{1 + m}{1 - m}$



$y = 2m \qquad r = (1 + m^2)$

$x^2 = (1 + m^2)^2 - 4m^2$

$x^2 = 1 + 2m^2 + m^4 - 4m^2$

$x^2 = 1 - 2m^2 + m^4$

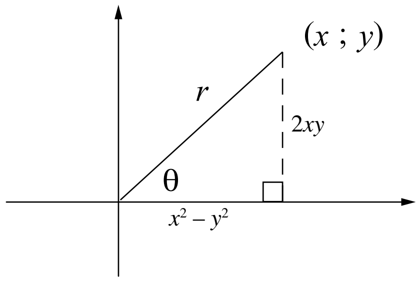
$x^2 = (1 - 1m^2)^2$

$x = 1 - 1m^2$

10. $\tan \theta = \dfrac{2xy}{x^2 - y^2}$

$\dfrac{2xy}{x^2 + y^2} + \dfrac{x^2 - y^2}{x^2 + y^2}$

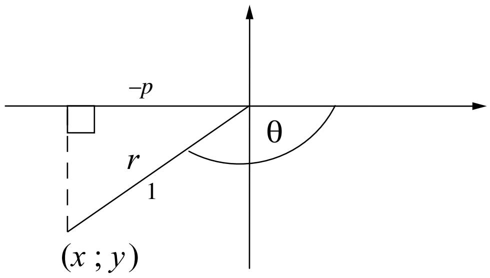
$= \dfrac{2xy + x^2 - y^2}{x^2 + y^2}$



$y = 2xy \quad x = x^2 - y^2 \quad r = x^2 + y^2$

11. $\cos\theta = \frac{p}{1}$

$\tan\theta = \frac{-\sqrt{1-p^2}}{p}$



$x = pr = 1y = -\sqrt{1-p^2}$

Lesson 14

1. $\frac{\sin^2\theta}{\cos^2\theta} - \frac{1}{\cos^2\theta} = \frac{\sin^2\theta - 1}{\cos^2\theta} = \frac{-\cos^2\theta}{\cos^2\theta} = -1$

2. $(1 - \sin^2 x) + \cos^2 x = \cos^2 x + \cos^2 x = 2\cos^2 x$

3. $\frac{1}{\sin A} - \cos A \times \frac{\cos A}{\sin A} = \frac{1 - \cos^2 A}{\sin A} = \frac{\sin^2 A}{\sin A} = \sin A$

4. $\frac{\sin^2\alpha}{\cos^2\alpha} \times \cos^2\alpha + \sin^2\alpha \times \frac{\cos^2\alpha}{\sin^2\alpha} = 1$

5. LHS $\frac{\sin A \times \cos^2 A}{\cos A \sin^2 A}$ $= \frac{\cos A}{\sin A}$ RHS $= \frac{\cos A}{\sin A}$

6. LHS $\cos^2\theta\left(1 + \frac{\sin^2\theta}{\cos^2\theta}\right) = \frac{\cos^2\theta}{1}\left(\frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta}\right)$
$= 1$

7. $(\sin\ \alpha + \cos\alpha)^2$
$= \sin^2\alpha + 2\sin\alpha\cos\alpha + \cos^2\alpha$
$= 1 + 2\sin\alpha\cos\alpha =$ RHS

8. LHS

$(2\cos^2\theta - 1)\left(\frac{1 + \sin^2\theta}{\cos^2\theta}\right)$

$= (2\cos^2\theta - 1)\left(\frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta}\right)$

$= (2\cos^2\theta - 1)\left(\frac{1}{\cos^2\theta}\right)$

$= 2 - \frac{1}{\cos^2\theta} - \frac{2\cos^2\theta - 1}{\cos^2\theta}$

RHS

$\frac{1 - \sin^2\theta}{\cos^2\theta = \cos^2\theta - \sin^2\theta/\cos^2\theta}$

$= \frac{\cos^2\theta - (1 - \cos^2\theta)}{\cos^2\theta}$

$= \frac{2\cos^2\theta - 1}{\cos^2\theta}$

LHS = RHS

9. LHS $\frac{1 - \sin\alpha + 1 + \sin\alpha}{(1 + \sin\alpha)(1 - \sin\alpha)} = \frac{2}{1 - \sin^2\alpha} = \frac{2}{\cos^2\alpha} =$ RHS

10. LHS $\sqrt{\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta} = \sqrt{(\sin\theta + \cos\theta)^2} = \sin\theta + \cos\theta$

11. LHS $\frac{\frac{1 - \sin x}{\cos x}}{\frac{1 + \sin x}{\cos x}}$ (× all terms by $\cos x$)
$= \frac{\cos x - \sin x}{\cos x + \sin x} =$ RHS

12. LHS $\frac{\frac{\sin^2\alpha}{\cos^2\alpha}}{1 + \frac{\sin^2\alpha}{\cos^2\alpha}}$ (× $\cos^2\alpha$)
$= \frac{\sin^2\alpha}{\cos^2\alpha + \sin^2\alpha} = \sin^2\alpha =$ RHS

13. LHS

$\frac{\sin^2 A + (1 + \cos A)^2}{(1 + \cos A)\sin A}$

$= \frac{\sin^2 A + 1 + 2\cos A + \cos^2 A}{(1 + \cos A)\sin A}$

$= \frac{2 + 2\cos A}{(1 + \cos A)\sin A}$

$= \frac{2(1 + \cos A)}{(1 + \cos A)\sin A} = \frac{2}{\sin A}$ = RHS

14. LHS

$\left(\frac{1 - \sin^2 x}{\sin x}\right)^2$

$= \left(\frac{\cos^2 x}{\sin x}\right)^2$

$= \frac{\cos^4 x}{\sin^2 x}$

RHS

$\frac{\cos^2 x}{\sin^2 x} - \frac{\cos^2 x}{1}$

$= \frac{\cos^2 x - \cos^2 x \sin^2 x}{\sin^2 x}$

$= \frac{\cos^2 x(1 - \sin^2 x)}{\sin^2 x}$

$= \frac{\cos^2 x \cos^2 x}{\sin^2 x}$

$= \frac{\cos^4 x}{\sin^2 x}$

15. RHS $\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta}$

$= \frac{\sin^2\theta - \cos^2\theta}{\cos\theta\sin\theta}$

$= \frac{1 - \cos^2\theta - \cos^2\theta}{\cos\theta\sin\theta}$

$= \frac{1 - 2\cos^2\theta}{\cos\theta\sin\theta}$

= LHS (Sometimes choose the RHS)

16. LHS $\frac{1}{\sin^2 A} - \frac{\cos A}{\sin^2 A}$

$= \frac{1 - \cos A}{\sin^2 A}$

$= \frac{1 - \cos A}{1 - \cos^2 A}$

$= \frac{(1 - \cos A)}{(1 - \cos A)(1 + \cos A)} = \frac{1}{(1 + \cos A)}$ = RHS

17. LHS

$\frac{\sin^2\alpha}{\cos^2\alpha} - \frac{\sin^2\alpha}{1}$

$= \frac{\sin^2\alpha - \sin^2\alpha\cos^2\alpha}{\cos^2\alpha}$

$= \frac{\sin^2\alpha(1 - \cos^2\alpha)}{\cos^2\alpha}$

$= \frac{\sin^2\alpha\sin^2\alpha}{\cos^2\alpha}$

$= \frac{\sin^4\alpha}{\cos^2\alpha}$

RHS

$\frac{\sin^2\alpha}{\cos^2\alpha} \times \sin^2\alpha$

$= \frac{\sin^4\alpha}{\cos^2\alpha}$

18. LHS $\frac{1 - \cos^2\alpha}{1 + \cos\alpha} = \frac{(1 - \cos\alpha)(1 + \cos\alpha)}{1 + \cos\alpha}$

$= 1 - \cos\alpha$ = RHS

19. LHS $\frac{\sin x}{1 - \cos x}$ (× top and bottom by $\sin x$)

$= \frac{\sin^2 x}{(1 - \cos x)\sin x}$

$= \frac{1 - \cos^2 x}{(1 - \cos x)\sin x} = \frac{(1 - \cos x)(1 + \cos x)}{(1 - \cos x)\sin x}$

$= \frac{1 + \cos x}{\sin x}$

20. LHS $\left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right)^2 = \left(\frac{1 - \sin x}{\cos x}\right)^2$

$= \frac{(1 - \sin x)^2}{\cos^2 x}$

$= \frac{(1 - \sin x)^2}{1 - \sin^2 x}$

$= \frac{(1 - \sin x)^2}{(1 - \sin x)(1 + \sin x)}$

$= \frac{1 - \sin x}{1 + \sin x} =$ RHS

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Lesson 15

Activity 1

A.

1) $1 - \frac{\sin^2 A}{1 - (-\cos A)}$

$= 1 - \frac{1 - \cos^2 A}{1 + \cos A}$

$= \frac{1 + (1 + \cos A)(1 - \cos A)}{(1 + \cos A)}$

$= \cos A$

2) $\frac{(\sin\theta)(\sin\theta)(-\cos\theta)}{(\cos\theta)\left(\frac{\sin\theta}{\cos\theta}\right)(\sin\theta)} = \cos\theta$

3) $= \frac{1}{-\left(\frac{\sin\theta}{\cos\theta}\right)(-\cos\theta)} - \frac{\cos^2\theta}{\sin\theta}$

$= \frac{1}{\sin\theta} - \frac{\cos^2\theta}{\sin\theta}$

$= \frac{1 - \cos^2\theta}{\sin\theta} = \frac{\sin^2\theta}{\sin\theta}$

$= \sin\theta$

4) $\frac{\sin^2\theta + \cos^2\theta}{-\left(\frac{\sin\theta}{\cos\theta}\right)/\cos\theta}$

$= \frac{1}{\left(\frac{\sin\theta}{\cos\theta}\right)\left(\frac{\cos\theta}{1}\right)}$

$= +\frac{1}{\sin\theta}$

5) $\frac{\sin A}{\cos A} \cdot \frac{1}{\sin A} - \frac{\sin^2 A}{\cos A}$

$= \frac{1 - \sin^2 A}{\cos A}$

$= \frac{\cos^2 A}{\cos A} = \cos A$

6) $\frac{\tan\theta - \sin\theta}{1 - \cos\theta}$

$= \frac{\frac{\sin\theta}{\cos\theta} - \sin\theta}{1 - \cos\theta}$

$= \frac{\sin\theta - \sin\theta\cos\theta}{\cos\theta(1 - \cos\theta)}$

$= \frac{\sin\theta(1 - \cos\theta)}{\cos\theta(1 - \cos\theta)}$

$= \tan\theta$

7) $\frac{(\sin\theta)\sin\theta - \cos^2\theta}{(\tan\theta) + \frac{1}{\tan\theta}}$

$= \frac{\sin^2\theta - \cos^2\theta}{\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta}}$

$= \frac{(\sin^2\theta - \cos^2\theta)}{1}$

$\times \frac{\sin\theta\cos\theta}{(\sin^2\theta - \cos^2\theta)}$

$= \sin\theta\cos\theta$

B.

1) LHS

$\frac{1}{\sin A + 1} - \frac{1}{\sin A - 1}$

$= \frac{\sin A - 1 - (\sin A + 1)}{(\sin A + 1)(\sin A - 1)}$

$= \frac{-2}{\sin^2 A - 1}$

$= \frac{2}{1 - \sin^2 A}$

$= \frac{2}{\cos^2 A}$

2) LHS

$2(-\cos x)(-\sin x)$

$= 2 \sin x \cos x$

$\therefore$ LHS = RHS

RHS

$\frac{2(\tan x)}{1 + \tan^2 x} = \frac{\frac{2 \sin x}{\cos x}}{1 + \frac{\sin^2 x}{\cos^2 x}}$

$= \frac{2\sin x \cos x}{\cos^2 x + \sin^2 x}$

$= 2 \sin x \cos x$

3) LHS

$\left(\frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta}\right)^2$

$= \left(\frac{1 - \sin \theta}{\cos \theta}\right)^2$

$= \frac{(1 - \sin \theta)^2}{\cos^2 \theta}$

$= \frac{(1 - \sin \theta)^2}{(1 - \sin^2 \theta)}$

$= \frac{(1 - \sin \theta)^2}{(1 - \sin \theta)(1 + \sin \theta)}$

$= \frac{1 - \sin \theta}{1 + \sin \theta}$

$\therefore$ LHS = RHS

RHS

$\frac{1 + (-\sin \theta)}{1 - (-\sin \theta)}$

$= \frac{1 - \sin \theta}{1 + \sin \theta}$

4) LHS

$\left(\frac{1}{\cos \theta}\right) - (-\tan \theta)$

$= \frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta}$

$= \frac{1 + \sin \theta}{\cos \theta}$

RHS

$\frac{\cos \theta}{1 - \sin \theta}$

$= \frac{\cos \theta \times \cos \theta}{(1 - \sin \theta) \times \cos \theta}$

$= \frac{\cos^2 \theta}{\cos \theta(1 - \sin \theta)}$

$= \frac{1 - \sin^2 \theta}{\cos \theta(1 - \sin \theta)}$

$= \frac{(1 - \sin \theta)(1 + \sin \theta)}{\cos \theta(1 - \sin \theta)}$

$= \frac{1 + \sin \theta}{\cos \theta}$

Activity 2

A

1) $\frac{(-\sin A)(\tan A)(\cos A)}{(-\cos A)(\sin A)(-\tan A)} = -1$

2) $\frac{(\cos \theta)(-\sin \theta)(-\sin \theta)}{(-\tan \theta)(\cos \theta)(\tan \theta)}$

$= -\frac{\sin^2 \theta}{1} \times \frac{\cos^2 \theta}{\sin^2 \theta}$

$= -\cos^2 \theta$

3) $\frac{(-\sin \alpha)(\cos \alpha)(-\sin \alpha)}{(-\cos \alpha)(-\tan \alpha)(\cos \alpha)}$

$= \frac{\sin^2 \alpha \cos\alpha}{\cos \alpha \sin \alpha}$

$= \sin \alpha$

4) $\sin \theta \frac{\sin \theta}{\cos \theta} \cos \theta - \cos \theta(-\cos \theta)$

$= \sin^2 \theta + \cos^2 \theta$

$= 1$

5) $\dfrac{(\sin\beta)(-\tan\beta)(\cos^2\beta)}{(\cos\beta)(\sin\beta)(\cos\beta)}$

$= -\tan\beta$

6) $\dfrac{(\tan\theta)(\sin\theta)}{\cos\theta} - \dfrac{(-\cos\theta)}{\cos\theta}$

$= \dfrac{\sin^2\theta}{\cos^2\theta} + 1$

$= \dfrac{\sin^2\theta + \cos^2\theta}{\cos^2\theta}$

$= \dfrac{1}{\cos^2\theta}$

B

1) LHS $\dfrac{\sin^2 x + \cos^2 x}{(\tan x)(-\cos x)}$

$= \left(\dfrac{1}{\sin x/\cos x}\right)(-\cos x)$

$= -\dfrac{1}{\sin x}$

2) LHS $\left(\dfrac{(\cos\alpha)(-\cos\alpha)}{\sin\alpha}\right)^2$

$= \dfrac{\cos^4\alpha}{\sin^2\alpha}$

RHS $\dfrac{\cos^2\alpha}{\sin^2\alpha} - \cos^2\alpha$

$= \dfrac{\cos^2\alpha - \sin^2\alpha\cos^2\alpha}{\sin^2\alpha}$

$= \dfrac{\cos^2\alpha(1-\sin^2\alpha)}{\sin^2\alpha}$

$= \dfrac{\cos^4\alpha}{\sin^2\alpha}$

3) LHS $\dfrac{(-\sin x)(\tan x)(\cos x)}{(-\sin x)(\cos x)}$

$= \tan x$

4) LHS $\dfrac{1}{\sin A + 1} - \dfrac{1}{\sin A - 1}$

$= \dfrac{\sin A - 1 - \sin A - 1}{(\sin A + 1)(\sin A - 1)}$

$= \dfrac{-2}{\sin^2 A - 1}$

$= \dfrac{-2}{-\cos^2 A}$

$= \dfrac{2}{\cos^2 A}$

5) LHS $\cos x\left(\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}\right)$

$= \dfrac{\cos x}{1}\left(\dfrac{\sin^2 x + \cos^2 x}{\sin x\cos x}\right)$

$= \dfrac{1}{\sin x}$

Activity 3

1) a) $\sin 250°$
$= -\sin 70°$
$= -\cos 20°$

b) $\cos 340°$
$= \cos 20°$

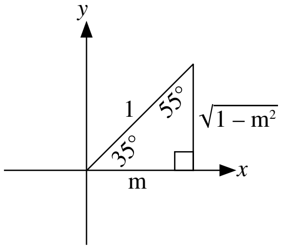
c) $\tan 160°$
$= -\tan 20°$

2) $\cos 35° = m$ $\quad \sin 55° = m$ $\quad \sin 35° = \sqrt{1-m^2}$

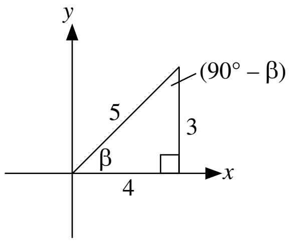
a) $\sin 305°$
$= -\sin 55°$
$= -m$

b) $\sin 245°$
$= -\sin 55°$
$= -m$

c) $\cos 245°$
$= -\cos 55°$
$= -\sqrt{1-m^2}$



3) $x = 3 \quad y = 4 \quad r = 5$ $\qquad x = 4 \quad y = 3 \quad r = 5$



$\cos \beta = \frac{4}{5}$

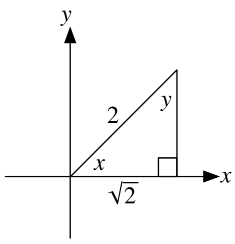
$\sin \beta - \tan \alpha$

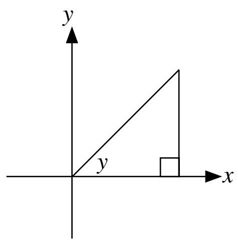
$= \frac{4}{5} - \frac{4}{3}$

$= \frac{12 - 20}{15}$

$= -\frac{8}{15}$

4) $\cos x = \frac{\sqrt{2}}{2}$ $\qquad \sin y \frac{\sqrt{2}}{2}$





$x = \sqrt{2} \quad r = 2 \quad y^2 = 4 - 2$ $\qquad y = \sqrt{2} \quad r = 2 \quad x = \sqrt{2}$

$y^2 = 2$

$y = \sqrt{2}$

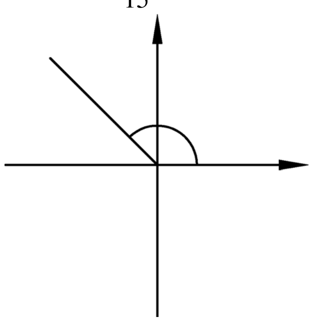
a) $\cos^2 x \circ \sin^2 y = 0$

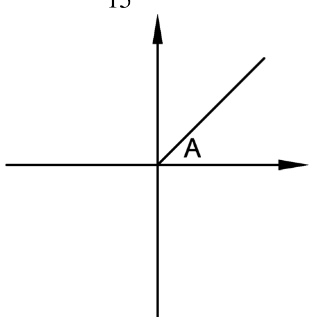
b) $\tan x \cdot \tan y$

$= 1 \times 1$

$= 1$

5) $\tan B = -\frac{8}{15}$ $\qquad \tan A = \frac{8}{15}$





$x = {}^\circ 15 \quad y = +8 \quad r = 17$ $\qquad x = 15 \quad y = 8 \quad r = 17$

$\sin A + \cos B = \frac{8}{17} + \frac{15}{17} = \frac{23}{17}$

6) $\cos\alpha \frac{12}{13}$ $\cos\beta \frac{12}{13}$

$x = +12$ $y = +5$ $r = +13$ $x = -12$ $y = +5$ $r = +13$

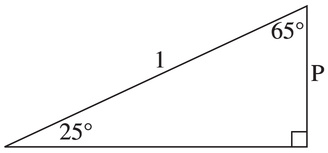
$\tan\beta - \sin\alpha$

$= -\frac{5}{12} - \frac{5}{13}$

$= \frac{-65-60}{156}$

$= \frac{-125}{156}$

7) $\sin 25° = p$ $\cos 25° = \sqrt{1-p^2}$ $\cos 65° = p$



a) $\cos 245°$

$= -\cos 65°$

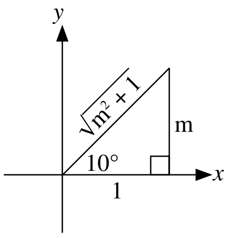
$= -p$

b) $\cos 155°$

$= -\cos 25°$

$= -\sqrt{1-p^2}$

8) $\tan 10° = \frac{m}{1}$



a) $\sin 10° = \frac{m}{\sqrt{m^2+1}}$

b) $\cos 10° = \frac{1}{\sqrt{m^2+1}}$

c) $\tan 90° = \frac{\sin 1\,080°}{80°}$

$= \frac{\frac{1}{\sqrt{m^2+1}}}{\frac{m}{\sqrt{m^2+1}}}$

$= \frac{1}{m}$

$x = 1$ $y = m$ $r^2 = m^2 + 1$ $r = \sqrt{m^2+1}$

9) a) LHS $\frac{2\sin 10°}{\sin 10°} = 2$ b) LHS $\frac{\cos 20°}{2\sin 70°} = \frac{\sin 70°}{2\sin 70°} = \frac{1}{2}$

c) $\frac{(\sin 70°)(-\cos 5°)}{(\cos 20°)(-\cos 5°)} = 1$ d) $\frac{\cos^2 20° + \sin^2 20°}{\sin^2 40°(1 + \frac{\sin^2 50°}{\cos^2 50°})}$

$\sin 70°$

Lessons 16–18

Activity 1

1. $\theta = \pm 55{,}5° + k\cdot 180°\ k \in \mathbb{Z}$
2. $\theta = -194{,}5° = k\cdot 360°$ or $\theta = 94.5° + k\cdot 360°\ \ k \in \mathbb{Z}$

   $\theta \in \{-265{,}5°\ ; -194{,}5°\ ; 94{,}5°\ ; 165{,}5°\}$
3. $\theta = \pm 90° + k\cdot 360°\ \ k \in \mathbb{Z}$
4. $\theta = \pm 180° + k\cdot 360°\ \ k \in \mathbb{Z}$
5. $\theta = 7° + k\cdot 90$ or $\theta = 76° + k\cdot 180°\ \ k \in \mathbb{Z}$

   $\theta \in \{-173°\ ; -104°\ ; -83°\ ; 7°\ ; 76°\ ; 97°\}$
6. $\theta = \pm 30° + k\cdot 180°\ ; k \in \mathbb{Z}$

7. $\cos 2\theta = \cos(90° - \theta + 40°)$

$2\theta = \pm(130° - \theta) + k \cdot 360°$; $k \in \mathbb{Z}$

$3\theta = 130° + k \cdot 360°$ or $\theta = -130° + k \cdot 360°$

$\theta = 43{,}3° + k\ 180°$ or $\theta = -130° + k \cdot 360°$ $k \in \mathbb{Z}$

Activity 2

1. $\theta = -10° + k \cdot 60°$
   $k \in \mathbb{Z}$ $\theta \in \{-10° ; 50°\}$

2. $\theta = 55° + k \cdot 90°$
   $k \in \mathbb{Z}$ $\theta \in \{-125° ; -35° ; 55° ; 145°\}$

3. $\theta = -40° + k \cdot 180°$
   $k \in \mathbb{Z}$ $\theta \in \{-220° ; -40°\}$

4. $\theta = 63{,}4° + \text{k}\ 180°$
   $k \in \mathbb{Z}$ $\theta \in \{-296{,}6° ; -116{,}6° ; 63{,}4° ; 243{,}4°\}$

5. $\theta = -26{,}6° + k \cdot 180°$
   $k \in \mathbb{Z}$ $\theta \in \{-26{,}6° ; 153{,}4°\}$

6. $\theta = -71{,}6° + k \cdot 180°$
   $k \in \mathbb{Z}$ $\theta \in \{-71{,}6° ; 108{,}4°\}$

Activity 3

1. $\tan \theta = \frac{1}{\sqrt{3}}$

   $\therefore \theta = -30° + k \cdot 180°$; $k \in \mathbb{Z}$

2. $\cos(90° - 3\theta) = \cos(\theta + 62°)$

   $\therefore 90° - 3\theta = \theta + 62° + k \cdot 360°$ or $90° - 3\theta = -\theta - 62° + k \cdot 360°$

   $\therefore -4\theta = -28° + k \cdot 360°$ or $-2\theta = -152° + k \cdot 360°$

   $\therefore \theta = 7° - k \cdot 90$ or $\theta = 76° - k \cdot 180°$ $k \in \mathbb{Z}$

3. $\cos 2\theta = -0{,}357$

   $2\theta = \pm 110{,}92° + k \cdot 360°$; $k \in \mathbb{Z}$

   $\theta = \pm 55{,}5° + k \cdot 180°$

4. $\sin 2\theta = -1$

   $\theta = -90° + k \cdot 360$ or $2\theta = 180° - (-90°) + k \cdot 360°$

   $\theta = -45° + k \cdot 180°$ $k \in \mathbb{Z}$

5. $\sin(3\theta + 24°) = -0{,}279$

   $3\theta + 24° = -16{,}2° + k \cdot 360°$ or $3\theta + 24° = 180° - (-16{,}2°) + k \cdot 360°$

   $3\theta = -40{,}2° + k \cdot 360°$ or $3\theta = -172{,}2° + k \cdot 360°$

   $\theta = -13{,}4° + k \cdot 120°$ or $\theta = 57{,}4° + k \cdot 120°$ $k \in \mathbb{Z}$

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6. $\cos(\theta + 50°) = -0{,}814$

$\theta + 50° = 144{,}49° + k \cdot 360°$ or $\theta + 50° = -144{,}49° + k \cdot 360°$ $k \in \mathbb{Z}$

$\theta = 94{,}49° + k \cdot 360°$ or $\theta = -194{,}49° + k \cdot 360°$ $k \in \mathbb{Z}$

7. $\tan^2 3\theta = 3$

$\therefore \tan 3\theta = \pm\sqrt{3}$

$\therefore 3\theta = \pm 60° + k \cdot 180°$

$\therefore \theta = \pm 20° + k \cdot 60°$ $k \in \mathbb{Z}$

8. $\cos 2x = \cos(90° - x + 40°)$

$\therefore \cos 2x = \cos(130° - x)$

$\therefore 2x = \pm(130° - x) + k \cdot 360°$

$\therefore 2x = 130° - x + k \cdot 360°$ or $2x = -130° + x + k \cdot 360°$

$\therefore 3x = 130° + k \cdot 360°$ or $x = -130° + k \cdot 360°$

$\therefore x = 43{,}3° + k \cdot 120°$ or $x = -130° + k \cdot 360°$ $k \in \mathbb{Z}$

9. $\tan(90° - x) = \tan(2x + 60°)$

$\therefore 90° - x = 2x + 60° + k \cdot 180°$

$\therefore -3x = -30° + k \cdot 180°$

$\therefore x = 10° + k \cdot 60°$ $k \in \mathbb{Z}$

10. $\cos\theta = \frac{-1}{\sqrt{2}}$

$\therefore \theta = \pm 135° + k \cdot 360°$ $k \in \mathbb{Z}$

11. $\sin 2\theta = -\frac{1}{\sqrt{2}}$

$2\theta = -45° + k \cdot 360°$ or $2\theta = 180° - (-45°) + k \cdot 360°$

$\therefore \theta = -22{,}5° + k \cdot 180°$ or $\theta = 112{,}5° + k \cdot 180°$ $k \in \mathbb{Z}$

Activity 4

1. $2\sin^2\theta - 3\cos^2\theta = 2(\sin^2\theta + \cos^2\theta)$

$2\sin^2\theta - 3\cos^2\theta = 2\sin^2\theta + 2\cos^2\theta$

$0 = 2\cos^2\theta + 3\cos^2\theta$

$\cos^2 = 0$

$\theta = \pm 90° + k\ 360°$ $k \in \mathbb{Z}$

$\{\pm 90° ; \pm 270°\}$

2. $2\sin\theta\cos\theta - \sin\theta + 2\cos\theta - 1 = 0$

$\sin\theta(2\cos\theta - 1) + (2\cos\theta - 1) = 0$

$(2\cos\theta - 1)(\sin\theta + 1) = 0$

$\cos\theta = \frac{1}{2}$ or $\sin\theta = -1$

$\theta = \pm 60° + k \cdot 360°$ or $\theta = -90° + k \cdot 360°$ $k \in \mathbb{Z}$

3. $7\sin^2\theta + 4\sin\theta\cos\theta = 3\sin^2\theta + 3\cos^2\theta$

$4\sin^2\theta + 4\sin\theta\cos\theta - 3\cos^2\theta = 0$

$(2\sin\theta - \cos\theta)(2\sin\theta + 3\cos\theta) = 0$

$2\sin\theta = \cos\theta$ or $2\sin\theta = -3\cos\theta$

$\tan\theta = \frac{1}{2}$ or $\tan\theta = -\frac{3}{2}$

$\theta = 26{,}6° + k\cdot 180°$ or $\theta = -56{,}3° + k\cdot 180°\ k \in \mathbb{Z}$

$\{26{,}6° ; -153{,}4° ; -56{,}3° ; 123{,}7° ; 206{,}6° ; 303{,}7°\}$

4. $\cos(5\theta - 40°) = \cos(90° - 2\theta + 20°)$

$\cos(5\theta - 40°) = \cos(110° - 2\theta)$

$5\theta - 40° = 110° - 2\theta + k\cdot 360°$ or $5\theta - 40° = 2\theta - 110° + k\cdot 360°$

$7\theta = 150° + k\cdot 360$ or $3\theta = -70° + k\cdot 360°$

$\theta = \frac{150°}{7} + \frac{k360°}{7}$ or $\theta = -23{,}3° + k\cdot 120°\ k \in \mathbb{Z}$

5. $8\cos x = 4(1 - \cos^2) - 7$

$8\cos x = 4 - 4\cos^2 x - 7$

$4\cos^2 x + 8\cos x + 3 = 0$

$(2\cos x + 1)(2\cos x + 3) = 0$

$\cos x = -\frac{1}{2}$ or $\cos x = -\frac{3}{2}$

invalid

$x = \pm 120° + k\cdot 360°$

$\{-120° ; -240°\}$

6. $5\sin^2\theta + 2\cos\theta - 5\cos\theta\sin\theta - 2\sin\theta = 0$

$5\sin\theta(\sin\theta - \cos\theta) + 2(\cos\theta - \sin\theta) = 0$

$(\sin\theta - \cos\theta)(5\sin\theta - 2) = 0$

$\sin\theta = \cos\theta$ or $\sin\theta = \frac{2}{5}$

$\tan\theta = 1$

$\theta = 45° + k\cdot 180°$ or $\theta = 23{,}6° + k\cdot 360°$

or $\theta = 156{,}4° + k\cdot 360°$

7. $\sin 2A = \frac{\sin 2A}{\cos 2A}$

$\sin 2A \cos 2A = \sin 2A$

$\sin 2A \cos 2A - \sin 2A = 0$

$\sin 2A(\cos 2A - 1) = 0$

$\sin 2A = 0$ or $\cos 2A = 1$

$2A = 0 + k\cdot 360°$ or $2A = 180° + k\cdot 360°$

$A = 0 + k\cdot 180°$ or $A = 90° + k\cdot 180°\quad k \in \mathbb{Z}$

$\{0 ; 180° ; -180° ; 90° ; -90°\}$

8. $4 \sin^2 x + 2(1 - \sin^2 x) = 5 \text{ in } x$

$4 \sin^2 x + 2 - 2 \sin^2 x - 5 \sin x = 0$

$2 \sin^2 x - 5 \sin x + 2 = 0$

$(2 \sin x - 1)(\sin x - 2) = 0$

$\sin x = 1/2$ or $\sin x = 2$

Invalid

$x = 30° + k \cdot 360°$ or $x = 150° + k \cdot 360°$ $k \in \mathbb{Z}$

Activity 5

1. $\cos 2A = \cos (180° - 3A)$

$2A = 180° \pm 3A + k \cdot 360°$

$5A = 180° + k \cdot 360°$ or $-A = 180° + k \cdot 360°$

$A = 36° + k \cdot 72°$ or $A = -180° + k \cdot 360°$, $k \in \mathbb{Z}$

2. $2(1 - \sin^2 x) = 3 \sin x + 3$

$2 - 2 \sin^2 x = 3 \sin x + 3$

$2 \sin^2 x + 3 \sin x + 1 = 0$

$(2 \sin x + 1)(\sin x + 1) = 0$

$\sin x = -\frac{1}{2}$ or $\sin x = -1$

$x = -30° + k \cdot 360°$ or $x = 210° + k \cdot 360°$ or $x = -90° + k \cdot 360°$

$\{-150° ; -90° ; -30° ; 210° ; 270° ; 330°\}$

3. $10 \cos^2\theta - 10 \sin \theta \cos \theta + 2 \sin^2\theta + 2 \cos^2\theta = 0$

$12 \cos^2\theta - 10 \sin \theta \cos \theta + 2 \sin^2\theta = 0$

$6 \cos^2\theta - 5 \sin \theta \cos \theta + \sin^2\theta = 0$

$(3 \cos \theta - \sin \theta)(2 \cos \theta - \sin \theta) = 0$

$3 \cos \theta = \sin \theta$ or $2 \cos \theta = \sin \theta$

$\tan \theta = 3$ or $\tan \theta = 2$

$\theta = 71{,}6°$ or $\theta = 63{,}4° + k \cdot 180°$ $k \in \mathbb{Z}$

4. $\cos(\theta - 30°) = \cos 40°$

$\theta - 30° = 40° + k \cdot 360°$ or $\theta - 30° = -40° + k \cdot 360°$

$\theta = 70° + k \cdot 360°$ or $\theta = -10° + k \cdot 360°$ $k \in \mathbb{Z}$

5. $\cos 3x = \sin (-4x)$

$\cos 3x = \cos (90° \pm 4x)$

$\therefore 3x = 90° \pm 4x + k \cdot 360°$

$\therefore 3x = 90° + 4x + k \cdot 360°$

$\therefore -x = 90° + k \cdot 360°$

$\therefore x = -90° + k \cdot 360°$ $k \in \mathbb{Z}$

or

$3x = 90° - 4x + k \cdot 360°$

$7x = 90° + k \cdot 360°$

$x = 12{,}9° + k \cdot 51{,}4°$

6. $3\sin^2 x = 5\cos^2 x$

$\tan^2 x = \frac{5}{3}$

$\tan x = \sqrt{\frac{5}{3}}$ or $\tan x = -\sqrt{\frac{5}{3}}$

$x = \pm 52{,}2 + k \cdot 180° \quad k \in \mathbb{Z}$

7. $(1 - \sin^2 x) - \sin^2 x - 3\sin x - 2 = 0$

$-2\sin^2 x - 3\sin x - 1 = 0$

$2\sin^2 x + 3\sin x + 1 = 0$

$(2\sin x + 1)(\sin x + 1) = 0$

$\sin x = -\frac{1}{2}$ or $\sin x = -1$

$x = -30° + k\ 360°$ or $x = 210° + k \cdot 360°$ or $x = -90° + k \cdot 360° \quad k \in \mathbb{Z}$

8. $\cos^2\theta - 2\sin\theta\cos\theta - (\sin^2\theta + \cos^2\theta) = 0$

$2\sin\theta\cos\theta + \sin^2\theta = 0$

$\sin\theta(2\cos\theta + \sin\theta) = 0$

$\sin\theta = 0$ or $2\cos\theta = -\sin\theta$

$\theta = 0° + k \cdot 360°$ or $-2 = \tan\theta$

$\theta = -63{,}4° + k \cdot 180°$ or $\theta = 180° + k \cdot 360° \quad k \in \mathbb{Z}$

9. $2\sin^2 x - \cos x + 2\sin x\cos x = \sin x$

$2\sin^2 x + 2\sin x\cos x - \cos x - \sin x = 0$

$2\sin x(\sin x + \cos x) - (\cos x + \sin x) = 0$

$(\sin x + \cos x)(2\sin x - 1) = 0$

$\sin x = -\cos x$ or $\sin x = \frac{1}{2}$

$\tan x = -1$

$x = -45° + k \cdot 180°$ or $x = 30° + k \cdot 360°$ or $x = 150° + k \cdot 360°$

$\{-45° ; 30° ; 135° ; 150°\}$

Lesson 19

Activity 1

(a) 4; 7; 10; 13; ……………………………..

$T_n = 4 + (n - 1)3$

$\therefore T_n = 4 + 3n - 3$

$\therefore T_n = 3n + 1$

$T_n = 3n + 1$

$\therefore T_{150} = 3(150) + 1 = 301$

(b) 0; 4; 8; 12; ………………………………

$T_n = 0 + (n - 1)4$

$\therefore T_n = 0 + 4n - 4$

$\therefore T_n = 4n - 4$

$T_n = 4n - 4$

$\therefore T_{150} = 4(150) - 4 = 596$

(c) 10; 6; 2; ………………………………..

$T_n = 10 + (n - 1)(-4)$

$\therefore T_n = 10 - 4n + 4$

$\therefore T_n = -4n + 14$

$T_n = -4n + 14$

$\therefore T_{150} = -4(150) + 14 = -586$

Activity 2

1. (a) 3; 12; 27; 48; ……………………………..

3 12 27 48 75

9 15 21 27

6 6 5

$2a = 6$ | $3a + b = 9$ | $a + b + c = 3$

$\therefore a = 3$ | $\therefore 3(3) + b = 9$ | $\therefore 3 + 0 + c = 3$

$\therefore b = 0$ | $\therefore c = 0$

$\therefore T_n = 3n^2$

(b) 7; 24; 51; 88; ……………………………

7 24 51 88 135

17 27 37 47

10 10 10

$2a = 10$ | $3a + b = 17$ | $a + b + c = 7$

$\therefore a = 5$ | $\therefore 3(5) + b = 17$ | $\therefore 5 + 2 + c = 7$

$\therefore b = 2$ | $\therefore c = 0$

$\therefore T_n = 5n^2 + 2n$

(c) 0; 3; 8; 15; ………………………………..

0 3 8 15 24

3 5 7 9

2 2 2

$2a = 2$	$3a + b = 3$	$a + b + c = 0$
$\therefore a = 1$	$\therefore 3(1) + b = 3$	$\therefore 1 + 0 + c = 0$
	$\therefore b = 0$	$\therefore c = -1$

$\therefore T_n = n^2 - 1$

(d) 6; 17; 34; 57; …………………….

6 17 34 57

11 17 23 → First difference

6 6 → Second difference

$2a = 6$	$3a + b = 11$	$a + b + c = 6$
$\therefore a = 3$	$\therefore 3(3) + b = 11$	$\therefore 3 + 2 + c = 6$
	$\therefore b = 2$	$\therefore c = 1$

$\therefore T_n = 3n^2 + 2n + 1$

2. (a) 1; 3; 6; 10; 15; ……………………….

1 3 6 10

2 3 4 → First difference

1 1 → Second difference

$2a = 1$	$3a + b = 2$	$a + b + c = 1$
$\therefore a = \frac{1}{2}$	$\therefore 3\left(\frac{1}{2}\right) + b = 2$	$\therefore \frac{1}{2} + \frac{1}{2} + c = 1$
	$\therefore b = \frac{1}{2}$	$\therefore c = 0$

$S_n = \frac{1}{2}n^2 + \frac{1}{2}n$

(b) $S_{200} = \frac{1}{2}(200)^2 + \frac{1}{2}(200) = 20\ 100$

Lesson 20

Activity 1

(a) 4; 7; 10; 13; ……………………………..

$T_n = T_n - 1 + 3$

(b) 0; 4; 8; 12; ………………………………

$T_n = T_{n-1} + 4$

(c) 10; 6; 2; ………………………………..

$T_n = T_{n-1} - 4$

Activity 2

1. (a) 7; 24; 51; 88; ………………………

$T_n = T_{n-1} + 10(n - 1) + 7$

$\therefore T_{n-1} + (10n - 3)$

(b) 3; 12; 27; 48; ……………………….

$T_n = T_{n-1} + 6(n - 1) + 3$

$\therefore T_n = T_{n-1} + (6n - 3)$

(c) 6; 17; 34; 57; …………………….

$T_n = T_{n-1} + 6(n - 1) + 5$

$\therefore T_n = T_{n-1} + (6n - 1)$

Lesson 21

Activity 1

1. a) $T_n = 4\left(-\frac{1}{2}\right)^{n-1}$ b) $T_n = -\frac{1}{8}(4)^{n-1}$

c) $T_n = 32(2)^{n-1}$ d) $T_n = 3a(2a)^{n-1}$

2. a) $T_n = \left(\frac{1}{3}\right)^{n-1}$ b) $T_8 = \left(\frac{1}{3}\right)^7 = \frac{1}{243}$

3. a) $T_n = \left(\frac{3}{2}\right)^{n-1}$ b) $\left(\frac{3}{2}\right)^{n-1} = \frac{243}{32}$

$\left(\frac{3}{2}\right)^{n-1} = \left(\frac{3}{2}\right)^5$

$n = 6$

4. $120;\ 120\left(\frac{9}{10}\right);\ 120\left(\frac{9}{10}\right)^2$

$T_8 = 120\left(\frac{9}{10}\right)^7$

$= 0{,}3$ cm

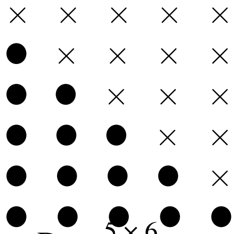
Activity 2

Rubric

Topic	1	2
1. Historical facts	Very few supplied	
2. The Fibonacci		

Activity 3

1. a)



Add the same number of crosses to form a rectangle 5×6

$\therefore$ Dots $\frac{5 \times 6}{2}$

b) $\frac{n(n+1)}{2}$

2. a)

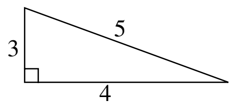
put T_3 or T_4 you get S_4

$\therefore S_4 = T_3 + T_4$

b) $S_n = T_{n-1} + T_n$

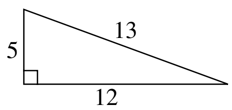
3. a) 1;4; 9;16; 25; 36; 49; 64; 81; 100

b) $9 + 16 = 25$



c) $6^2 + 8^2 = 10^2$

$= 36 + 64 = 100$



$9^2 + 12^2 = 15^2$

$81 + 144 = 225$

$25 + 144 = 169$

4. Square numbers can only end in

1; 4;9; 6;5

$\therefore$ c is not a square number

$(a + b)^2 = a^2 + 2(a)(b) + b^2$

a) 441

$= 400 + 40 + 1$

$= (20)^2 + 2(20)(1) + 1^2$

$= (20 + 1)^2$ square number

b) If 2001 is a square number then the number must end in 1 or 9.

$40^2 = 1600$ $50^2 = 2500$

$41^2 = (40 + 1)^2$

$= 1600 + 80 + 1 \neq 2001$

$49^2 = (40 + 9)^2$

= 1600 + 98 +81

= 1779

∴ 2001 is not a perfect square

c) No

d) 4096

$(60)^2 = 3600$ $(70)^2 = 4900$

Between 60 and 70 to end in 6 must be 64 or 66.

64(2) = (40 + 4)2

= 3600 + 480 + 16

4096

∴4096 is a square number

5. Let the numbers be x and $x + 1$

∴ $(x +1)2 - x^2$

$= x^2 + 2x + 1 - x^2$

$= 2x + 1$

∴ $2x$ is even ∴ $2x + 1$ is odd

6. a) $S_7 - S_4$

= 49 – 16

= 33 m of 3

c) $S_8 - S_5$

= 64 – 25 = 39 m of 3

d) $S_{(x+y)} - S_x$ is a multiple of y.

e) $64 = (2^2)^3$

$664 = (2^3)^2$

So

$(3^2)^3 = (9)^3 = 729$

$(3^3)^2 = (27)^2 = 729$

$(4^2)^3 = (16)^3 = 4096$

$(4^3)^2 = (64)^2 = 4096$

Generalisation

$(x^2)^3 = (x^3)^2$

7. a) Factors of a prime number (p) are p and 1.

∴$t(p) = 2$

b) Factors of 2^n are

1; 2; 2^2; … 2^n

$\therefore$ $(n + 1)$ factors

$\therefore$ $t(p^n) = (n + 1)$

c) $t(8 \times 4) = 6$

$t(80 \times t(4) = 4 \times 3 = 12$

$t(8 \times 4) \neq t(8) \times t(4)$

$t(5 \times 6) = 8$

$t(5) \times \mathrm{t}(6) = 2 \times 4 = 8$

$t(5 \times 6) = t(5) \times t(6)$

(This only occurs when the factors are consecutive counting numbers – co prime)

8. Your investigation. What did you find.

9. $1^3 + 2^3 + \ldots n^3 = (1 + 2 + \ldots + n)^2$

10. a) $\mathrm{LD}(n) \times \mathrm{LD}(m)$

b) 1;4; 5;6; 9

c) ends in 7

11. a) 20; 24; 30 (The next prime numbers plus 1)

b) 14; 4;4 (The product of each number)

c) Pentagonal numbers

1;5; 12; 22 …

4;7; 10

3 3

$T_n = \frac{3}{2}n^2 + an + b$

$1 = \frac{3}{2} + a + b$

$2 = 3 + 2a + 2b$

$2a + 2b = -1$

$5 = 6 + 2a + b$

$2a + b = -1$

$2a + 2b = -1$

$-b = 0$

$b = a$

$2a = -1 - 0$

$\mathrm{A} = -\frac{1}{2}$

$T_n = \frac{3}{2}n^2 - \frac{1}{2}$

Or

1 5 12 22 35

4 7 10 13

3 3 3

	T_1	T_2	T_3	T_4
$\frac{3}{2}n^2$	$\frac{3}{2}$	6	$\frac{27}{2}$	24
want	1	5	12	22
short	$-\frac{1}{2}$	$-\frac{2}{2}$	$-\frac{3}{2}$	$-\frac{4}{2}$

So $\mathrm{T}_n = \frac{3}{2}n^2 - \frac{n}{2}$

$= \frac{n}{2}(3n - 1)$

Lesson 22

Activity 2

1. a) $\frac{\sin \hat{C}}{3} = \frac{\sin 120°}{4}$

$\therefore \sin \hat{C} = 0{,}6495$

$\hat{C} = 40{,}5°$

$\therefore \hat{B} = 19{,}5°$

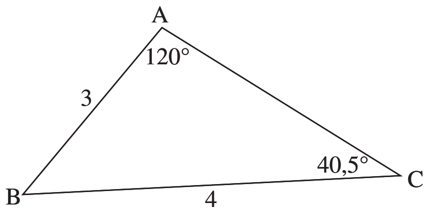
so area $= \frac{1}{2}(3)(4) \sin 19{,}5°$

$= 2\ u^2$

also

$\frac{AC}{\sin B} = \frac{4}{\sin 120°}$

$\therefore AC = \frac{4 \sin 19{,}5}{\sin 120°} = 1{,}54$



2. a) Area $\Delta XYZ = \frac{1}{2}(3)(3) \sin 102°$

$= 4{,}4\ u^2$

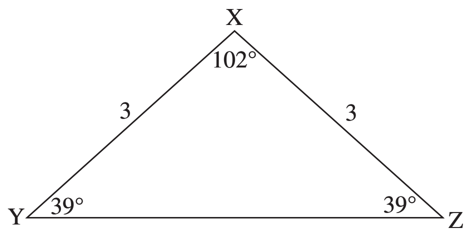
Further $\hat{Y} = \hat{Z} = \frac{180 - 102°}{2}$

$= 39°$

Now area $\Delta XYZ = \frac{1}{2}(3)(YZ) \sin 39°$

$\therefore 4{,}4 = \frac{3}{2}\ YZ \sin 39°$

$\therefore YZ = 4{,}66$



b) Volume = base area × height

$= 4{,}4 \times 12$

$= 52{,}8\ u^3$

$\hat{Q} = 180° - (151° + 12°)$

$\hat{Q} = 17°$

Now $\frac{PR}{\sin 17°} = \frac{3}{\sin 12°}$

$\therefore PR = 4{,}22$

and $\frac{PQ}{\sin 151} = \frac{3}{\sin 12°}$

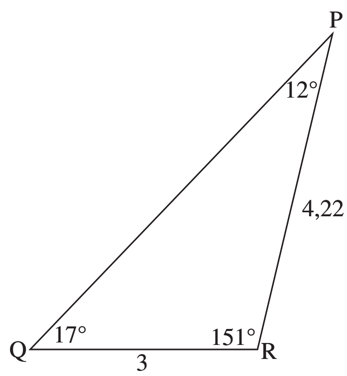
$\therefore PQ = 6{,}995 \approx 7$

Lastly area $\Delta PQR = \frac{1}{2}(7)(3) \sin 17°$

$= 3{,}07\ u^2$

alternatively: area $\Delta PQR = \frac{1}{2}(4{,}22)(3) \sin 151$

$= 3{,}07\ u^2$



3. $B_1 = 180° - 81° = 99°$

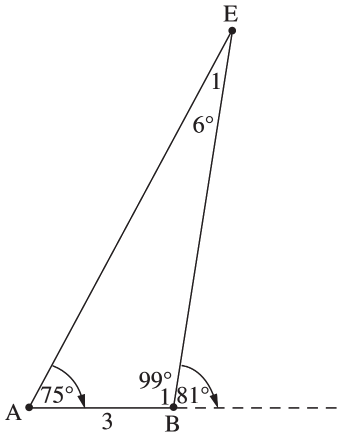
Now $\hat{E}_1 = 180° - 99° - 75°$

$\hat{E}_1 = 6°$

So: $\frac{EB}{\sin 75°} = \frac{3}{\sin 6°}$

$\therefore EB = \frac{3 \sin 75°}{\sin 6°}$

$EB = 27{,}72$ m



4. a) $\frac{\sin \hat{A}}{5{,}3} = \frac{\sin 42°}{5{,}9}$

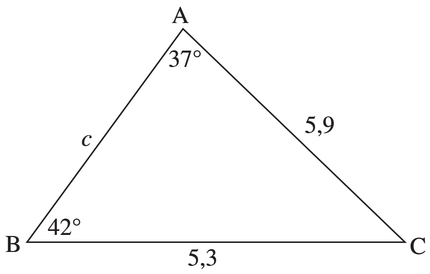
$\therefore \sin \hat{A} = 0{,}601$

$\hat{A} = 36{,}95° \approx 37°$

$\therefore \hat{C} = 101°$

$\therefore \frac{AB}{\sin 101} = \frac{5{,}3}{\sin 37°}$

$\therefore AB = 8{,}65$



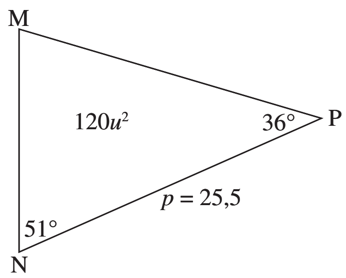
b) Area $= \frac{1}{2}p.(16) \sin 36° = 120$

$\therefore p = 25{,}5$

$\hat{M} = 93°$

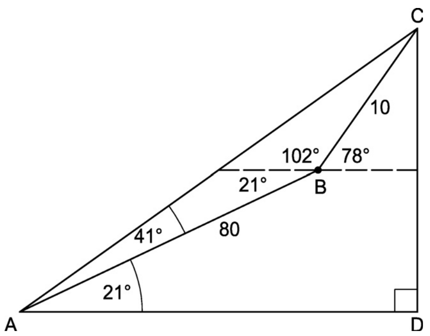
$\therefore \frac{16}{\sin 93°} = \frac{MN}{\sin 36°}$

$MN = 9{,}4$



Activity 3

1.



$\frac{AC}{\sin 123} = \frac{80}{\sin 16}$

$\therefore \frac{80 \sin 123}{\sin 16}$

$= 243{,}4$

$\frac{CD}{\sin 62°} = \frac{243{,}4}{\sin 90°}$

$\therefore BD$

$= \frac{10 \sin 43°}{\sin 310°}$

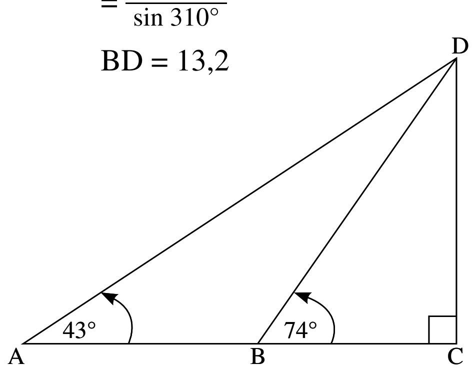
$BD = 13{,}2$

2. $D = 31°$

$\frac{BD}{\sin 43} = \frac{10}{\sin 31}$ $\therefore BD = \frac{10 \sin 43°}{\sin 310}$

$BD = 13{,}2$

$\frac{DC}{\sin 74°} = \frac{13{,}2}{\sin 90°}$ $DC = 12{,}7$ m



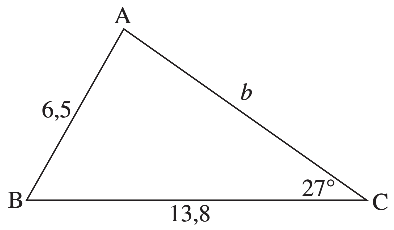
Lesson 23

Activity 4

1. a) $\frac{\sin \hat{A}}{13,8} = \frac{\sin 27^\circ}{6,5}$

$\therefore \sin \hat{A} = 0,96^\circ$

$\hat{A} = 74,5^\circ$ or $\hat{A} = 105,5^\circ$

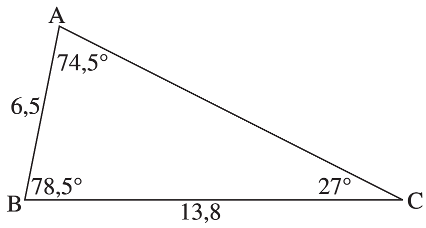


If $\hat{A} = 74,5^\circ$

$\hat{B} = 180^\circ - 74,5^\circ - 27^\circ = 78,5^\circ$

$\therefore \frac{AC}{\sin 78,5^\circ} = \frac{6,5}{\sin 27^\circ}$

$\therefore AC = 14$

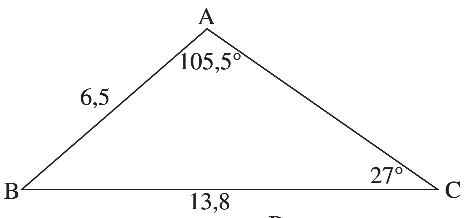


If $\hat{A} = 105,5^\circ$

$\hat{B} = 180^\circ - 105,5^\circ - 27^\circ = 47,5^\circ$

$\therefore \frac{AC}{\sin 47,5} = \frac{6,5}{\sin 27}$

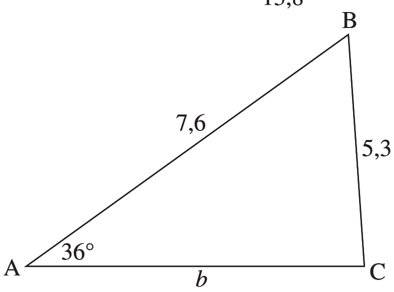
$AC = 10,6$



b) $\frac{\sin \hat{C}}{7,6} = \frac{\sin 36}{5,3}$

$\sin \hat{C} = 0,842$

$\hat{C} = 57,4^\circ$ or $\hat{C} = 122,6^\circ$



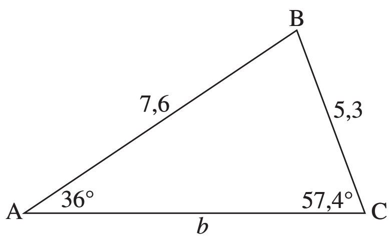
If $\hat{C} = 57,4^\circ$:

$\hat{B} = 180^\circ - 36^\circ - 57,4^\circ$

$= 86,6^\circ$

$\therefore \frac{AC}{\sin 86,6^\circ} = \frac{7,6}{\sin 57,4^\circ}$

$\therefore AC = 9$

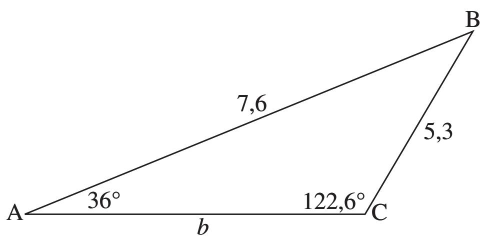


If $\hat{C} = 122,6^\circ$

$\hat{B} = 21,4^\circ$

$\frac{AC}{\sin 21,4^\circ} = \frac{7,6}{\sin 122,6}$

$AC = 3,3$



c) $\frac{\sin \hat{C}}{40} = \frac{\sin 36°}{30}$

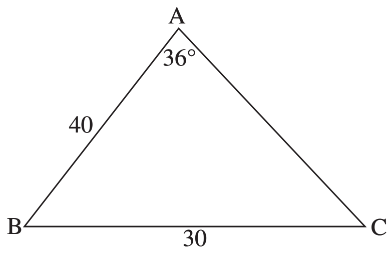
$\sin \hat{C} = 0{,}7837$

$\hat{C} = 51{,}6°$ or $\hat{C} = 128{,}4°$

If $\hat{C} = 128{,}4°$ $\quad\quad \hat{B} = 15{,}6°$

$\frac{AC}{\sin 15{,}6°} = \frac{30}{\sin 36°}$

$AC = 13{,}7$



2. $Q\hat{P}S = 180° - (75° + 8°) = 97°$

Now PS: (In ΔPQS)

$\frac{PS}{\sin 75°} = \frac{800}{\sin 97°}$

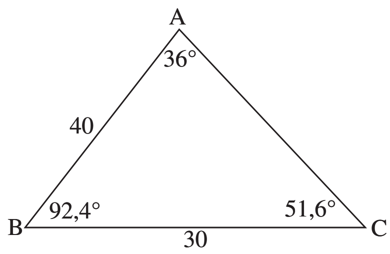
$\therefore PS = 778{,}5$ m

In ΔPRS: ($\hat{R} = 90°$)

$\therefore \sin 8° = \frac{h}{PS}$

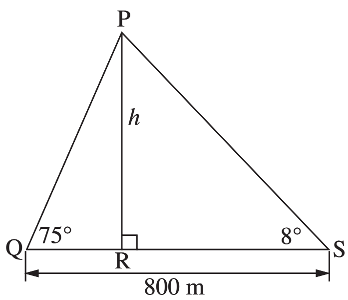
$\therefore h = 778{,}5 \times \sin 8$

$= 108{,}3$ m



3. $\frac{x}{\sin 62} = \frac{20}{\sin 81}$

$x = \frac{20 \sin 62}{\sin 81} = 17{,}8$ m



4. a) $4^2 = 2^2 + 3^2 - 2(2)(3) \cos \hat{E}$

$\therefore \cos \hat{E} = \frac{2^2 + 3^2 - 4^2}{4.3} = -0{,}25$

$\therefore \hat{E} = 104{,}5°$

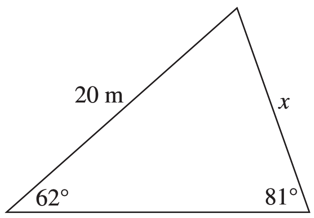
and $\frac{4}{\sin 104{,}5°} = \frac{2}{\sin F}$

$\therefore \sin F = 0{,}48$

$\therefore \hat{F} = 29°$

$\therefore D = 180° - 29° - 104{,}5$

$= 46{,}5°$



b) $RQ = \sqrt{(2{,}2)^2 + (3{,}9)^2 - 2(2{,}2)(3{,}9) \cos 49}$

$= 2{,}96$

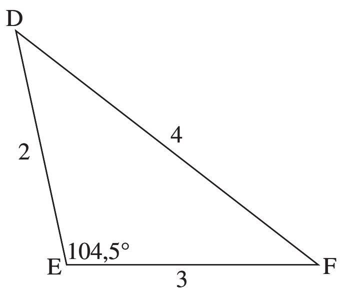
$\frac{\sin \hat{Q}}{2{,}2} = \frac{\sin 49}{2{,}96}$

$\sin \hat{Q} = 0{,}56$

$\therefore \hat{Q} = 34{,}1°$

Then $\hat{R} = 180° - 34{,}1° - 49°$

$\hat{R} = 96{,}9°$



c) $BC = \sqrt{9^2 + 8^2 - 2(9)(8)\cos 60}$

$= 8,54$

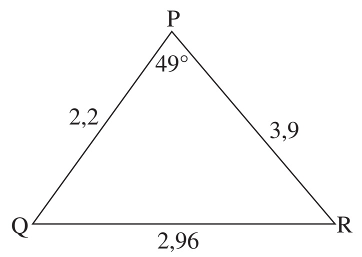
so $\frac{\sin \hat{C}}{9} = \frac{\sin 60}{8,54}$

$\therefore \sin \hat{C} = 0,9126$

$\hat{C} = 65,9°$

Then $B = 180° - 65,9° - 60°$

$B = 54,1°$



Activity 5

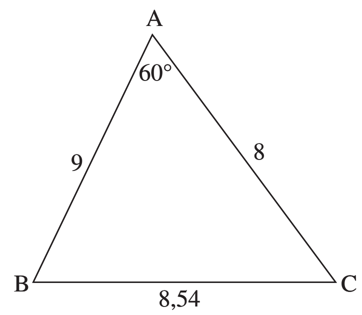
1. a) AC = 33 cm

$\cos \hat{B} = \frac{19^2 + 25^2 - 33^2}{2(19)(25)} = -0,108$

$\therefore \hat{B} = 96,2° = \hat{D}$ (opp ∆s equal)

Then $A = C = (180° - 96,2°)$ co-int ∆s)

$= 83,8°$

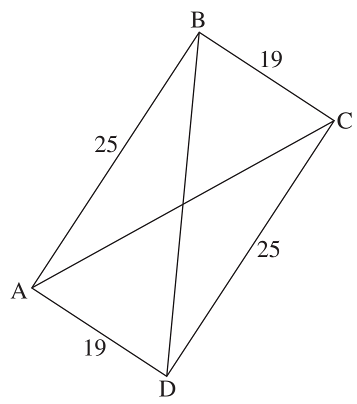


b) $BD = \sqrt{19^2 + 25^2 - 2(19)(25)\cos 83,8°}$

$= 29,7$ cm

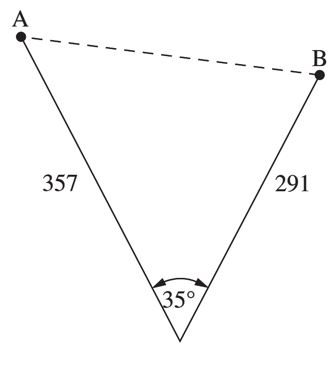
2. $AB = \sqrt{291^2 + 357^2 - 2(291)(357)\cos 35}$

$= 204,8$ m



3. a) Area $\Delta ABC = \frac{1}{2}(8,4)(5,1)\sin 55$

$= 17,55\ u^2$



b) Area ∆AIB + Area ∆BIC + Area ∆AIC

$= \frac{1}{2}AB\cdot r + \frac{1}{2}BC\cdot r + \frac{1}{2}AC\cdot r$

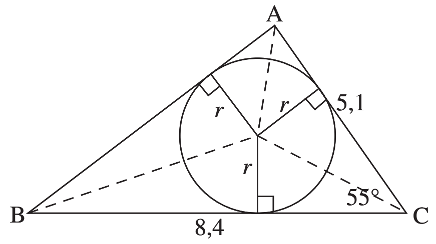
$= \frac{1}{2}r(AB + BC + AC)$

But $AB = R8,4^2 + 5,1^2 - 2(5,1)(8,4)\cos 55$

$= 6,89$

$\therefore 17,55 = \frac{1}{2}r(6,89 + 5,1 + 8,4)$

$\therefore r = 1,72$



Lesson 24

Activity 1

1. a) $DB = \sqrt{152^2 + 120^2 - 2(152)(120)\cos 130°}$

$= 79,5$

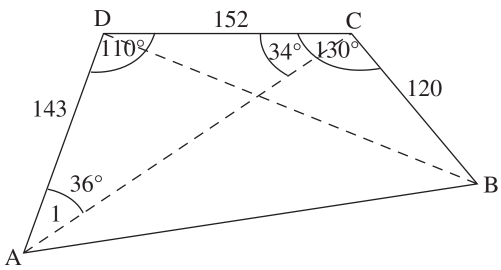
$AC = \sqrt{143^2 + 152^2 - 2(143)(152)\cos 110°}$

$= 241,7$

Now: $\frac{\sin A_1}{152} = \frac{\sin 110°}{241,7}$

$\therefore \sin A_1 = 0,591$

$\therefore \hat{A}_1 = 36°$



2. In ΔOPQ:

$P\hat{Q}O = 140°$

so $OP = \sqrt{400^2 + 50^2 - 2(50)(400)\cos 140}$

$= 439,5$ km/h

For the hearing:

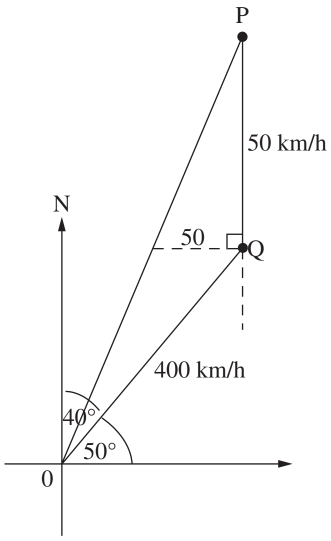
$\frac{\sin Ø}{50} = \frac{\sin 140}{439,5}$

$\therefore \sin Ø = \frac{50 \times \sin 140}{439,5}$

$\sin Ø = 0,073$

$Ø = 4,1934°$

So the bearing was $40° - 4,19° = 35,8°$



3.1. 30 km/h × 4h = 120 km

3.2 500 km/h × 4h = 2 000 km

3.3 If AB + BC = 2 000 km, then if AB = x, BC = 2 000 – x.

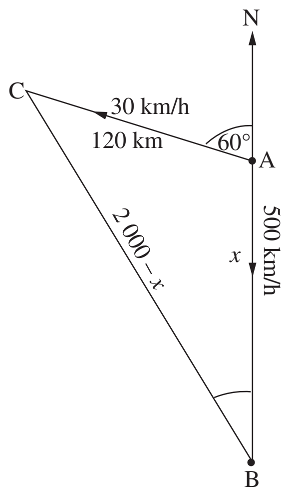
So:

$(2\,000 - x)^2 = 500^2 + x^2 - 2(500)(x)\cos 120°$

$4\,000\,000 - 4\,000x + x^2 = 250\,000 + x^2 + 500x$

$\therefore -4\,500x = -3\,750\,000$

$\therefore x = 833$ km



3.4 $\frac{\sin B}{120} = \frac{\sin 120}{2\,000 - 833}$

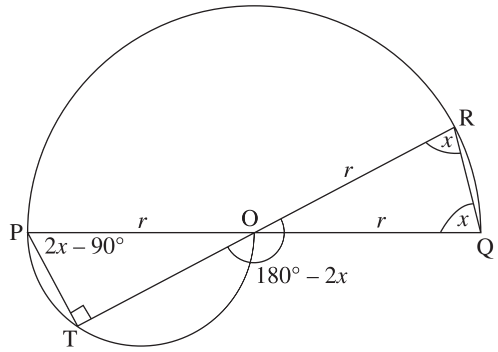
$\therefore \sin B = 0,08905$

$\hat{B} = 5{,}1°$

$\therefore$ Direction is N5°W or a bearing of 355°.

4. $\hat{P} = 90° - 180° + 2x$

$= 2x - 90°$



4.1 $\frac{PQ}{\sin(180 - 2x)} = \frac{r}{\sin x}$

$\therefore$ RQ $= r \sin x.\sin 2x$

$= 2r \sin^2 (180 - 2x)$

4.2 Area $= \frac{1}{2}r^2 \sin(180 - 2x)$

$= \frac{1}{2}r^2 \sin 2x$

4.3 $\sin(180° - 2x) = \frac{PT}{r}$

PT = r sin 2x

and $\cos 180° - 2x = \frac{OT}{r}$

$\therefore$ OT $= -r \cos 2x$ Thus area ΔPOT $= \frac{1}{2}$PT.OT

$= -\frac{1}{2}r^2 \sin 2x \cos 2x$

4.4 $\frac{\text{Area } \Delta ROQ}{\text{Area } \Delta POT}$

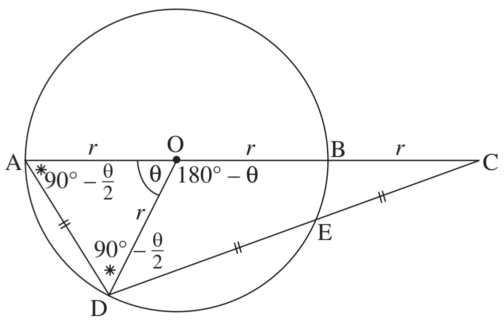
$= \frac{\frac{1}{2}r^2 \sin 2x}{-\frac{1}{2}r^2 \sin 2x \cos 2x}$

$= \frac{-1}{\cos 150°}$

$= \frac{-1}{\frac{-\sqrt{3}}{2}}$

$= \frac{2}{\sqrt{3}}$

5.1 In ΔAOD



$AD^2 = r^2 + r^2 - 2r^2 \cos\theta$

$= 2r^2 - 2r^2 \cos\theta$

$= 2r^2(1 - \cos\theta)$ …(1)

In ΔDOC: DC = 2AD $\Rightarrow DC^2 = 4AD^2$

$\therefore 4AD^2 = r^2 + (2r)^2 - 2(2r)(r)\cos(180 - \theta)$

$= 5r^2 + 4r^2 \cos\theta$

$\therefore AD^2 = \frac{1}{4}(5r^2 + 4r^2 \cos\theta)$…(2)

Now: (1) = (2)

$2r^2(1 - \cos\theta) = \frac{1}{4}(5r^2 + 4r^2\cos\theta)$

$8r^2 - 8r^2\cos\theta = 5r^2 + 4r^2\cos\theta$

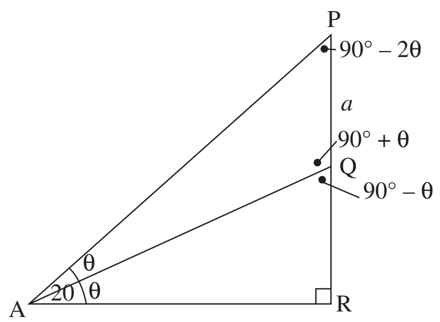
$3r^2 = 12r^2\cos\theta$

$\therefore \cos\theta = \frac{1}{4}$

5.2 Area $\Delta AOD = \frac{1}{2}r^2\sin\theta$

$= \frac{1}{2}r^2\frac{\sqrt{15}}{4}$

$= \frac{r^2\sqrt{15}}{8}$



6.1 $\frac{AP}{\sin(90° + \theta)} = \frac{a}{\sin\theta} \Rightarrow AP = \frac{a\cos\theta}{\sin\theta} = \frac{a}{\frac{\sin\theta}{\cos\theta}} = \frac{a}{\tan\theta}$

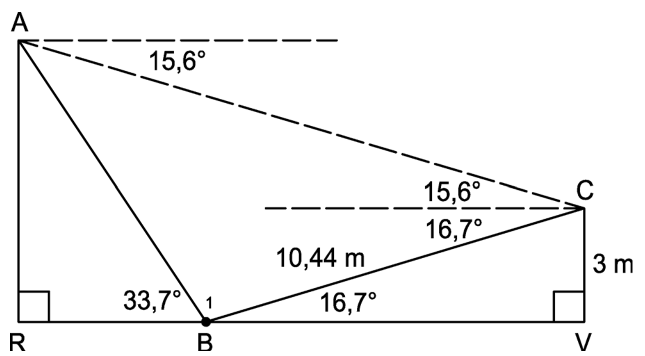
6.2 $\sin 2\theta = \frac{PR}{AP}$

$\therefore PR = AP\sin 2\theta = \frac{a\cos\theta}{\sin\theta}\cdot 2\sin\theta\cos\theta$

$= 2a\cos^2\theta$

6.3 $\frac{PQ}{PR} = \frac{a}{2a\cos^2\theta} = \frac{2}{(\cos 30)^2} = \frac{2}{\left(\frac{\sqrt{3}}{2}\right)^2} = \frac{2}{\frac{3}{4}} = \frac{8}{3}$

7.



$\frac{BC}{\sin 90°} = \frac{3}{\sin 16,7}$ $\therefore$ BC = 10,44

$\hat{B}_1 = 180° - 33,5° - 16.7°$
$= 128,8°$

$\hat{A}_1 = 180° - 129,8° - 16,7° - 15,6°$
$= 17,9°$

$\frac{AB}{\sin 32,3} = \frac{10,44}{\sin 17,9°}$ $\therefore AB = \frac{10,44\sin 32.3°}{\sin 17,9} = 18,2$

$\frac{AR}{\sin 33,5°} = \frac{18,2}{\sin 90°}$ $AR = \frac{18,2\sin 33,5}{1}$

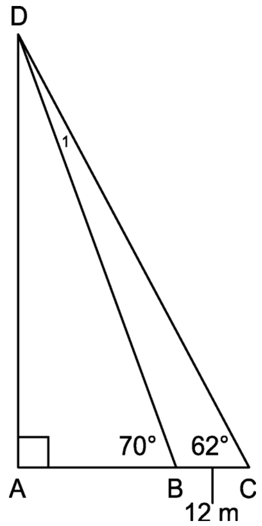
= 10 m (9,9 m)

8. $\hat{D}_1 = 8°$

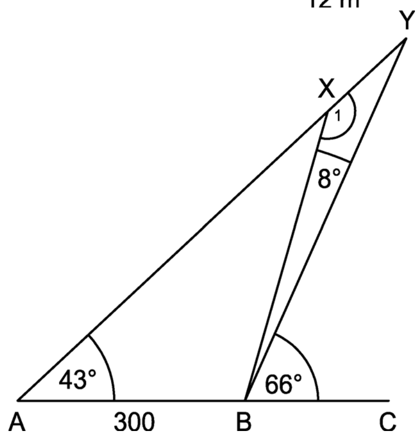
$\frac{BD}{\sin 62°} = \frac{12}{\sin 8}$

$BD = \frac{12 \sin 62}{\sin 8} = 76{,}1$

$\frac{AD}{\sin 70°} = \frac{76{,}1}{\sin 90°}$ $\therefore AD = 71{,}5$



9. $\hat{Y} = 23°$ $\hat{X}_1 = 149°$



$\frac{YB}{\sin 43} = \frac{800}{\sin 23}$

$\therefore YB = \frac{800 \sin 43°}{\sin 23°} = 1\ 396{,}3$

$XY = \frac{1\ 396{,}3 \sin 8°}{\sin 149°} = 377{,}3$

Lesson 25

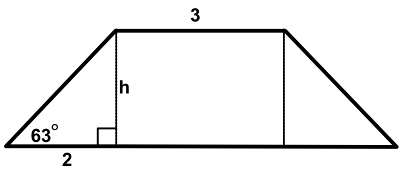
Activity 1

1. Volume:	a)	4 cm^3	b)	5 cm^3
	c)	8 cm^3	d)	7 cm^3
	e)	14 cm^3	f)	10 cm^3
Surface area;	a)	18 cm^2	b)	20 cm^2
	c)	28 cm^2	d)	24 cm^2
	e)	48 cm^2	f)	32 cm^2

2. a) 10 m^3 (b) 107 cm^3 (c) 3,30 cm^3

d) 113,1 cm^3 (e) 392,7 cm^3

f)



$\therefore h = \frac{2 \sin 63}{\sin 23}$ $\quad$ V = (0,50(10)(4,6) × 6

= 4,6 $\quad$ = 1 380 cm^3

$\frac{h}{\sin 63} = \frac{2}{\sin 23}$

g) $\frac{1}{2}\pi(36)15$ (h) $36 = 9 + 16 - 2(3)(4) \cos \alpha$

= 848,2 cm^3 $\quad$ $\cos \alpha = \frac{25 - 36}{24}$

$\alpha = 117,3°$

$V = \frac{1}{2}(4)(3) \sin 117,3 \times 3$

= 16 cm^3

3. a) $197 = 80 + 10h + 16h$ (b) Volume P = $\pi(16)4$

$117 = 26h$ $\quad$ = 201,1 cm^3

$h = 4,5$ cm $\quad$ Volume Q = $\pi(9)(7)$

V = 8 x 5 x 4,5 $\quad$ = 197,9 cm^3

= 180 cm^3 $\quad$ ∴ Tin P holds the most cat food.

c) Volume of water (d) $V = \pi r^2 h$

= $\pi(9)(8)$ $\quad$ $346 = \pi(3,6)2h$

= 226,2 cm^3 $\quad$ $h = 8,5$

$226,2 = \pi(16)h$ $\quad$ Surface area = $2\pi r^2 + 2\pi rh$

$h =$ 4,5 cm $\quad$ = $2\pi(3,6)2 + 2\pi(3,6)(8,5)$

= 273,7 cm^2

e) a) Inside area = $2\pi(15)(150)$

= 14 137,2 cm^2

b) Outside curved area = $2\pi(20)(150)$

= 18 849,6 cm^2

Activity 2

1. To find the weight of the pyramid, we need the volume of the pyramid.

   Volume of the pyramid = $\frac{1}{3}$ × (Base Area) × Height = $\frac{1}{3}(776)^2 \times 481$ = 96 548 885,33 ft^3

   The weight of a block of limestone is 167 pounds per cubic foot. So the weight of the pyramid is 96 548 885,33 ft^3 × 167 p/ft^3 = 1,612 366 385 × 10^{10} pounds.

   The weight in kilogram will be 1,612 366 385 × 10^{10} × 0,453 592 37 = 7 313 570 899 kg

2. a. Slant height = $\sqrt{64 + 36} = \sqrt{100} = 10$

   Surface area of pyramid

   = Area of base + $\frac{1}{2}$(perimeter of base × slant height)

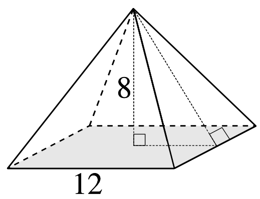
   = $144 + \frac{1}{2}(48 \times 10)$

   = 384 cm^2

   Volume = $\frac{1}{3}$ × (Base Area) × Height

   = $\frac{1}{3} \times (144) \times 8$

   = 384 cm^3



   b. Slant height = $\sqrt{15^2 - 10^2} = \sqrt{125} = 5\sqrt{5}$

   Surface area of pyramid

   = Area of base + $\frac{1}{2}$(perimeter of base × slant height)

   = $400 + \frac{1}{2}(80 \times 5\sqrt{5})$

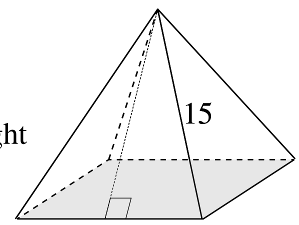
   = 847,21 cm^2

   Height of pyramid = $\sqrt{(5\sqrt{5})^2 - 10^2} = 5$

   Volume = $\frac{1}{3}$ × (Base Area) × Height

   = $\frac{1}{3} \times (400) \times 5$

   = 666,67 cm^3



3. From Pythagoras: BC = $\sqrt{25 - 9} = \sqrt{16} = 4$

   So the base will have length $4\sqrt{2}$.

   Slant height = $\sqrt{(2\sqrt{2})^2 + 9} = \sqrt{17}$

   Surface area of pyramid

   = Area of base + $\frac{1}{2}$ (perimeter of base × slant height)

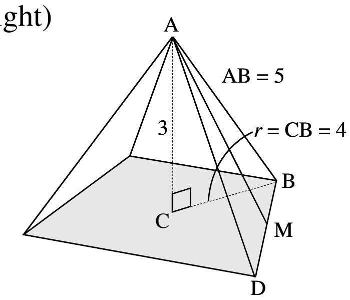
   = $(4\sqrt{2})^2 + \frac{1}{2}(4 \times 4\sqrt{2} \times \sqrt{17})$

   = 78,65 cm^2

   Volume = $\frac{1}{3}$ × (Base Area) × Height

   = $\frac{1}{3} \times (32) \times 3$

   = 32 cm^3



Lesson 26

1. a) $56{,}6\ m^3$ (b) $56{,}6\ m^2$
2. a) $2\ 450{,}4\ m^2$ (b) $13\ 627{,}18\ m^3$
3. a) $5 \times 10^{-3}\ m^3$ (b) $79{,}2\ m^3$
   c) 15 714 bricks
4. a) 8,5 cm (b) $25{,}5\ cm^2$
   c) $426\ cm^2$ (d) $528\ cm^3$
5. 148,2 million
6. Volume: $1\ 466{,}08\ cm^3$
   Surface area: $837{,}07\ cm^2$
7. (a) 18,85 cm (b) 3 cm
   (c) 11,62 cm (d) $109{,}52\ cm^3$

Lesson 27

Activity 5

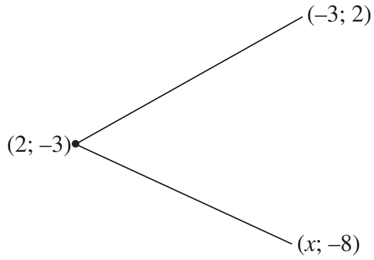
1. a) $\sqrt{(0-2)^2 + (y-0)^2} = \sqrt{40}$

$\therefore 4 + y^2 = 40$

$\therefore y^2 = 36$

$\therefore y = \pm 6$

1. (b)



$(2+3)^2 + (-3-2)^2 = (x-2)^2 + (-8+3)^2$

$5^2 + 5^2 = (x-2)^2 + 5^2$

$\therefore (x-2)^2 = 25$

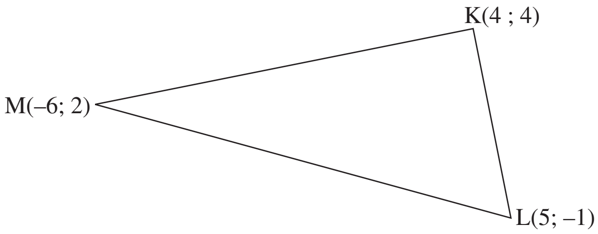
$x - 2 = \pm 5$

$\therefore x = 2 \pm 5$

$\therefore x = 7$ or $x = -3$

1. c) (3; 2) d) $y = -6$ or $y = 2$ 2. D(19;–11)

3.



$M_{KL} = \frac{y_K - y_L}{x_K - x_L} = \frac{4 - (-1)}{4 - 5} = \frac{5}{-1} = -5$

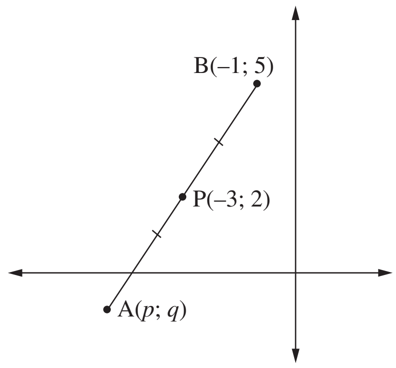
$M_{KM} = \frac{y_K - y_M}{x_K - x_M} = \frac{4 - 2}{4 - (-6)} = \frac{2}{10} = \frac{1}{5}$

Thus $M_{KL} \cdot M_{KM} = -5\left(\frac{1}{5}\right) = -1$

$\therefore$ KL$\perp$KM

Therefore ΔKLM is right angled at K.

4.



P is the midpoint of AB

$x_P = \frac{x_A + x_B}{2}$ and $y_P = \frac{y_A + y_B}{2}$

$\therefore -3 = \frac{p + (-1)}{2}$ and $2 = \frac{q + 5}{2}$

$\therefore -6 = p - 1$ and $4 = q + 5$

$\therefore p = -5$ and $q = -1$

Thus A(p; q) = A(–5; –1)

5. $M_{DE} = \frac{y_D - y_E}{x_D - x_E} = \frac{3 - (-1)}{2 - (-3)} = \frac{4}{5}$ and $M_{DF} = \frac{y_D - y_F}{x_D - x_F} = \frac{3 - (-2)}{2 - 6} = \frac{5}{-4}$

$\therefore (M_{DE}) \times (M_{DF}) = \left(\frac{4}{5}\right)\left(-\frac{5}{4}\right) = -1$

$\therefore$ DE$\perp$DF

$\therefore \hat{D} = 90°$.

Furthermore: P is the midpoint of DE and Q the midpoint of DF

Thus M_{dpt} DE: $x_P = \frac{1}{2}(x_D + x_E)$ and $y_P = \frac{1}{2}(y_D + y_E)$

$\therefore x_P = \frac{1}{2}(2 + (-3))$ $\quad y_P = \frac{1}{2}(3 + (-1))$

$\therefore x_P = \frac{1}{2}(-1)$ $\quad y_P = \frac{1}{2}(2)$

$\therefore x_P = -\frac{1}{2}$ $\quad y_P = 1$

$\therefore P\left(-\frac{1}{2}; 1\right)$

M_{dpt} DF: $x_Q = \frac{1}{2}(x_D + x_F)$ and $y_Q = \frac{1}{2}(y_D + y_F)$

$\therefore x_Q = \frac{1}{2}(2 + 6)$ and $y_Q = \frac{1}{2}(3 + (-2))$

$\therefore x_Q = \frac{1}{2}(8)$ and $y_Q = \frac{1}{2}(1)$

$\therefore x_Q = 4$ and $y_Q = \frac{1}{2}$

$\therefore Q\left(4; \frac{1}{2}\right)$

Now: PQ will be parallel to DE if and only if $M_{PQ} = M_{DE}$

$M_{PQ} = \frac{y_P - y_Q}{x_P - x_Q} = \frac{1 - \frac{1}{2}}{-\frac{1}{2} - 4} \cdot \frac{2}{2} = \frac{2 - 1}{-1 - 8} = -\frac{1}{9}$ and $M_{EF} = \frac{y_E - y_F}{x_E - x_F} = \frac{-1 - (-2)}{-9} = -\frac{1}{9}$

$\therefore M_{PQ} = M_{EF}$

$\therefore$ PQ||EF

Now for the lengths:

$PQ = \sqrt{(x_P - x_Q)^2 + (y_P - y_Q)^2}$ $\qquad EF = \sqrt{(x_E - x_F)^2 + (y_E - y_F)^2}$

$= \sqrt{\left(-\frac{1}{2} - 4\right)^2 + \left(1 - \frac{1}{2}\right)^2}$ $\quad = \sqrt{(-3-6)^2 + (-1-(-2))^2}$

$= \sqrt{\left(-\frac{9}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$ $\quad = \sqrt{(81)^2 + (1)^2}$

$= \sqrt{\frac{82}{4}}$ $\quad = \sqrt{82}$

$= \frac{\sqrt{82}}{2}$

thus 2PQ = EF

6. $y = 2x + 1$ $\quad y = 3x + 6$ $\quad 2y = -6x + 7$

$y = -3x + \frac{7}{2}$

and $y = 3x + 1$

Two lines have gradients that are equal to 3, so these will be parallel.

For ① = ②: $2x + 1 = 3x + 6$ $\quad -3x + \frac{7}{2} = 3x + 6$

$x = -5$ $\quad -6x + 7 = 6x + 12$

Then $y = -10 + 1$ $\quad -12x = 5$

$= -9$ $\quad x = -\frac{5}{12}$

So A(–5; –9) $\quad$ Then $y = 3\left(\frac{-5}{12}\right) + 6 = \frac{19}{4}$

$B\left(-\frac{5}{12}; \frac{19}{4}\right)$

$3x + 1 = 2x + 1$ $\quad 3x + 1 = -3x + \frac{7}{2}$

$x = 0$ $\quad 6x + 2 = -6x + 7$

Then $y = 1$ $\quad 12x = 5$

$c(0; 1)$ $\quad x = \frac{5}{12}$

$\therefore D\left(\frac{5}{12}; \frac{9}{4}\right)$

Then $y = 3\left(\frac{5}{12}\right) + 1$

$= \frac{5}{4} + \frac{4}{4}$

$= \frac{9}{4}$

Lesson 28

1. a) $M_{AB} = \frac{8-0}{6+3} = \frac{8}{9}$

$\tan\theta = \frac{8}{9}$

$\theta = 41{,}6°$

b) $M_{AB} = \tan\theta$

$\therefore \theta = \tan^{-1}\left(\frac{\sqrt{2}-1}{-4-7}\right)$

$= \tan^{-1}(-0{,}0376)$

$= 177{,}8°$

2. a) $m = -\frac{1}{3}$ $\tan\theta = -\frac{1}{3}$

$\theta = 161{,}6°$

b) $\tan\theta = \frac{1}{2}$ $\quad \therefore \theta = 23{,}6°$

c) $\tan\theta = -4$ $\quad \therefore \theta = 104°$

d) $\tan\theta = 2$ $\quad \therefore \theta = 63{,}4°$

3. $M_{DE} = \frac{-2-7}{5+3} = -\frac{9}{8}$

$M_{EF} = \frac{5-7}{2+3} = \frac{-2}{5}$

$M_{DF} = \frac{5+2}{2-5} = \frac{7}{-3}$

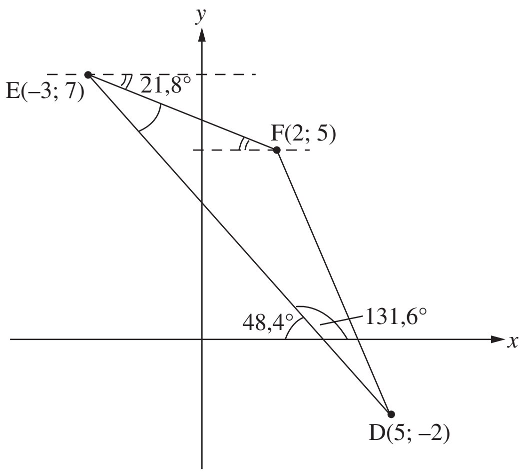
$\theta_{DE} = \tan^{-1}\left(-\frac{9}{8}\right)$

$= 131{,}6°$

$\theta_{EF} = \tan^{-1}\left(-\frac{2}{5}\right)$

$= 158{,}2°$

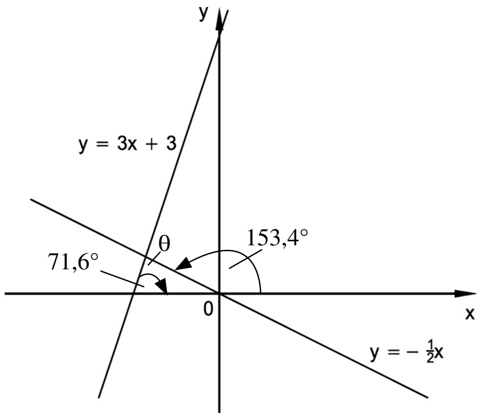
$\therefore$ DEF $= 48{,}4° - 21{,}8° = 26{,}6°$



4. Draw the graph

$\alpha = 71{,}6°$

$\beta = -26{,}6° + 180° = 153{,}4°$



Then: $\theta = 153{,}4° - 71{,}6°$ (ext ∠ of △)

$= 81{,}8°$

OR $\theta' = 180° - 81{,}8° = 98{,}2°$

5. $\tan\theta = \frac{1}{4}$ $\quad \theta = 14°$

$\tan\alpha = -2$ $\quad \alpha = 63{,}4°$

$\therefore \alpha = 102{,}6°$ $\quad \beta = 77{,}4°$

6. $\tan\alpha = 1$ $\quad \alpha = 45°$

$\therefore \beta = 90°$ and the gradient is undefined.

7. $\tan\beta = -2 \therefore \beta = 116{,}6°$

$\therefore \alpha = 54{,}6°$

m = 1,4

8. $A\hat{O}B = 53{,}1°$ $C\hat{O}B = 26{,}6°$

$\tan 26{,}6° = \frac{1}{2}$ $\quad$ gradient of OC $= \frac{1}{2}$

9. $\hat{P} = 101{,}3°$ $\quad \hat{Q} = 54{,}3°$ $\quad \hat{R} = 24{,}4°$

Lesson 29

1. a) $m_{CD} = m_{AB}$

$\frac{t + 1 - 3}{-\frac{1}{3} - 2} = \frac{3}{2}$

$2(t - 2) = 3(-\frac{1}{3} - 2)$

$2t - 4 = -1 - 6$

$2t = -3$

$t = -\frac{3}{2}$

b) $m_{AB} = m_{AE}$

$\frac{3}{2} = \frac{3}{r - 3}$

$3(r - 3) = 6$

$r - 3 = 2$

$r = 5$

2. $(3 - 2k)x + (k + 1y) = 12$

or $(k + 1)y = -(3 - 2k)x + 12$

$y = \frac{(-3 + 2k)x}{k + 1} = \frac{12}{k + 1}$

a) $\frac{-3 + 2k}{k + 1} = 4$

$-3 + 2k = 4k + 4$

$-7 = 2k$

$k = -\frac{7}{2}$

b) $m = \frac{1}{-3}$

$m_{perp} = 3$

$\therefore \frac{-3 + 2k}{k + 1} = 3$

$-3 + 2k = 3k + 3$

$-6 = k$

c) Substitute $x = -3$ $y = 4$

$(3 - 2k)(-3) = (k + 1)\ 4 = 12$

$-9 + 6k + 4k + 4 = 12$

$10k = 17$

$k = \frac{17}{10}$

d) $3 - 2k = 0$

$2k = 3$

$k = \frac{3}{2}$

e) $k + 1 = 0$

$k = -1$

3. a) $y = 7$ or $y = -5$

b) $x = 3$ or $x = -5$

Lesson 30

1. $m_{MN} = 1$

$\tan a = 1$ $\alpha = 45°$

$m_{PQ} = -\frac{3}{2}$ $\beta = 180° - 56{,}3°$

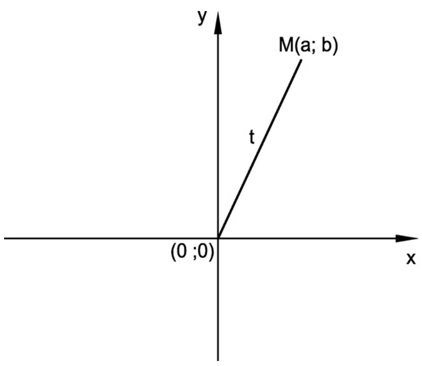
$= 123{,}7°$

$\theta + \alpha = \beta$

$\theta = \beta - \alpha$

$= 78{,}7°$

2. $a^2 + b^2 = t^2$



Point is on the line, so $a + \sqrt{3}\ b = 2t$

$b = \frac{2t - a}{\sqrt{3}}$ Substitute

$a^2 + \left(\frac{2t - a}{\sqrt{3}}\right)^2 = t^2$

$a^2 + \frac{4t^2 - 4at + a^2}{3} = t^2$

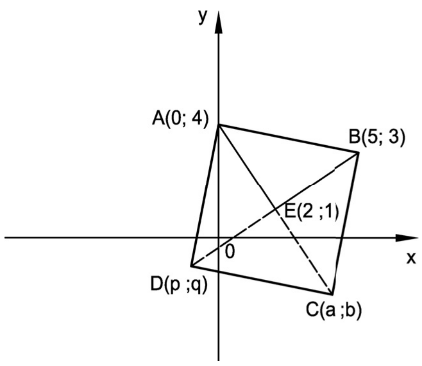
$3a^2 + 4t^2 - 4at + a^2 = 3t^2$

$4a^2 - 4at + t^2 = 0$

$(2a - t)(2a - t) = 0$

$a = \frac{t}{2}\quad b = \frac{3t}{2\sqrt{3}}$

3. a) $m_{AE} + \frac{3}{-2}$ $\quad m_{BE} = \frac{2}{3}$ $\quad \therefore \hat{E} = 90°$



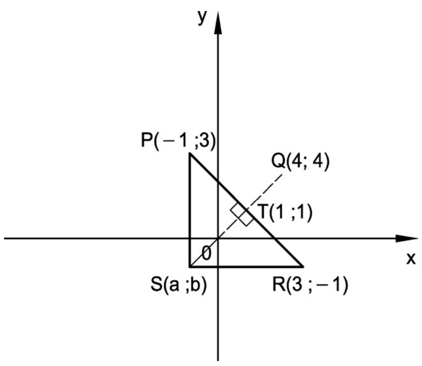
b) $2 = \frac{a + 0}{2}$ $\quad 1 = \frac{b + 4}{2}$

$\therefore a = 4$ $\quad b = -2$ $\quad$ C(4 ; –2)

$\frac{p + 5}{2} = 2$ $\quad \frac{q + 3}{2} = 1$

$p = -1$ $q = -1$ D(–1 ; –1)

4. a) T(1 ; 1)



b) T(1 ; 1) gradient PR = –1

gradient perp. bisector = 1

$y - 1 = x - 1\quad y = x$

If S is on the line $a = b$

c) $\frac{1}{2}$ PR × TS = 12

$\frac{1}{2}\sqrt{32}.\sqrt{2(1 - a)^2} = 12$

$32.2(1 - a)^2 = 24^2$

$(1 - a)^2 = 9$

$a^2 - 2a - 8 = 0$

$(a - 4)(a + 2) = 0$

$a = 4$ or $a = -2$

S(–2 ; –2)

d) Prove diagonals bisect at 90°

i) Mid-point QS is (1 ; 1)

Mid-point PR is T(1 ; 1)

ii) PR ⊥ QS

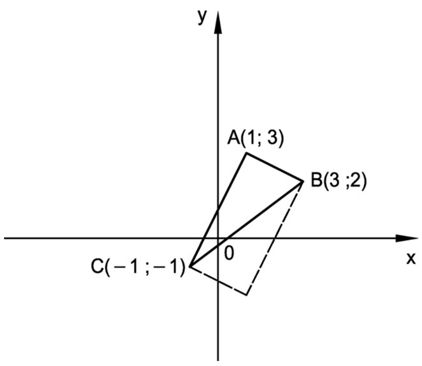
$\therefore$ PQRS is a rhombus

5. a) $a = \frac{-2+8}{2}$ $a = 3$

b) $d^2 = 25$

$e^2 + 16 = 25$

$e^2 = 9 \therefore e = \pm 3$



c) i) pt(–1 ; –1) $m_{AB} = -\frac{1}{2}$

Equation: $y + 1 = -\frac{1}{2}(x + 1)$

$2y + 2 = -x - 1$

$2y = -x - 3$

ii) Pt(3 ; 2) $m_{AB} = -\frac{1}{2}$

$m_{perp} = 2$

Equation : $y - 2 = 2(x - 3)$

$y = 2x - 4$

iii) $y = 2x - 4$

$2(2x - 4) = -x - 3$

$4x - 8 = -x - 3$

$5x = 5$

$x = 1$ $y = -2$

(1 ; –2)

6. a) $m_{AD} = m_{BC}$

$\frac{4}{-2} = \frac{6-y}{x-4}$

$-2(x - 4) = 6 - y$

$-2x + 8 = 6 - y$

$y = 2x - 2$

$BC^2 = 4AD^2$

$(x - 4)^2 + (6 - y)^2 = 4[4 + 16\}$

$(x - 4)^2 + (6 - y)^2 = 80$

Solve simultaneously.

$(x - 4)^2 + (6 - 2x + 2)^2 = 80$

$x^2 - 8x + 16 + 64 - 32x + 4x^2 = 80$

$5x^2 - 40x = 0$

$5x(x - 8) = 0$

$x = 0$ or $x = 8$ $(x < 4)$

$\therefore x = 0$ $y = -2$

b) A(–4 ; 3) B(0 ; 6)

$m = \frac{3}{4}$ eq. $y = \frac{3}{4}x + 6$

C(4 ; –2) D(–2 ; –1)

$m = -\frac{1}{6}$ equation $y + 1 = -\frac{1}{6}(x + 2)$

$y + 1 = -\frac{1}{6}x - \frac{1}{3}$

Solve simultaneously.

$\frac{3}{4}x + 6 + 1 = -\frac{1}{6}x - \frac{1}{3}$

$9x + 84 = -2x - 4$

$11x = -88$

$x = -8$ $y = 0$ (–8 ; 0)

7. a) 18,4°

b) Gradient BK is $\frac{8}{4} = 2$

$\therefore B\hat{K}X = 63{,}4°$

$\therefore T\hat{A}C = B\hat{A}C = 45°$ By geometry

c) Equation BAK is $y = 2x + 4$

$\therefore$ K(–2 ; 0) eq. KH is gradient $\frac{1}{\sqrt{3}}$

Equation : $y = \frac{1}{\sqrt{3}}(x + 2)$

$\therefore y = \frac{1}{\sqrt{3}}(8) = \frac{8}{\sqrt{3}}$

Lesson 31

1. Let AP = $5k$

$\therefore$ PB = $4k$

PQ || BC

$\therefore \frac{AP}{PB} = \frac{AQ}{QC} = \frac{5\text{ m}}{4\text{ m}}$

AQ = 5 m QC = 4 m

$\therefore$ 24 = 4 m

m = 6

$\therefore$ AQ = 30 cm

2. PR || BC

$\therefore \frac{AP}{PB} = \frac{AR}{RC}$ Line || one side of △

RQ || AB

$\therefore \frac{AR}{RC} = \frac{BQ}{QC}$ Line || one side of △

$\therefore \frac{AP}{PB} = \frac{BQ}{QC}$

3. i) $\frac{BS}{SE} = \frac{2}{1}$ S is the centroid

ii) In △BEF SF || EF

$\therefore \frac{BS}{SE} = \frac{BD}{DF} = \frac{2k}{k}$ Line || one side of △

BD = $2k$ and DF = k

But BD = DC AD is a median

$\therefore$ DC = $2k$ $\therefore$ FC = k

$\therefore \frac{BF}{FC} = \frac{3k}{k} = \frac{3}{1}$

4. In △PQM

NK || PM

$\therefore \frac{PN}{NQ} = \frac{MK}{KQ} = \frac{3\text{ m}}{5\text{ m}}$ Line || one side of △

let MK = 3 m and KQ = 5 m

$\therefore$ MQ = 8 m

But MQ = SM = 8 m Diagonals of a parallelogram

i) $\therefore \frac{KQ}{SQ} = \frac{5\text{ m}}{16\text{ m}} = \frac{5}{16}$

ii) $\therefore \frac{SM}{MK} = \frac{8\text{ m}}{3\text{ m}} = \frac{8}{3}$

iii) In △NSK TM || NK

$\frac{SM}{MK} = \frac{ST}{TN} = \frac{8t}{3t}$

ST = $8t$

8t = 32 $\therefore t = 4$

$\therefore$ SN = $11t$

$\therefore$ SN = 44 cm

5. a) In △ABC XY || BC

$\therefore \frac{AX}{XB} = \frac{AY}{YC}$ Line || one side of △

In △ACP: XZ || CP

$\therefore \frac{AY}{YC} = \frac{AZ}{ZP}$ Line || one side of △

$\therefore \frac{AX}{XB} = \frac{AZ}{ZP}$

$\therefore$ ZP || BP Sides in proportion

b) In △AMP KZ || MP

$\therefore \frac{AK}{KM} = \frac{AZ}{ZP}$ Line || one side of △

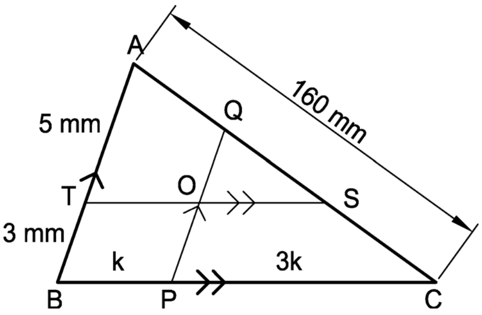
In △ACP YZ || CP

$\therefore \frac{AZ}{ZP} = \frac{AY}{YC}$ Line || one side of △

$\therefore \frac{AK}{KM} = \frac{AY}{YC}$

LC Maths G11 WB.indb 336 2008/08/09 08:28:45 AM

9. a) TS ∥ BC



$\therefore \frac{AS}{SC} = \frac{AT}{TB} = \frac{5t}{3t}$

∴ AC = 8t

160 = 8t

t = 20

∴ AS = 100

SC = 60

AB ∥ QP

$\therefore \frac{AQ}{AC} = \frac{BP}{BC} = \frac{1}{4} = \frac{m}{4m}$

4m = 160 ∴ m = 40

∴ AQ = 40 mm and QS = 60 mm

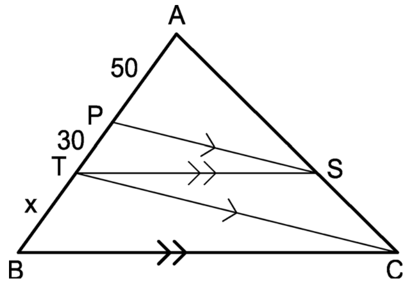
b) ∴ QS = SC = 60

In △QPC: QS = SC

OS ∥ PC

∴ QO = OP

10. PS ∥ TC



$\frac{AP}{PT} = \frac{AS}{SC} = \frac{5}{3}$

TS ∥ BC

$\frac{AT}{TB} = \frac{AS}{SC} = \frac{80}{x}$

$\therefore \frac{5}{3} = \frac{80}{x}$

∴ $5x = 240$

$x = 48$

Lesson 32

1. a) In △ABD and △AEC

$\hat{A}^1 = \hat{A}^2$ (given)

$\hat{B} = \hat{E}$ (given)

3rd angle = 3rd angle (angles in triangle)

∴ △ABD ||| △AEC

b) $\frac{AB}{AE} = \frac{AD}{AC}$

AB.AC = AD.AE

= AD(AD + DE)

= AD^2 + AD.DE

Now prove AD.DE = BD.DC

In △s ADB & DEC

$\hat{D}^1 = \hat{D}^2$ Vertically opposite angles

$\hat{B} = \hat{E}$ given

3rd angle = 3rd angle Angles in triangle

∴ △DBA ||| △DEC

$\therefore \frac{DB}{DE} = \frac{DA}{DC}$

∴ DB.DC = DA.DE

∴ AB.AC = AD2 + BD.DC

2. a) In $\triangle$s GDF and GED

$\frac{GD}{GE} = \frac{6}{9} = \frac{2}{3}$

$\frac{DF}{DE} = \frac{3}{4,5} = \frac{2}{3}$

$\frac{GF}{GD} = \frac{4}{6} = \frac{2}{3}$ the sides are in proportion

$\therefore \triangle GDF \,|||\, \triangle GED$

b) $\hat{G} = \hat{G}$

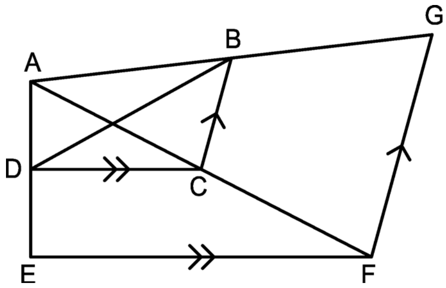
$F\hat{D}G = \hat{E}$

$D\hat{F}G = E\hat{D}G$

3. a) In $\triangle$AEF: $\frac{AC}{CF} = \frac{AD}{DE}$ (DC || EF)

$\frac{AC}{5} = \frac{12}{3}$

AC = 20cm



b) $\triangle ADV \,|||\, \triangle AEF$

$\therefore \frac{AD}{AE} = \frac{DC}{EF}$

$\therefore \frac{12}{15} = \frac{14}{EF}$

EF = 17,5 cm

c) AB = 14cm

In $\triangle$AGF: $\frac{BG}{AB} = \frac{CF}{AC}$ (BC || GF)

$\frac{BG}{14} = \frac{5}{20}$

BG = 3,5 cm

4. a) By Pythagoras QR = 52 cm

b) In $\triangle$s PQR and TSR

$\hat{R} = \hat{R}$ Common

$\hat{Q} = \hat{S}$ (90°)

$\therefore \hat{P} = \hat{T}$ Angles in triangle

$\therefore \triangle RPQ \,|||\, \triangle RST$

$\therefore \frac{RQ}{RS} = \frac{QP}{ST} = \frac{RP}{RT}$

$\therefore \frac{32}{20} = \frac{24}{ST} = \frac{40}{RT}$

$ST = \frac{24 \times 20}{32}$

= 15 cm

c) $RT = \frac{40 \times 20}{32} = 25$

$\therefore$ QT = 7 cm

5. In $\triangle$s ABD and BCD

$\hat{D} = \hat{D}$ (common)

$\hat{A} = \hat{B}$ (given)

$\therefore \hat{B} = \hat{C}$ Angles in triangle

$\therefore \triangle DAB \,|||\, DBC$

$\therefore \frac{AD}{DB} = \frac{AB}{BC} = \frac{BD}{CD}$

$\frac{P + 6}{15} = \frac{15}{6}$

$P = \frac{15 \times 15}{6} - 6$

P = 31,5

$\frac{AB}{5} = \frac{15}{6}$

$AB = 12\frac{1}{2}$

6. a) In $\triangle$s BCA and BCN

$\hat{C} = \hat{C}$ (common)

$\hat{B} = \hat{N}$ (given)

$\therefore \hat{A} = \hat{B}$ ($\angle$s in $\triangle$)

$\therefore \triangle CBA \parallel\mid \triangle CNB$

$\therefore \frac{CB}{CN} = \frac{AC}{CB}$

$\therefore BC^2 = AC.CN$

b) In $\triangle$s AMN and NCB

$\hat{M}^1 = \hat{N}$ ($\hat{M}1 = \hat{B}$ Corresponding angles MN $\parallel$ BC

$\hat{N}^1 = \hat{C}$ Corresponding angles MN $\parallel$ BC

$\therefore \hat{A} = \hat{B}$ Angles in triangle

$\therefore \triangle MNA \parallel \triangle NCB$

$\therefore \frac{MN}{NC} = \frac{AM}{NB}$

$\therefore MN.NB = AM.NC$

7. In $\triangle$s BMN and BAC

$\hat{B} = \hat{B}$ (common)

$\hat{M} = \hat{A}$ Corresponding angles AC $\parallel$ MN

$\hat{N} = \hat{C}$ Corresponding angles AC $\parallel$ MN

$\therefore \triangle BMN \parallel\mid \triangle BAC$

$\frac{BM}{BA} = \frac{MN}{AC} = \frac{BN}{BC}$

$\frac{6}{18} = \frac{7,5}{AC}$

$AC = 22,5$

$\frac{x}{x+9} = \frac{1}{3}$

$2x = x + 9$

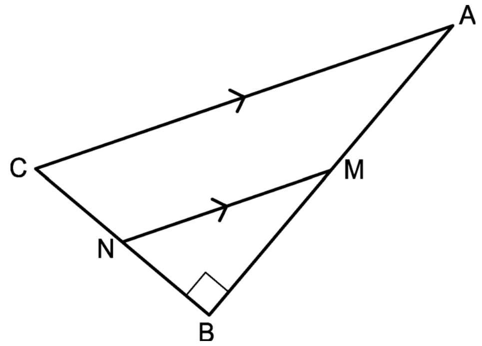
$x = 4,5$

$AC^2 = AB^2 + BC^2$

$\therefore$ The $\triangle$ is right angled

(Interesting – the diagram is not an accurate sketch but it is only a sketch and this often happens in mathematics.)

Possibly should look like this.



8. $\frac{BC}{AC} = \frac{CD}{AD}$

$\frac{BC}{AC} = \frac{BD}{CD}$

$\therefore \frac{BC}{AC} = \frac{CD}{AD} = \frac{BD}{CD}$

$\triangle BCD$ is similar to $\triangle ACD$

$\therefore \hat{D}^2 = \hat{D}^1$

$\hat{B} = \hat{C}_1 = x$

$\hat{C}_2 = \hat{A} = y$

$\therefore 2x + 2y = 180°$

$x + y = 90°$

$\therefore \hat{C} = 90°$

By Pythagoras

$AB^2 = AC^2 + BC^2$

Lesson 33

1. $PQ^2 = QS.QR$ (perp. from right < of $\triangle$ to hyp.)

$PR^2 = RS.RQ$ (perp. from right < of $\triangle$ to hyp.)

$\therefore \frac{PQ^2}{PR^2} = \frac{QS.QR}{RS.RQ} = \frac{QS}{RS}$

2. i) $RT \times RP$ (perp. from right < to hyp.)

ii) $QT^2 + TR^2$ (Pythagoras)

iii) $PR^2 - PQ^2$ (Pythagoras)

iv) $PT \times PR$ (perp. from right < to hyp.)

v) $QT^2 + PT^2$ (Pythagoras)

vi) $PR^2 - QR^2$ (Pythagoras)

vii) $TR \times TP$ (perp. from right < to hyp.)

3. In $\triangle DAC$: $DB^2 = BA.BC$ (perp. from right-angle to hypotenuse)

$\therefore 4 = BA.4$

$\therefore BA = 1$

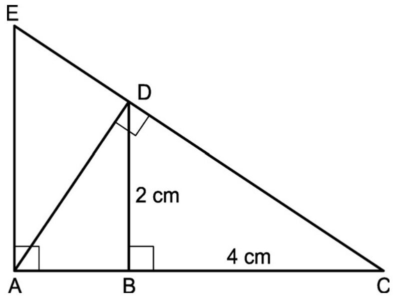
$BD \parallel AE$ ((B)1 = (A) (angles)

$\therefore \triangle CDB\ ///\ \triangle CEA$

$\therefore \frac{DB}{AE} = \frac{CB}{AC}$

$\therefore \frac{2}{AE} = \frac{4}{5}$

$\therefore AE = \frac{10}{4} = \frac{5}{2} = 2\frac{1}{2}$ units



4. $DK = x \qquad KC = 3x$

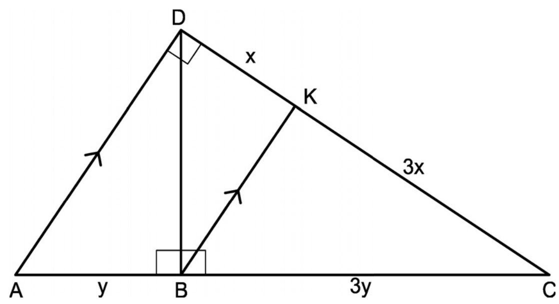
$\frac{DK}{KC} = \frac{AB}{BC} = \frac{1}{3}$

$AB = y \qquad BC = 3y$

$DA^2 = AB.AC$ (perp. from right < to hyp.)

$DA^2 = y.4y$

$DA2 = 4y^2$



$DA = 2y$

$2DA = 4y$

5. $\frac{TS}{SR} = \frac{PQ}{QR} = \frac{1}{4}$ (PT ∥ SQ)

$PQ = y$ $QR = 4y$

In △ TPR

$TP^2 = PQ.PR$ (perp. from right < to hyp.)

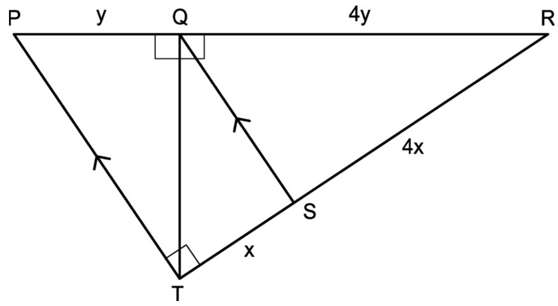
$TP^2 = y.5y$

$= 5y^2$

$TP = \sqrt{5y}$

$4TP = \sqrt{5}\ 4y$

$4TP = \sqrt{5}\ QR$



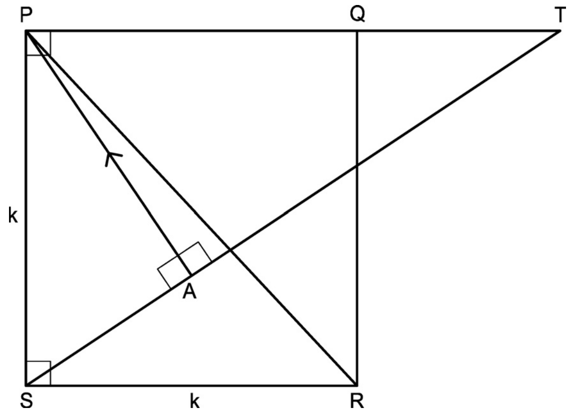
6. let $PS = k$ ∴ $SR = k$ (square)

i) ∴ $PR^2 = k^2 + k^2$ (Pythagoras)

∴ $PR = R^2 K$

∴ $PR = R^2 PS$

ii) In △ PTS: $PT^2 = TA\cdot TS$ (perp. from right < to hyp.)



∴ $PR^2 = TA\cdot TS$ (PT = PR)

∴ $2PS^2 = TA\cdot TS$ ($PR = R^2 PS$ from (i)

∴ $PS^2 = \frac{TA\cdot TS}{2}$

but $PS^2 = SA.ST$ (perp. from right < to hyp.)

∴ $AS\cdot TS = \frac{AT\cdot TS}{2}$

∴ $2AS\cdot TS = AT\cdot TS$

∴ $2AS = TA$

7. i) $BC^2 = BD\cdot BA$ (perp. from right < to hyp.)

$AC^2 = AD\cdot AB$ (perp. from right < to hyp.)

∴ $\frac{BC^2}{AC^2} = \frac{BD\cdot BA}{AD\cdot BA}$

$= \frac{BD}{AD}$

ii) $\frac{BC^4}{AC^4} = \frac{BD^2}{AD^2}$ from above

$BD^2 = BE \cdot BC$ (perp. from right < to hyp.)

$AD^2 = AF \cdot AC$ (perp. from right < to hyp.)

$\therefore \frac{BC^4}{AC^4} = \frac{BE \cdot BC}{AF \cdot AC}$

$\therefore \frac{BC^3}{AC^3} = \frac{BE}{AF}$

Lesson 34

Activity 1

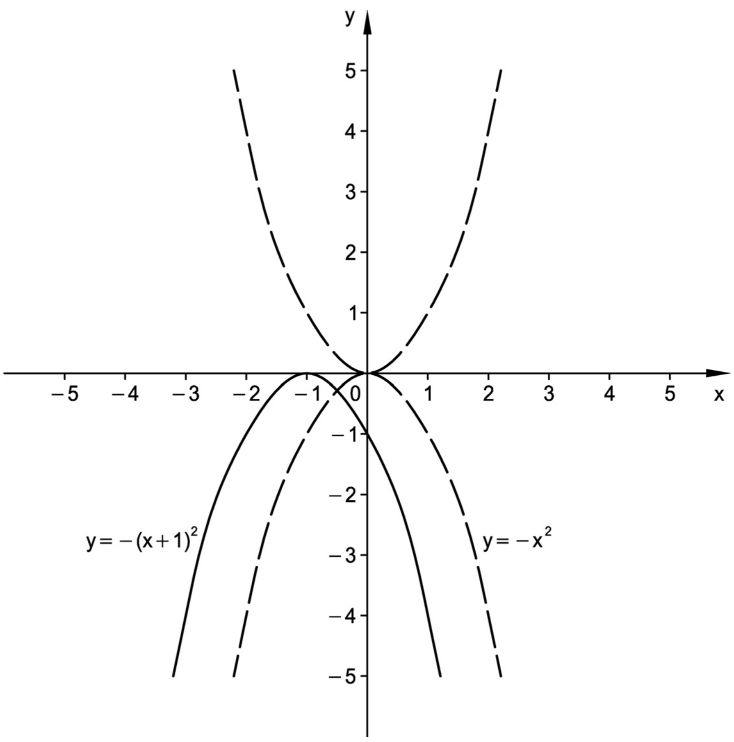
1. a) $y = 2(x + 3)^2$

$y = 2x^2$ is translated 3 units to the left horizontally.

b) $y = -3(x - 4)^2$

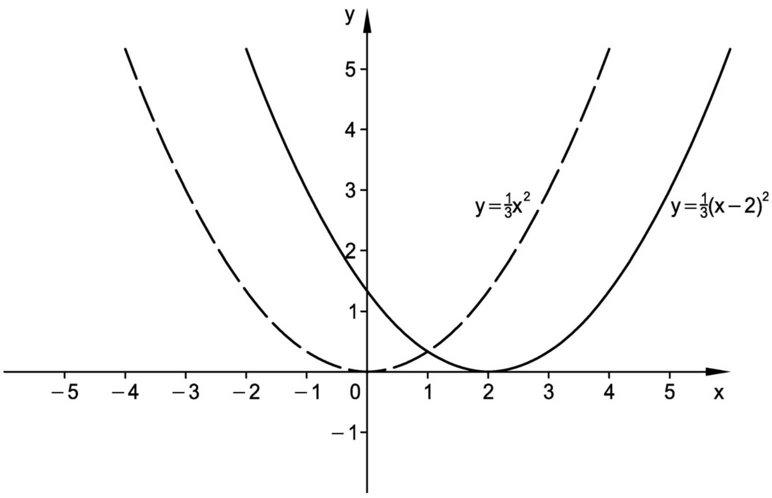
$y = 3x^2$ is reflected across the x-axis and translated 4 units to the right horizontally.

2. a)

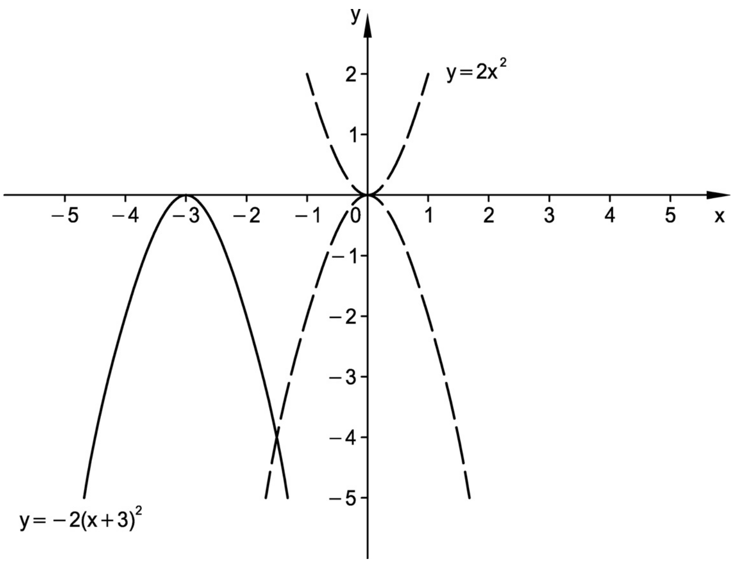


$y = x^2$ is reflected across the x–axis and translated 1 unit to the left horizontally.

b)



c)



$y = 2x^2$ is reflected across the x–axis and translated 3 units to the left.

Activity 2

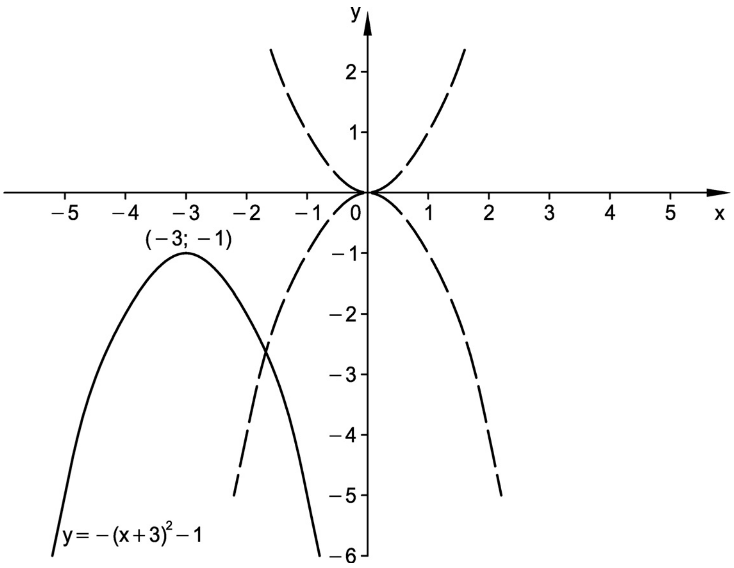
1. a) $y = 2(x - 2)^2 - 1$

   $y = 2x^2$ is translated 1 unit down and 2 units to the right horizontally.

   b) $y = -\frac{1}{3}(x - 3)^2 - 1$

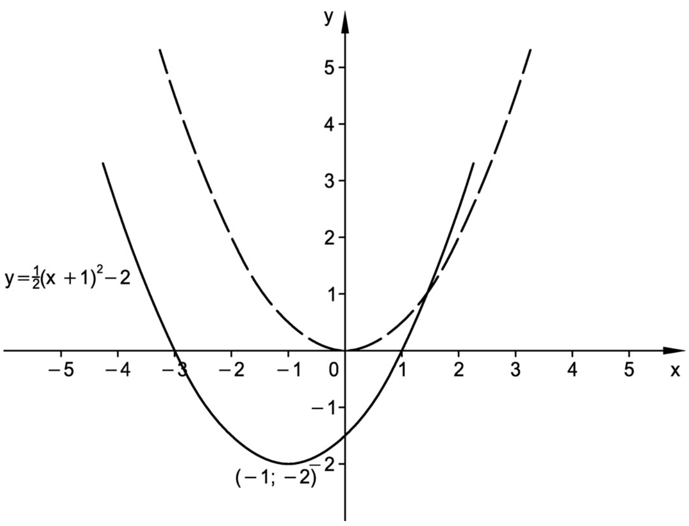
   $y = ax^2$ is reflected across the x–axis, translated 1 unit down and 3 units to the right.

2. a)



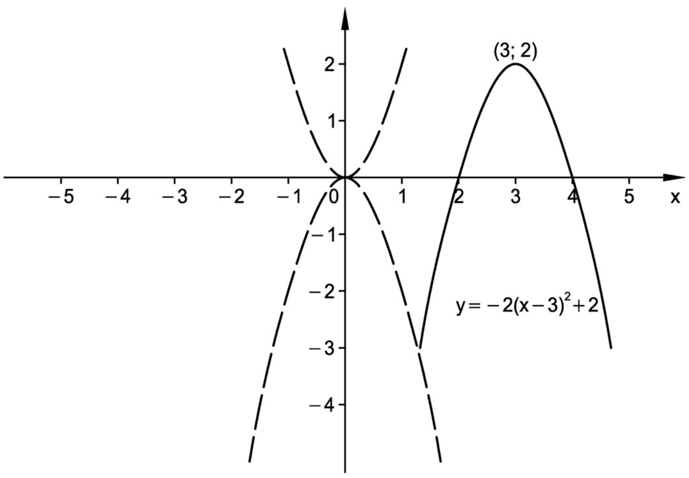
$y = x^2$ is reflected across the x–axis, translated 3 units to the left and 1 unit down.

b)



$y = \frac{1}{2}x^2$ is translated 1 unit to the left and two units down.

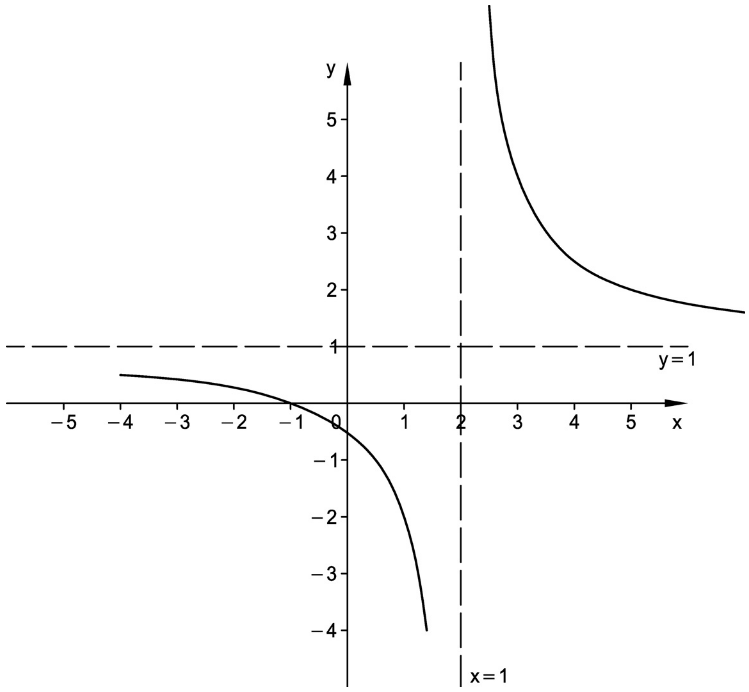
c)



$y = 2x^2$ is reflected across the x–axis, translated 3 units to the right and up 2 units.

Activity 3

1. a)

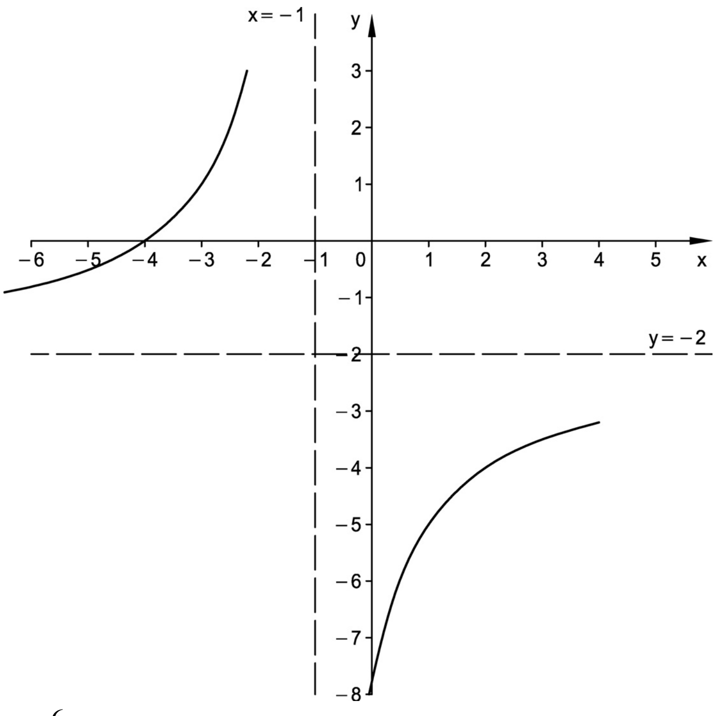


x int. $(-1\ ;\ 0)$

y int. $\left(0\ ;\ -\frac{1}{2}\right)$

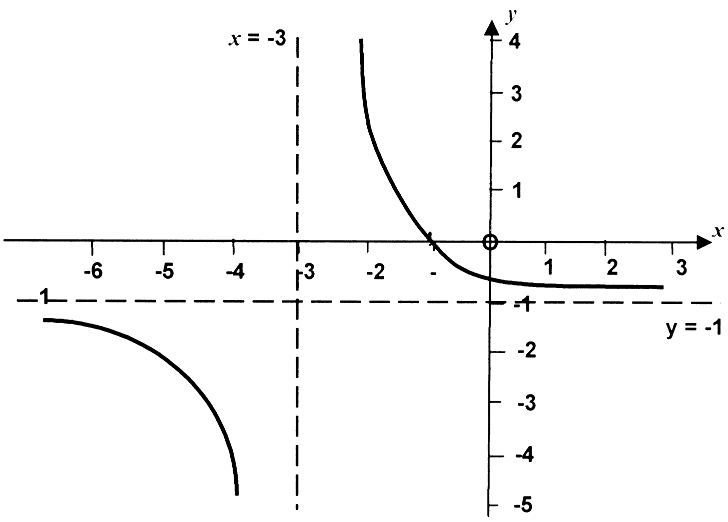
$y = \frac{3}{x}$ is translated 2 units to the right and 1 unit up.

b)



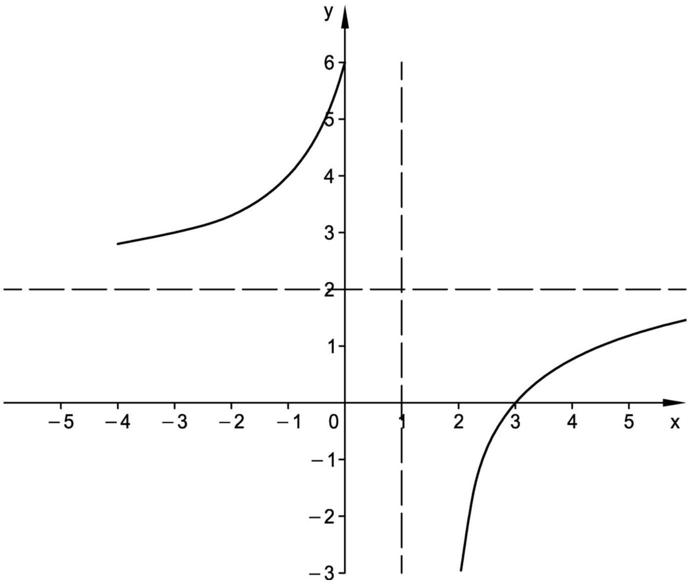
$y = \frac{6}{x}$ is reflected across the y–axis, translated 1 unit to the left and 2 units down.

c)



y int. = $\left(0\ ;-\frac{1}{3}\right)$

d)

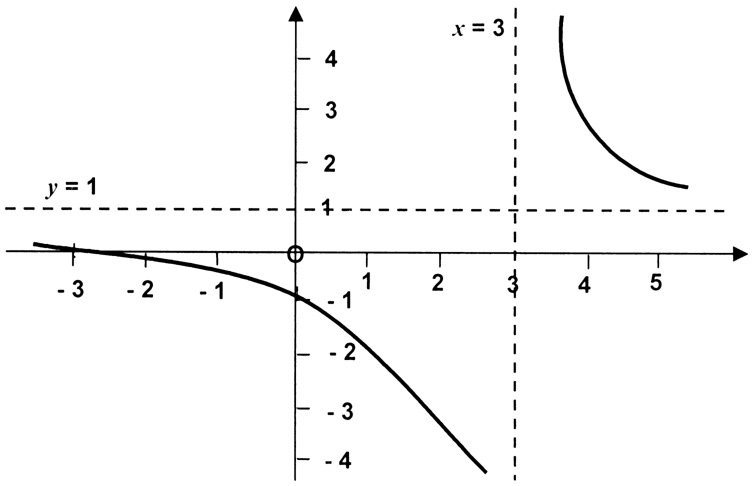


x int. (3 ; 0)

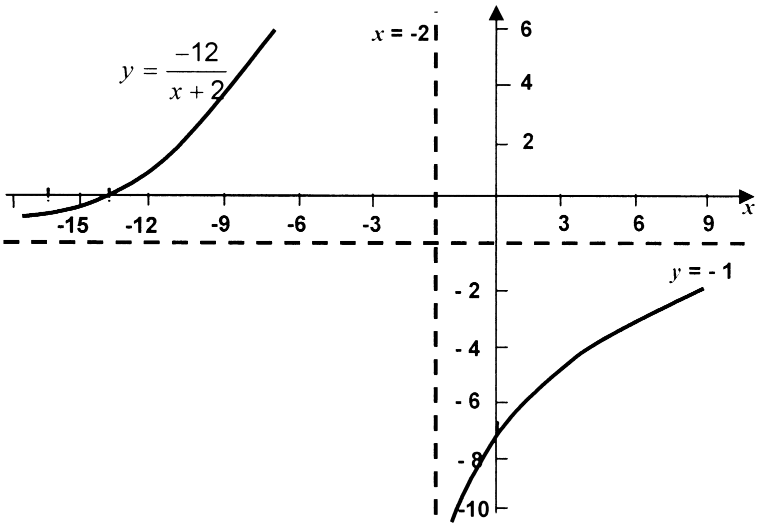
y int. (0 ; 0)

$y = \frac{4}{x}$ is reflected across the y–axis, translated right 1 unit and up 2 units.

2.



3.



$y = \frac{-12}{x+2}$

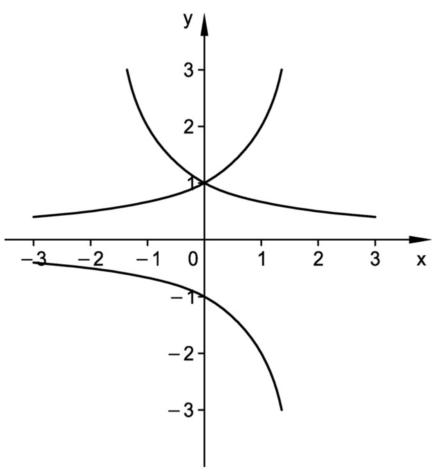
x int. $(-14 ; 0)$

y int. $(0 ; -7)$

Lesson 35

Activity 1

a)



Lesson 36

Activity 1

1. (a) $y - a(x-2)^2 + 1$ pt $(0 ; 7)$

$7 = a(4) + 1$

$6 = 4a$

$a = \frac{3}{2}$ $y = \frac{3}{2}(x-2)^2 + 1$

(b) $y = a(x+2)^2 + 8$ pt $(0 ; -8)$

$-8 = 4a + 8$

$4a = -16$

$a = -4$

$y = -4(x+2)^2 + 8$

(c) $y = a(x+1)^2 - 3$ pt $(1 ; 9)$

$9 - 4a - 3$

$4a = 12$

$a = 3$ $y = 3(x+1)^2 - 3$

(d) $y = a(x+2)^2 + 2$ pt $(0 ; 0)$

$0 = 4a + 2$

$a = -\frac{1}{2} y = -\frac{1}{2}(x+2)^2 + 2$

2. (a) $y = a(x-1)^2 - 1$ pt $(3 ; -5)$

$-5 = a(4) - 1$

$4a = -4$

$a = -1$

$y = -(x-1)^2 - 1$

(b) $y = -(x+3)^2 + 3$

Activity 2

1. $y = \frac{k}{x+1} + 2$ pt $(-2 ; 4)$

$4 = -k + 2$

$k = -2$

$y = \frac{-2}{x+1} + 2$

2. $y = \frac{k}{x-2} + 1$ pt $(6 ; 0)$

$0 = \frac{k}{4} + 1$

$k = -4$

$y = \frac{-4}{x-2}$ y int $(0 ; 3)$

3. $y = \frac{k}{x-3} - 1$ pt $(4 ; 3)$

$3 = k - 1$

$k = 4$

$y = \frac{4}{x-3} - 1$ x int $(7 ; 0)$

y int $\left(0 ; -\frac{7}{3}\right)$

4. $y = \frac{k}{x+2} + 4$ pt (2 ; 3_

$3 = \frac{k}{4} + 4$

$k = -4$

$y = \frac{-4}{x+2} + 4$ x int $(-1 ; 0)$

y int $(0 ; 2)$

Activity 3

1. pt $\left(0 ; \frac{1}{4}\right)$

$\frac{1}{4} = 2 - p$

$2 - 2 = 2 - p$

$\therefore p = 2$

$y = 2x - 2$

2. $y = 2^{x+p} + 3$ pt $(1 ; 4)$

$4 = 21 + p + 3$

$2(0) = 21 + p$

$p = -1$

$y = 2\text{x} - 1 + 3$

y int $\left(0 ; 3\frac{1}{2}\right)$

3. $y = 3^{x+p} + 1$ pt (1 ; 10)

$10 = 31 + p + 1$

$32 = 31 + p$

$p = 1$

$y = 3x + 1 + 1$

y int. (0 ; 4)

4. pt. (–1 ; 8) is on the graph so

$8 = a.b–1$

$\therefore\ 8 = \frac{a}{b}$

pt (0 ; 2) is on the graph so

$2 = a.b0$

$\therefore\ a = 2$

$\therefore\ 8 = \frac{2}{b}$

$\therefore\ b = \frac{1}{4}$

$y = 2.\left(\frac{1}{4}\right)^{x}$

Lesson 37

Activity 1

1. Shape 

$\frac{y}{2} = x^2 - \frac{7}{2}x + \left(\frac{7}{4}\right)^2 - \frac{49}{16} - 2$

$y = 2\left(x - \frac{7}{4}\right)^2 - 8\frac{1}{8}$

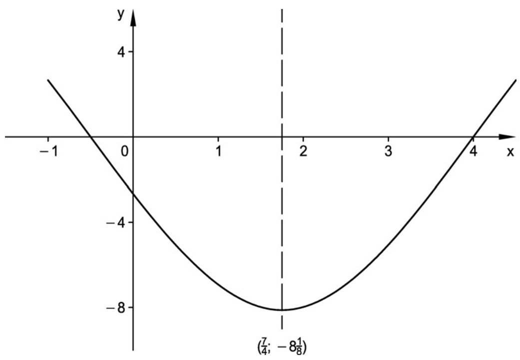
Turning points $\left(\frac{7}{4} ; -8\frac{1}{8}\right)$

y-intercepts (0 ; –4)

x-intercepts $0 = 2x^2 - 7x - 4$

$0 = (2x + 1)(x - 4)$

$\left(-\frac{1}{2} ; 0\right)$ (4 ; 0)



2. Shape 

$-y = x^2 - 8x + (-4)^2 - 16 + 12$

$y = -(x - 4)^2 + 4$

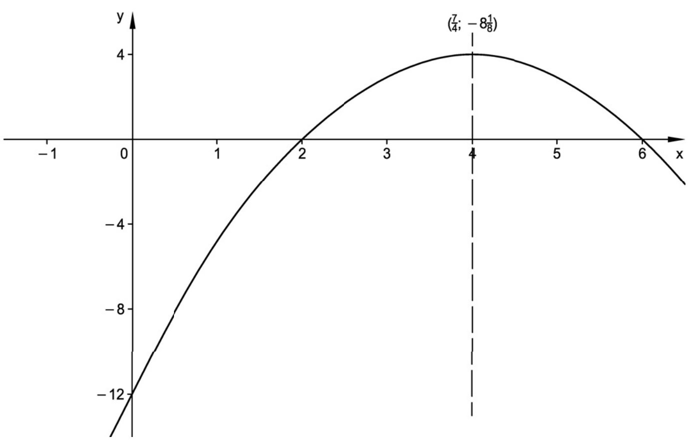
Turning points (4 ; 4)

y-intercepts (0 ; –12)

x-intercepts $0 = 2x^2 - 8x + 12$

$0 = (x - 6)(x - 2)$

(6 ; 0) (2 ; 0)



3. Shape 

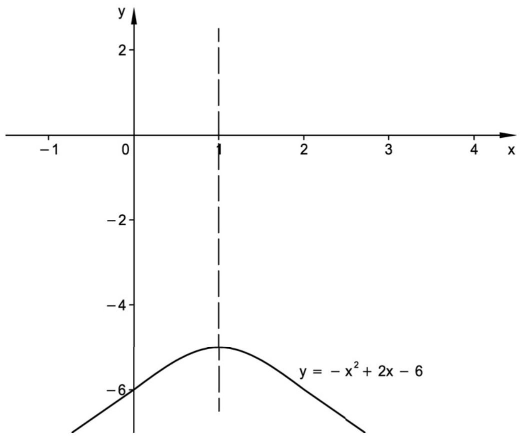
$-y = x^2 - 2x + (-1)^2 - 1 + 6$

$y = -(x - 1)^2 - 5$

Turning points $(1 ; -5)$

y-intercepts $(0 ; -6)$

no x-intercepts



Activity 2

1. (NB: If you prefer drawing the parabola by completing the square – do so.)

a) $y = -x^2 + 3x - 2$ Shape 

Axis of symmetry $x = \frac{-b}{2a}$

$x = \frac{-3}{-2} = \frac{3}{2}$

Maximum value $y = -\left(\frac{9}{4}\right) + \frac{9}{2} - \frac{2}{1}$

$y = \frac{-9 + 18 - 8}{4} = \frac{1}{4}$

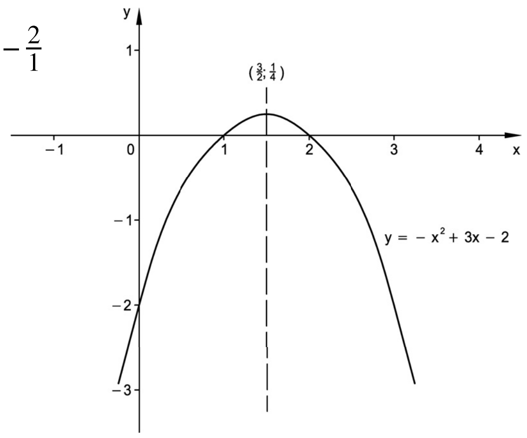
Turning points $\left(\frac{3}{2} ; \frac{1}{4}\right)$

y-intercepts $(0 ; -2)$

x-intercepts $0 = x^2 - 3x + 2$

$0 = (x - 2)(x - 1)$

$(2 ; 0)$ $(1 ; 0)$



b) $y = 2x^2 - 5x - 3$ Shape 

Axis of symmetry $x = \frac{5}{4}$

Minimum value $y = 2\left(\frac{25}{16}\right) - \left(\frac{5}{4}\right) - \frac{3}{1}$

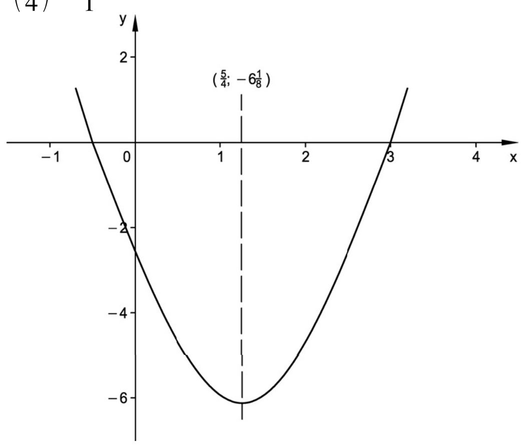
$y = \frac{50 - 100 - 48}{16}$

$= -\frac{98}{16}$

Turning points $\left(\frac{5}{4} ; -6\,\frac{1}{8}\right)$

y-intercepts $(0 ; -3)$

x-intercepts $\left(0 ; -\frac{1}{2}\right)$ $(3 ; 0)$



c) $y = -2x^2 + 5x$

$$\frac{y}{-2} = x^2 - \frac{5}{2}x + \left(-\frac{5}{4}\right)^2 - \frac{25}{16}$$
$$y = -2\left(x - \frac{5}{4}\right)^2 + \frac{25}{8}$$

Turning points $\left(\frac{5}{4}\,;-3\frac{1}{8}\right)$ Shape 

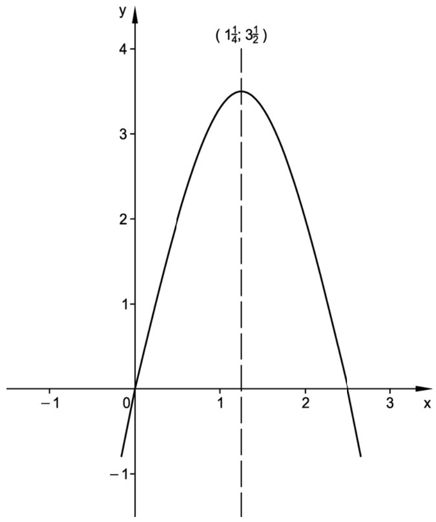
y-intercepts (0 ; 0)

x-intercepts $2x^2 - 5x = 0$

$x(2x - 5) = 0$

$x = 0$ or $x = \frac{5}{2}$

$(0\,;0)\ \left(2\frac{1}{2}\,;0\right)$



2. a) $y = a(x - 3)^2 - 1$

$2 = a(9) - 1$

$3 = 9a$

$\therefore\ a = \frac{1}{3}$

$y = \frac{1}{3}(x - 3)^2 - 1$

b) $y = a(x + 1)^2 + 8$

$7 = a(1)^2 + 8$

$a = -1$

$y = -(x + 2)^2 + 8$

c) $y = a(x + 2)^2 + 18$

$0 = a(3)^2 + 18$

$-18 = 9a$

$\therefore\ a = -2$

$y = -2(x - 2)^2 + 18$

3. a) $-x^2 - 6x + 7 = 0$

$x^2 + 6x - 7 = 0$

$(x + 7)(x - 1) = 0$

A(–7 ; 0)

b(1 ; 0) C(0 ; 7)

$y = -x^2 - 6x + 7$

$\frac{y}{-1} = x^2 + 6x + (5)^2 - 9 - 7$

$y = -(x + 3)^2 + 16$

D(–3 ; 16)

b) $-x^2 - 6x + k = 0$ has no x-intercepts if the original graph is moved 16 units or more down. k is the y–intercept. It was 7 so it moves 16 down, i.e. $k < -9$

c) $-x^2 - 6x + k = 0$ has two negative roots if $-9 \le k < 0$

4. a) $y = 2p^2x^2 - 2px + 1$

$$\frac{y}{2p^2} = x^2 - \frac{1}{p}x + \frac{1}{2p^2}$$
$$\frac{y}{2p^2} = x^2 - \frac{1}{p}x + \left(\frac{1}{2p}\right)^2 - \frac{1}{42p^2} + \frac{1}{2p^2}$$
$$\frac{y}{2p^2} = \left(x - \frac{1}{2p}\right)^2 + \frac{2 - 1}{4p^2}$$

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$y = 2p^2\left(x - \frac{1}{2p}\right)^2 + \frac{1}{2p^2}$

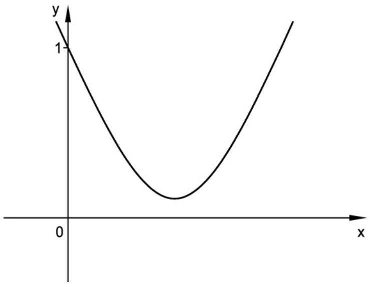
$2p^2 = \left(x - \frac{1}{2p}\right)^2 \geq 0$

$\therefore\ 2p^2\left(x - \frac{1}{2p}\right) + \frac{1}{2p^2} > 0$

Shape $\cup$ Turning points $\left(\frac{1}{2p}\,;\,\frac{1}{2p^2}\right)$

Always positive

$\therefore$ no x-intercepts



b) $\left(\frac{1}{2p}\,;\,\frac{1}{2p^2}\right)$

Lesson 38

1. a) $2x^2 - 4x - 16 = 0$

$x^2 - 2x - 8 = 0$

$(x - 4)(x + 2) = 0$

A(–2; 0) B(4; 0) C(0; –16) D(1; –18) E(2; –16)

b) $m_{AE} = \frac{16}{-4} = -4$

$y = -4(x + 2)$

$y = -4x - 8$ $m = -4$ $k = -8$

c) PQ = top – bottom

$= 2x^2 - 4x - 16 + 4x + 8$

$= 2x^2 - 8$

Substitute $x = -3$

PQ = 10 units

d) $\text{KL} = -4x - 8 - 2x^2 + 4x + 16$

$= -2x^2 - 8$

Maximum is 8 at $x = 0$

2. a) $y = a(x - 1)^2 + 1$

Substitute (0; –3)

$-3 = a(1) + 1$

$a = -4$

$y = -4(x - 1)^2 + 1$ or $y = -4x^2 + 8x - 3$

at C $y = 0$ $\quad 0 = 4x^2 - 8x + 3$

$0 = (2x - 1)(2x - 3)$

$x = \frac{1}{2}$ or $x = \frac{3}{2}$

$\text{C}\left(\frac{3}{2}; 0\right)$ T(0; –3)

$m = \frac{-3}{-\frac{3}{2}} = 2 \quad y = 2x - 3$

$f(x) = -4x^2 + 8x - 3$

$g(x) = 2x - 3$

b) $\frac{1}{2} < x < \frac{3}{2}$

c) $0 < x < \frac{3}{2}$

d) Subst. $x = \frac{1}{2}$ into $y = +2x - 3$

$= 1 - 3$

$= |-2|$

AD = 2 units

e) $PQ = -4x^2 + 8x - 3 - 2x + 3$

$= -4x^2 + 6x$

$\frac{PQ}{-4} = x^2 - \frac{3}{2}x + \left(\frac{3}{4}\right)^2 - \frac{9}{16}$

$PQ = -4\left(x - \frac{3}{4}\right)^2 + \frac{9}{4}$ maximum $2\frac{1}{4}$

3. Shape $\cup$ ax of sym $= \frac{2}{2} = 1$

minimum value $y = -4$ turning point $(1; -4)$

y int. $(0; -3)$

$x^2 - 2x - 3 = 0$

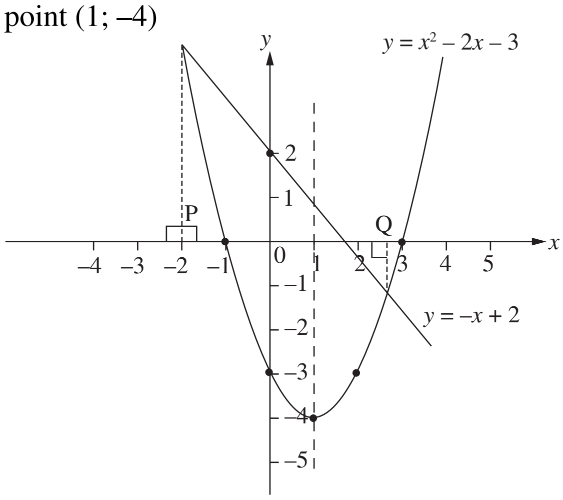
$(x - 3)(x + 1) = 0$

$(3; 0)(-1; 0)$

$x^2 - x - 5 = 0$

$x^2 - 2x - 3 = -x + 2$

draw $y = -x + 2$



4. a) $x = 2 \quad y = -1$

b) A(0; 1) B(–2; 0)

c) D(–2; 5) A(0; 1)

$m = \frac{4}{-2} = -2 \qquad y = -2x + 1$

$m = -2 \qquad k = 1$

d) $-2x + 1 = \frac{-4}{x - 2} - 1$

$-2x + 2 = \frac{4}{x - 2}$

$-2x(2x - 2) + 2(x - 2) = -4$

$-2x^2 + 4x + 2x - 4 = -4$

$0 = 2x^2 - 6x$

$0 = x^2 - 3x$

$0 = x(x - 3)$

C (3; –5)

e) D(–2; 5) C(3; –5)

$DC = \sqrt{25 + 100} = 5\sqrt{5}$

f) $y = -(x - 2) - 1$

$y = -x + 1$

5. a) $y = k^x$ $\quad 2 = \frac{a}{-1}$

$2 = k^{-1}$ $\quad a = -2$

$k = \frac{1}{2}$

$f(x) = \left(\frac{1}{2}\right)^x$ $\quad g(x) = \frac{-2}{x}$

b) A(0; 1) c) $y = -\left(\frac{1}{2}\right)^x$

d) $y = 2^x$ e) $y = \frac{2}{x}$

f) $y = \frac{2}{x}$ g) $h(x) = \frac{-2}{(x + 1)} - 2$

h) $y = -(x + 1) - 2$

$y = -x - 3$

6. a) $x^2 - 2x - 3 = 0$

$(x - 3)(x + 1) = 0$

B(–1; 0) $\quad \therefore$ AB = 3

b) $PQ = x + 4 + x^2 - 2x - 3$

$7 = x^2 - x + 1$

$0 = x^2 - x - 6$

$0 = (x - 3)(x + 2)$

$x = 3$ or $x = -2$

OR = 2 units

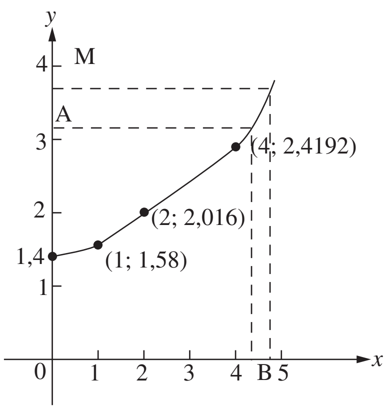
c) $CD = x^2 - x + 1$

$CD = x^2 - x + \left(\frac{1}{2}\right)^2 - \frac{1}{4} + 1$

$CD = \left(x - \frac{1}{2}\right)^2 + \frac{3}{4}$

minimum value $\frac{3}{4}$ at $x = \frac{1}{2}$

7. a)



b) At A

c) 3,18 kg

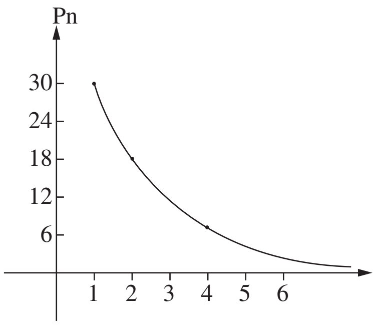
d) At B ≅ 4,9 months

e) $M = 1,8(1,2)^x$

8. a) Sequence

$36; 18; 9; \frac{9}{2}$

b)



c) The triangle will get very very small.

9. a) $\frac{4}{x-4} = -2$

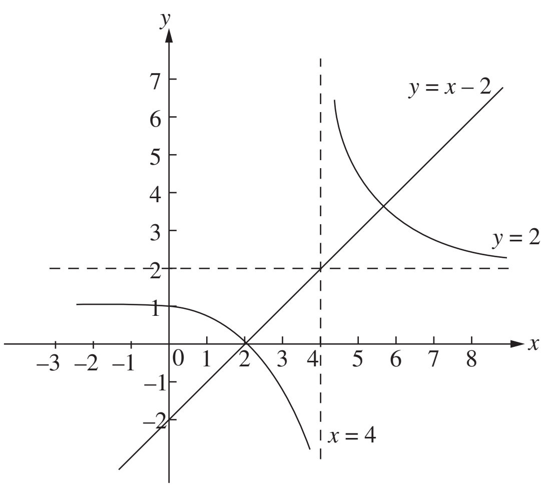
$4 = -2x + 8$

$2x = 4 \qquad \therefore x = 2$

b) $p = -1 + 2$

$p = 1$

c)



d) $y \in \Re - \{2\}$

e) $y = (x - 4) + 2$

$y = x - 2$

10. a) 2 in 6 hours (1 in 12 hours)

4 in 3 hours

b) $\frac{12}{0,4} = 5$

c)

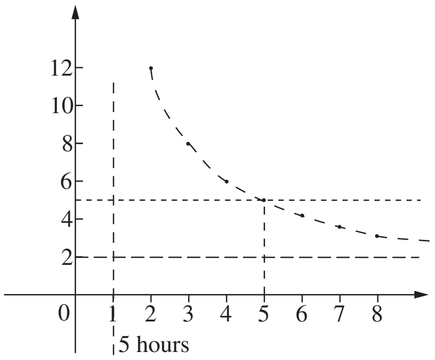
x	1	2	4	5
y (hrs)	12	6	3	2,4

d) $y = \frac{12}{x}$

e) $y = \frac{12}{x} + 2$

f) $y = \frac{12}{(x-1)} + 2$

g) Translated up 2 and to the right 1



i) On graph

Lesson 39

1. S = {a; b; c; d; e; f; g; h; i; j}

(i) $\frac{5}{10} = \frac{1}{2}$ (ii) $\frac{1}{5}$ $n(\text{S}) = 10$

(iii) $\frac{1}{10}$ (iv) $\frac{7}{10}$

(v) $\frac{3}{10}$ (vi) $\frac{1}{10}$

2. Let Event A: additional mathematics pupils
M: mathematics pupils
W: science pupils

$n(\text{S}) = 126$

a) $\frac{4}{126} = \frac{2}{63}$ b) $\frac{3}{31}$ c) $\frac{7}{63}$ d) $\frac{25}{63}$ e) $\frac{122}{126} = \frac{61}{63}$

3. a) 0,1 b) 0,2 c) 0,3 d) 0,9

4. a) 0,45 b) 0,15 c) 0,6 d) 0,7 e) 0,7
f) 0,75 g) 0,3

5. $0{,}48 - x + x + 0{,}5 - x = 0{,}72$

$0{,}98 - x = 0{,}72$

$0{,}26 = x$

$P(A \cap B) = 0{,}26$

6. $\frac{5}{8} - x + x + \frac{5}{6} - x = \frac{15}{16}$

$30 - 48x + 48x + 40 - 48x = 45$

$70 - 48x = 45$

$25 = 48x$

$x = \frac{25}{48}$

a) $\frac{25}{48}$ b) $\frac{5}{6} - \frac{25}{48} = \frac{40-25}{48} = \frac{15}{48}$ c) $\frac{1}{16}$

7. $\frac{16}{52} = \frac{4}{13}$

8. $0{,}7 - x + x + 0{,}3 - x = 0{,}8$

   $1 - x = 0{,}8$

   $0{,}2 = x$

   a) 0,2 b) 0,6 c) 0,2

9. Draw and put the intersection in first.

   $0{,}5 + 0{,}3 - x + 0{,}4 = 1$

   $-x = 1 - 1{,}2$

   $x = 0{,}2$

   (i) 0,2

   (ii) 0,3

10. $19 + 17 - x + 14 + 7 + 11 + 8 + 12 = 80$

    $-x = -8$

    $x = 8$

    a) 12

    b) 8

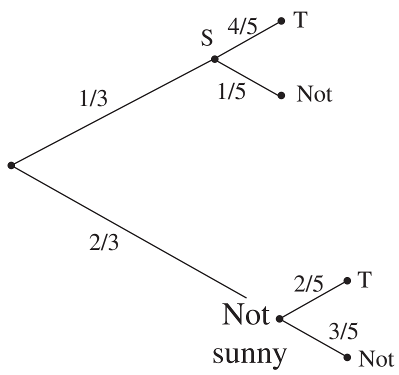
    c) $\frac{40}{80} = \frac{1}{2}$

Lesson 40

1. P(Jenny plays tennis)

   $= \left(\frac{1}{3}\right)\left(\frac{4}{5}\right) + \left(\frac{2}{3}\right)\left(\frac{2}{5}\right)$

   $= \frac{4}{15} + \frac{4}{15} = \frac{8}{15}$

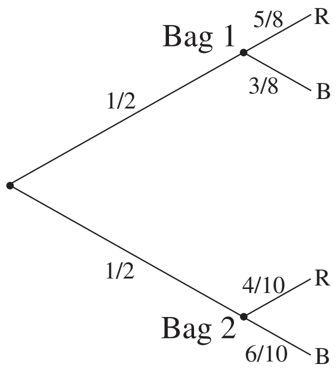


2. P(Blue)

$= \left(\frac{1}{2}\right)\left(\frac{3}{8}\right) + \left(\frac{1}{2}\right)\left(\frac{3}{5}\right)$

$= \frac{3}{16} + \frac{3}{10}$

$= \frac{15 + 24}{80} = \frac{39}{80}$



3. a) P(accident)

$= (0{,}25)(0{,}02) + (0{,}75)(0{,}004)$

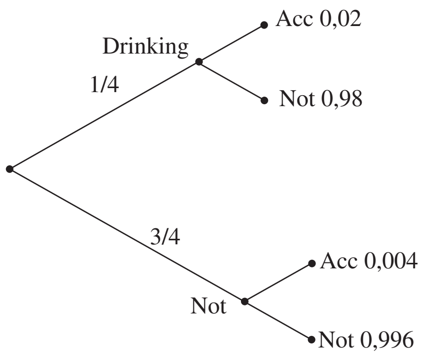
$= 0{,}008$

b) Not really on the syllabus but interesting.

$P(\text{drunk}) = \frac{P(\text{Drinking and Accident})}{P(\text{accident})}$

$= \frac{(0{,}25)(0{,}02)}{0{,}008}$

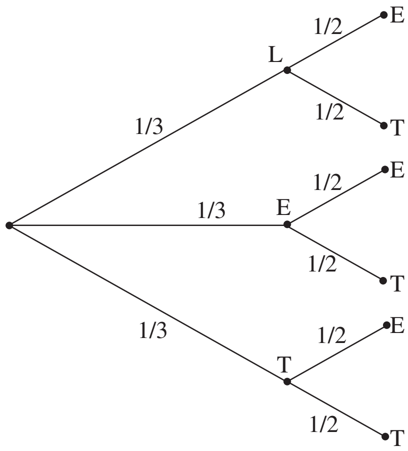
$= 0{,}625$



4. a) $\left(\frac{1}{3}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{3}\right)\left(\frac{1}{2}\right) = \frac{1}{3}$

b) $\left(\frac{1}{3}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{3}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{3}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{3}\right)\left(\frac{1}{2}\right)$

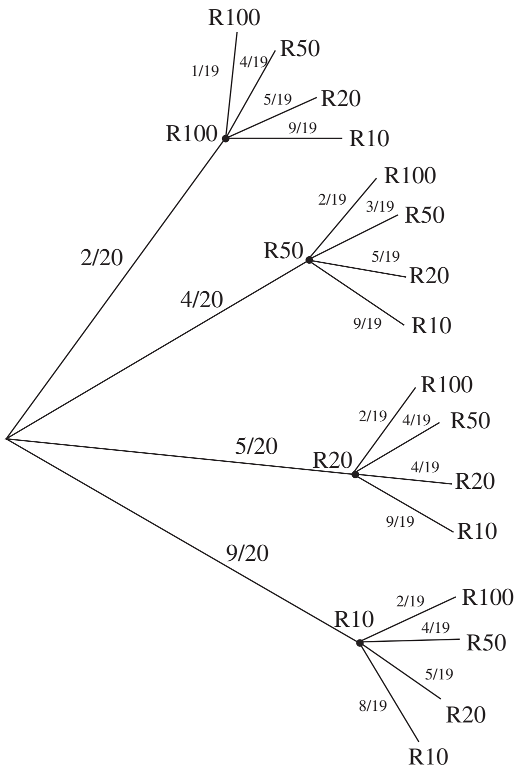
$= \frac{2}{3}$



5. a) $\frac{4}{20} \times \frac{5}{19}$

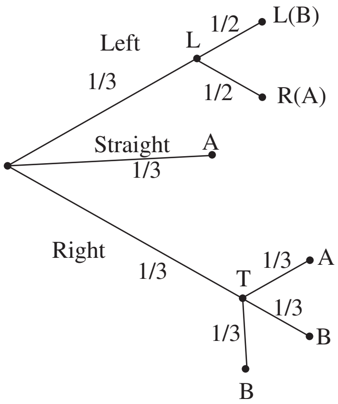
$= \frac{1}{19}$

b) $\left(\frac{4}{20}\right)\left(\frac{5}{19}\right) + \left(\frac{4}{20}\right)\left(\frac{9}{19}\right) + \left(\frac{5}{20}\right)\left(\frac{4}{19}\right) + \left(\frac{5}{20}\right)\left(\frac{9}{19}\right) + \left(\frac{9}{20}\right)\left(\frac{4}{19}\right) +$
$\left(\frac{9}{20}\right)\left(\frac{5}{19}\right) + \left(\frac{9}{20}\right)\left(\frac{8}{19}\right)$



6. $P(A) = \frac{1}{6} + \frac{1}{3} + \frac{1}{9} = \frac{11}{18}$

$P(B) = \frac{1}{6} + \frac{1}{9} + \frac{1}{9} = \frac{7}{18}$



Lesson 41

Activity 1

1. (a) $A = 4\ 000(1 + 0{,}14 \times 5) = R6\ 800$ (b) $A = 4\ 000(1 + 0{,}14)^5 = R7\ 701{,}66$

2. (a) $13\ 000 = P(1 + 0{,}12 \times 6)$

$\therefore\ 13000 = P(1{,}72)$

$\therefore \frac{13\ 000}{(1{,}72)} = P$

$\therefore\ P = R7\ 558{,}14$

(b) $13\ 000 = P(1 + 0{,}12)^6$

$\therefore 13\ 000 = P(1{,}12)^6$

$\therefore\ \frac{13\ 000}{(1{,}12)^6} = P$

$\therefore\ P = R6\ 586{,}20$

3. $A = 70\ 000(1 + 0{,}09 \times 3)\ .\ (1 + 0{,}08)^2$

$\therefore\ A = R103\ 692{,}96$

Activity 2

(a) R60 000 (b) R20 000 (c) 7 years (d) 8 years (e) zero rands

Activity 3

1. (a) $A = 250\ 000(1 - 0{,}12 \times 5) = R100\ 000$ (b) $A = 250\ 000(1 - 0{,}12)5 = R131\ 932{,}98$

2. (a) $A = P(1 - in)$

$\therefore\ 3\ 200 = P(1 - 0{,}12 \times 7)$

$\therefore\ 3\ 200 = P(0{,}16)$

$\therefore\ \frac{3\ 200}{0{,}16} = P$

$\therefore\ P = R20\ 000$

b) $A = P(1 - i)n$

$\therefore\ 3\ 200 = P(1 - 0{,}12)^7$

$\therefore\ 3\ 200 = P(0{,}88)7$

$\therefore\ \frac{3\ 200}{(0{,}88)^7} = P$

$\therefore\ P = R7\ 830{,}17$

3. (a) $19\ 000 = 140\ 000(1 - 10i)$

$19\ 000 = 140\ 000(1 - 10i)$

$\therefore\ 19\ 000 = 140\ 000 - 1\ 400\ 000i$

$\therefore\ 1\ 400\ 000i = 121\ 000$

$\therefore\ i = 0{,}08642857143$

$\therefore\ r = 8{,}6\%$

(b) $19\ 000 = 140\ 000(1 - i)10$

$\therefore\ \frac{19\ 000}{140\ 000} = (1 - i)10$

$\therefore \left(\frac{19\ 000}{140\ 000}\right)^{\frac{1}{10}} = 1 - i$

$\therefore\ i = 1 - \left(\frac{19\ 000}{140\ 000}\right)^{\frac{1}{10}}$

$\therefore\ i = 0{,}1810402522$

$\therefore\ r = 18{,}1\%$

4. $56\ 000 = P(1 - 0{,}14)^5$

$\therefore\ 56\ 000 = P(0{,}86)^5$

$\therefore\ \frac{56\ 000}{(0{,}86)^5} = Pw$

$\therefore\ P = R110\ 040{,}78$

5. $A = 400\ 000(1 - 0{,}16 \times 6)$

$\therefore\ A = 16\ 000$

6. $A = 750\ 000\ (1 - 0{,}13)6$

$\therefore\ A = R325\ 219{,}65$

Lesson 42

Activity 1

1. (a) $A = 20000(1 + 0{,}15)^6 = R46261{,}22$

(b) $A = 20000\left(1+ \frac{0{,}15}{2}\right)^{12} = R47635{,}59$

(c) $A = 20000\left(1 + \frac{0{,}15}{4}\right)^{24} = R48388{,}76$

(d) $A = 20000\left(1 + \frac{0{,}15}{12}\right)^{72} = R48918{,}41$

(e) $A = 20000\left(1 + \frac{0{,}15}{365}\right)^{2190} = R49182{,}97$

2. Annual compounding:

$A = 60000(1 + 0{,}12)^{15} = R328412{,}95$

Monthly compounding:

$A = 60000\left(1 + \frac{0{,}12}{12}\right)^{180} = R359748{,}12$

The monthly compounding would give you R31334,17 more.

3. $A = 30000\left(1 + \frac{0{,}16}{4}\right)^{16} \cdot \left(1+ \frac{0{,}15}{2}\right)^{10} = R115808{,}20$

4. A. $A = x\left(1 + \frac{0{,}14}{12}\right)^{12} = x(1{,}149342029)$

B. $A = x\left(1 + \frac{0,16}{4}\right)^4 = x(1,16985856)$

Option B is the better option.

5. Present value at T_0

$1200000 = P(1 + \frac{0,18}{12})^{72}$

$\therefore 1200000 = P(1,015)^{72}$

$\therefore \frac{1200000}{(1,015)^{72}} = P$

$\therefore P = R410796$

Activity 2

1. (a)

$1 + i_{eff} = \left(1 + \frac{i_{nom}}{n}\right)^n$

$\therefore 1 + i_{eff} = \left(1 + \frac{0,14}{2}\right)^2$

$\therefore i_{eff} = (1,07)^2 - 1$

$\therefore i_{eff} = 0,1449$

$\therefore r_{eff} = 14,5\%$

(b)

$1 + i_{eff} = \left(1 + \frac{i_{nom}}{n}\right)^n$

$\therefore 1 + i_{eff} = \left(1 + \frac{0,16}{4}\right)^4$

$\therefore i_{eff} = (1,04)^4 - 1$

$\therefore i_{eff} = 0,16985856$

$\therefore r_{eff} = 17\%$

(c)

$1 + i_{eff} = \left(1 + \frac{i_{nom}}{n}\right)^n$

$\therefore 1 + i_{eff} = \left(1 + \frac{0,12}{12}\right)^{12}$

$\therefore i_{eff} = (1,01)^{12} - 1$

$\therefore i_{eff} = 0,1268250301$

$\therefore r_{eff} = 12,7\%$

(d)

$1 + i_{eff} = \left(1 + \frac{i_{nom}}{n}\right)^n$

$\therefore 1 + i_{eff} = \left(1 + \frac{0,10}{365}\right)^{365}$

$\therefore i_{eff} = \left(1 + \frac{0,10}{365}\right)^{365} - 1$

$\therefore i_{eff} = 0,1051557816$

$\therefore r_{eff} = 10,5\%$

2. (a)

$1 + i_{eff} = \left(1 + \frac{i_{nom}}{n}\right)^n$

$\therefore 1 + 0,132 = \left(1 + \frac{i_{nom}}{4}\right)^4$

$\therefore 1,132 = \left(1 + \frac{i_{nom}}{4}\right)^4$

$\therefore (1,132)^{\frac{1}{4}} = 1 + \frac{i_{nom}}{4}$

$\therefore 4\left[(1,132)^{\frac{1}{4}} - 1\right] = i_{nom}$

$\therefore 0,1259275539 = i_{nom}$

$\therefore r_{nom} = 12.6\%$

$\therefore r = 12.6\%$ per ann c.q.

(b)

$1 + i_{eff} = \left(1 + \frac{i_{nom}}{n}\right)^n$

$\therefore 1 + 0,145 = \left(1 + \frac{i_{nom}}{2}\right)^2$

$\therefore 1,145 = \left(1 + \frac{i_{nom}}{2}\right)^2$

$\therefore (1,145)^{\frac{1}{2}} = 1 + \frac{i_{nom}}{2}$

$\therefore 2\left[(1,145)^{\frac{1}{2}} - 1\right] = i_{nom}$

$\therefore 0,1400934559 = i_{nom}$

$\therefore r_{nom} = 14\%$

$\therefore r = 14\%$ pa csa

$\therefore 1 + 0,105 = \left(1 + \frac{i_{nom}}{12}\right)^{12}$

$\therefore 1,105 = \left(1 + \frac{i_{nom}}{12}\right)^{12}$

$\therefore (1,105)^{\frac{1}{2}} = 1 + \frac{i_{nom}}{12}$

$\therefore 12\left[(1,105)^{\frac{1}{12}}\right] - 1 = i_{nom}$

$\therefore 0,1002618682 = i_{nom}$

$\therefore r_{nom} = 10\%$

(c) $1 + i_{eff} = \left(1 + \frac{i_{nom}}{n}\right)^n$

3. (a) $A = 24000\left(1 + \frac{0,16}{4}\right)^{48} = R157692,68$

(b) $1 + i_{eff} = \left(1 + \frac{0,16}{4}\right)^4$

$\therefore\ i_{eff} = (1,04)^4 - 1$

$\therefore\ i_{eff} = 0,16985856$

$\therefore\ r_{eff} = 17\%$

(c) $A = 24000(1 + 0,16985856)^{12} = R157692,68$

$\therefore\ 1 + 0,0446975070792 = \left(1 + \frac{i_{nom}}{12}\right)^{12}$

$\therefore\ 1 + 0,0446975070792 = \left(1 + \frac{i_{nom}}{12}\right)^{12}$

$\therefore\ (1 + 0,0446975070792)^{\frac{1}{12}} = \left(1 + \frac{i_{nom}}{12}\right)$^

$\therefore\ 12\left[(1 + 0,0446975070792)^{\frac{1}{12}} - 1\right] = i_{nom}$

$\therefore\ i_{nom} = 0,04380714361$

4. (a) $650000 = 500000 = (1 + i_{eff})^6$

(b) $1 + i_{eff} = \left(1 + \frac{i_{nom}}{n}\right)^n$

$\therefore\ \frac{650000}{500000} = (1 + i_{eff})^6$

$\therefore\ \left(\frac{65}{50}\right)^{\frac{1}{6}} = 1 + i_{eff}$

$\therefore\ \left(\frac{65}{50}\right)^{\frac{1}{6}} - 1 = i_{eff}$

$\therefore\ i_{eff} = 0,04469750792$

$\therefore\ r_{eff} = 4,5\%$

Lesson 43

Activity 1

1. $A = 5000\left(1 + \frac{0,07}{4}\right)^{16}\left(1 + \frac{0,08}{2}\right)^{12} = R10566,25$

2. $A = 2000\left(1 + \frac{0,18}{12}\right)^{48}\left(1 + \frac{0,24}{2}\right)^{6} = R8066,93$

3. $A = 3000\left(1 + \frac{0,13}{12}\right)^{60}\cdot\left(1 + \frac{0,14}{12}\right)^{36}\cdot\left(1 + \frac{0,12}{12}\right)^{24} = R11039,65$

4. (a) $1 + \mathrm{i}_{eff} = \left(1 + \frac{0,08}{12}\right)^{12}$ $\qquad$ $1 + \mathrm{i}_{eff} = \left(1 + \frac{0,10}{2}\right)^{2}$

$\therefore \mathrm{i}_{eff} = \left(1 + \frac{0,08}{12}\right)^{12} - 1$ $\qquad$ $\therefore \mathrm{i}_{eff} = \left(1 + \frac{0,10}{2}\right)^{2} - 1$

$\therefore\ \mathrm{i}_{eff} = 0,08299950681$ $\qquad$ $\therefore \mathrm{i}_{eff} = 0,1025$

(b) $A = 6000(1,08299950681)^7\cdot(1,1025)^5 = R17078,20$

Activity 2

1. $P = 13000\left(1 + \frac{0,12}{12}\right)^{-36}(1 + 0,09)^{-4} = R6436,77$

2. $P= 10000000\left(1 + \frac{0,20}{4}\right)^{-8}\left(1+ \frac{0,15}{12}\right)^{-72} = R2767217,60$

3. $P50000\left(1 + \frac{0,11}{12}\right)^{-24}\left(1 + \frac{0,14}{4}\right)^{-4} = R35002,50$

4. (a) $P = 100000\left(1 + \frac{0,13}{2}\right)^{-4}\left(1+ \frac{0,14}{12}\right)^{-24} = R58844,11$

   (b) $1 + i_{eff} = \left(1 + \frac{0,14}{12}\right)^{12}$ $\quad$ $1 + i_{eff} = \left(1 + \frac{0,13}{2}\right)^{2}$

   $\therefore\ i_{eff} = 1 + = \left(1 + \frac{0,14}{12}\right)^{12} - 1$ $\quad$ $\therefore\ i_{eff} = 1 + = \left(1 + \frac{0,13}{2}\right)^{2} - 1$

   $\therefore\ i_{eff} = 0,1493420292$ $\quad$ $\therefore\ i_{eff} = 0,134225$

   (c) $P = 100000(1 + 0,\ 134225)^{-2}(1 + 0,\ 1493420292)^{-2}$
   $= R\ R58844,11$

   The same amount for P as in 4(a) is obtained.

Lesson 44

Activity 1

1. $A = 3500\left(1 + \frac{0,08}{12}\right)^{24}\left(1 + \frac{0,10}{2}\right)^{8} + 4000\left(1 + \frac{0,10}{2}\right)^{6} = R11425,50$

2. $A = 5000\left(1 + \frac{0,13}{2}\right)^{8}\left(1 + \frac{0,14}{4}\right)^{12} + 4000\left(1 + \frac{0,13}{2}\right)^{2}\left(1 + \frac{0,14}{4}\right)^{12} + 6000$

   $\therefore\ A = R25359,63$

3. $A = 2000\left(1+ \frac{0,09}{12}\right)^{12}\left(1+ \frac{0,08}{12}\right)^{24} + 4000\left(1 + \frac{0,08}{12}\right)^{24} + 8000\left(1 + \frac{0,08}{12}\right)^{12}$

   $\therefore\ A = R15921,37$

4. $x\left(1+ \frac{0,18}{12}\right)^{60} + 2x\left(1 + \frac{0.18}{12}\right)^{36} + 3x = 60000$

   $\therefore\ x = \left[\left(1 + \frac{0,18}{12}\right)^{60} + 2\left(1 + \frac{0,18}{12}\right)^{36} + 3\right] = 6000$

   $\therefore\ x = 6000/\left(1 + \frac{0,18}{12}\right)^{60} + 2\left(1 + \frac{0,18}{12}\right)^{36} + 3$

   $\therefore\ x = R677,09$

Activity 2

1. $A = 15000\left(1 + \frac{0,13}{2}\right)^{12} - 3000\left(1 + \frac{0,13}{2}\right)^{4}$

   $\therefore\ A = R28077,04$

2. $A = 5000\left(1 + \frac{0,08}{12}\right)^{36}\left(1 + \frac{0,09}{4}\right)^{16} + 6000\left(1 + \frac{0,08}{12}\right)^{12}\left(1 + \frac{0,09}{4}\right)^{16} - 3000\left(1 + \frac{0,09}{4}\right)^{8}$

$\therefore$ A = R14759,27

3. $A = 4300\left(1 + \frac{0,13}{12}\right)^{36}\left(1 + \frac{0,14}{4}\right)^{16} + 7000\left(1 + \frac{0,14}{4}\right)^{16} - 2000\left(1 + \frac{0,09}{4}\right)^{8} + 1000$

$\therefore$ A = R21737,74

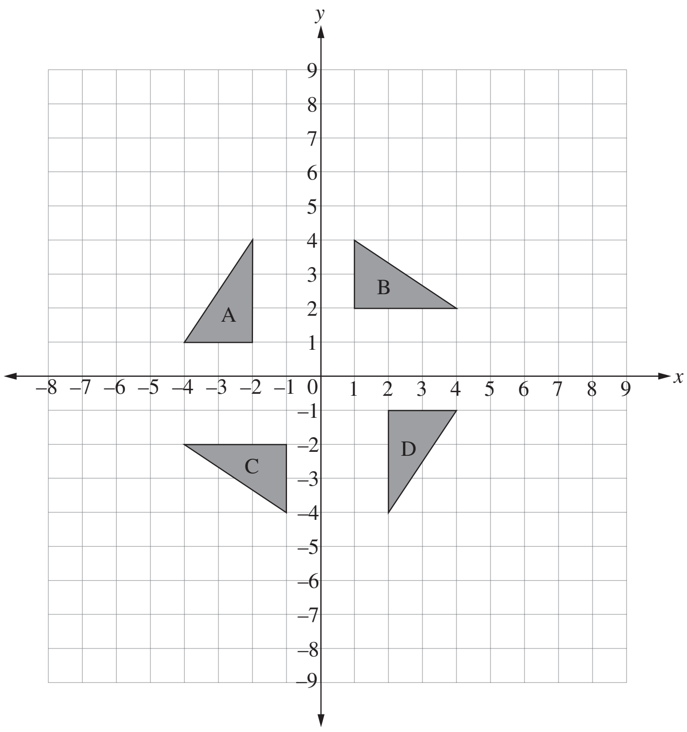
Activity 3

1. $P = 13000\left(1 + \frac{0,12}{4}\right)^{-12}\left(1 + \frac{0,09}{12}\right)^{-48} = R6369,92$
2. $P = 2300\left(1 + \frac{0,10}{2}\right)^{-8} + 4200\left(1 + \frac{0,12}{12}\right)^{-24}\left(1 + \frac{0,10}{2}\right)^{-10} = R3587,42$
3. $P = 6000 + 8000\left(1 + \frac{0,18}{12}\right)^{-24} + 900\left(1 + \frac{0,19}{12}\right)^{-24}\left(1 + \frac{0,18}{12}\right)^{-24} = R10319,06$
4. $A = 30000 + 20000\left(1 + \frac{0,18}{12}\right)^{-24} + 100000(1 + 0,32)^{-5}\left(1 + \frac{0,18}{12}\right)^{-36}$

$\therefore$ A = R58590,88

Lesson 46

Activity 1

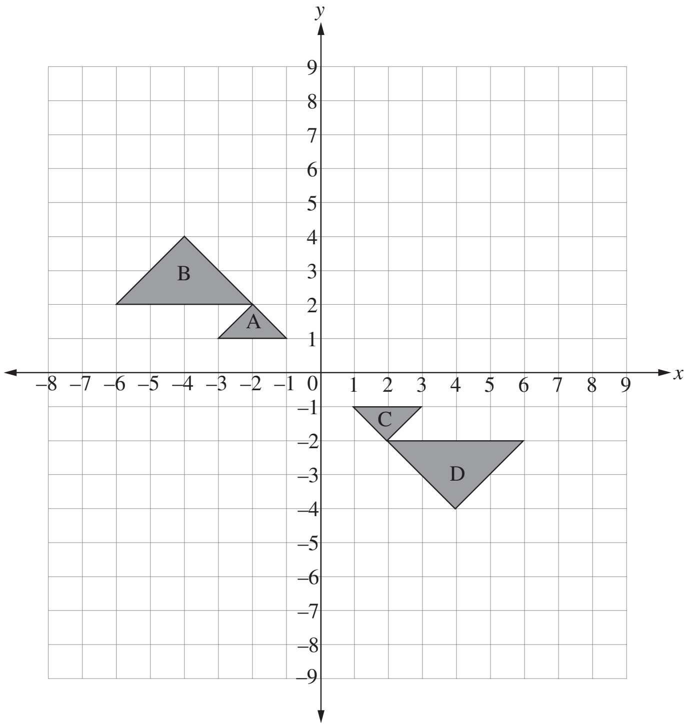


B: Rotation of 90° clockwise

C: Rotation of 90° anti-clockwise

D: Rotation of 180°

Activity 2

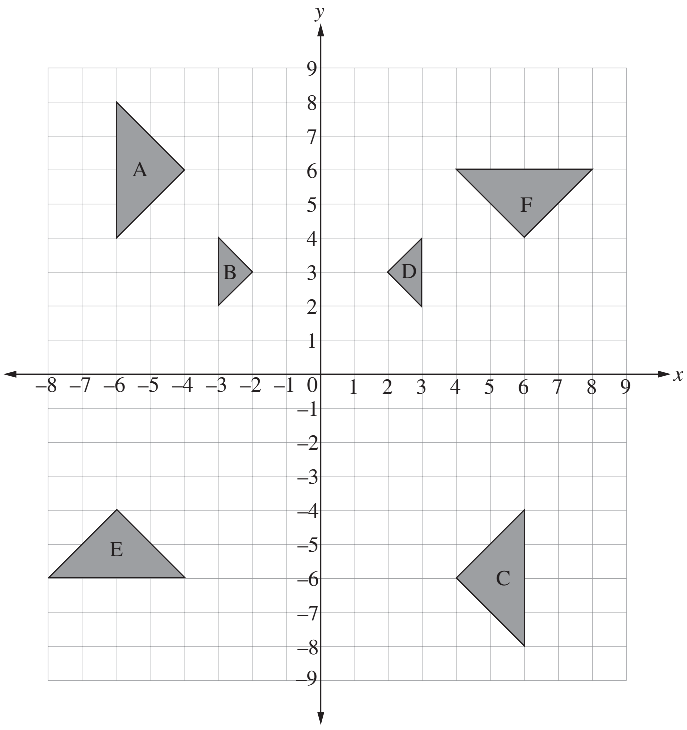


B: Enlargement by a scale factor of 2

C: Rotation of 180°

D: Combined rotation of 180° and enlargement

Activity 3



Lesson 47

Activity 1

1. A. The position of $Q_2 = \frac{1}{2}(19 + 1) = 10$.

The **Median** of the data is 16 (the 10th value).

The position of $Q_1 = \frac{1}{4}(19 + 1) = 5$

The **Lower Quartile** of the data is 9 (the 5th value). It is a part of the data set.

The position of $Q_3 = \frac{3}{4}(19 + 1) = 15$

The **Upper Quartile** of the data is 21 (the 15th value). It is part of the data set.

A	2	3	5	7	9	10	11	13	15	16	16	17	18	19	21	22	23	25	32	

B. The position of $Q_2 = \frac{1}{2}(17 + 1) = 9$th position.

The **Median** of the data is the 9th value:

$Q_2 = 15$

The position of $Q_1 = \frac{1}{4}(17 + 1) = 4{,}5$th position

The **Lower Quartile** of the data is the average between the 4th and 5th value:

$Q_1 = \frac{7 + 9}{2} = 8$ (not part of the data set)

The position of $Q_3 = \frac{3}{4}(17 + 1) = 13{,}5$th position

The **Upper Quartile** of the data is the average between the 13th and 14th value:

$Q_1 = \frac{18 + 19}{2} = 18{,}5$ (not part of the data set)

B	2	3	5	7	8	9	10	11	13	15	16	16	17	18	18,5	19	21	22	23

C. The position of $Q_2 = \frac{1}{2}(20 + 1) = 10{,}5$ (average of the 10th and 11th)

The **Median** of the data is $\frac{16 + 16}{2} = 16$ (not in data set)

Since n is even and since $\frac{n}{2} = \frac{20}{2} = 10$ which is even, the lower and upper quartiles will not be values in the data set.

The position of $Q_1 = \frac{1}{4}(20 + 1) = 5{,}25$ (average of the 5th and 6th value).

The **Lower Quartile** of the data is $\frac{9 + 10}{2} = 9{,}5$

The position of $Q_3 = \frac{3}{4}(20 + 1) = 15{,}75$ (average of the 15th and 16th).

The **Upper Quartile** of the data is $\frac{21 + 22}{2} = 21{,}5$

C	2	3	5	7	9	9,5	10	11	13	15	16	16	16	17	18	19	21	21,5	22	23	25	32	34

D. The position of $Q_2 = \frac{1}{2}(18 + 1) = 9{,}5$th position (average of the 9th and 10th value).

The **Median** of the data is $\frac{15 + 16}{2} = 15{,}5$

Since n is even and since $\frac{n}{2} = \frac{18}{2} = 9$ which is odd, the lower and upper quartiles will be values in the data set.

The position of $Q_1 = \frac{1}{4}(18 + 1) = 4{,}75$ (5th value)

The **Lower Quartile** of the data is 9.

The position of $Q_3 = \frac{3}{4}(18 + 1) = 14{,}25$ (14th value).

The **Upper Quartile** of the data is

D	2	3	5	7	9	10	11	13	15	15,5	16	16	17	18	19	21	22	23	25

2. First arrange in ascending order.

10A	11	12	14	15	15	16	16	16	16	16	16	17	17	19	22	22	23	24	26	–
10B	8	9	10	13	14	14	14	14	16	17	18	19	20	20	23	24	27	28	29	30
10C	2	5	7	12	12	12	14	14	14	14	14	14	14	20	21	21	24	26	–	–

(a) Mean for 10A: $\frac{333}{19} = 17{,}5$

Mean for 10B: $\frac{367}{20} = 18{,}4$

Mean for 10C: $\frac{260}{18} = 14{,}4$

(b) Mode for 10A: 16

Mode for 10B: 14

Mode for 10C: 14

(c) Median for 10A: 16

Median for 10B: 17,5

Median for 10C: 14

(d) Range for 10A: 26 – 11 = 15

Range for 10B: 30 – 8 = 22

Range for 10C: 26 – 2 = 24

(e) Lower quartile for 10A: $Q_1 = 15$

Lower quartile for 10B: $Q_1 = 14$

Lower quartile for 10C: $Q_1 = 12$

(f) Upper quartile for 10A: $Q_3 = 22$

Upper quartile for 10B: $Q_3 = 23{,}5$

Upper quartile for 10C: $Q_3 = 20{,}5$

(g) Interquartile range for 10A: $Q_3 - Q_1 = 22 - 15 = 7$

Interquartile range for 10B: $Q_3 - Q_1 = 23{,}5 - 14 = 9{,}5$

Interquartile range for 10C: $Q_3 - Q_1 = 20{,}5 - 12 = 8{,}5$

(h) Semi-interquartile range for 10A: $\frac{Q_3 - Q_1}{2} = \frac{22 - 15}{2} = 3{,}5$

Semi-interquartile range for 10B: $\frac{Q_3 - Q_1}{2} = \frac{23{,}5 - 14}{2} = 4{,}75$

Semi-interquartile range for 10C: $\frac{Q_3 - Q_1}{2} = \frac{20{,}5 - 12}{2} = 4{,}25$

3.

1	9	10	4	7	4	4	10
7	3	9	3	8	9	3	7
7	3	9	4	5	8	6	6
1	3	10	2	2	7	8	7
7	2	7	6	2	8	7	6

<table>
<tr><th>Marks (10)</th><th>Tally</th><th>Frequency (f)</th><th>Mark x f</th></tr>
<tr><td>1</td><td>|</td><td>2</td><td>2</td></tr>
<tr><td>2</td><td>||||</td><td>4</td><td>8</td></tr>
<tr><td>3</td><td>|||| 卌</td><td>5</td><td>15</td></tr>
<tr><td>4</td><td>||||</td><td>4</td><td>16</td></tr>
<tr><td>5</td><td>|</td><td>1</td><td>5</td></tr>
<tr><td>6</td><td>||||</td><td>4</td><td>24</td></tr>
<tr><td>7</td><td>卌 ||||</td><td>9</td><td>63</td></tr>
<tr><td>8</td><td>||||</td><td>4</td><td>32</td></tr>
<tr><td>9</td><td>||||</td><td>4</td><td>36</td></tr>
<tr><td>10</td><td>|||</td><td>3</td><td>30</td></tr>
<tr><td></td><td></td><td>Total: 40</td><td>Total: 231</td></tr>
</table>

Mean = $\frac{231}{40} = 5{,}8$

Activity 2

1. (a) A: $\overline{x} = \frac{79}{13} = 6{,}1$ B: $\overline{x} = \frac{87}{12} = 7{,}3$

C: $\overline{x} = \frac{52}{11} = 4{,}7$ D: $\overline{x} = \frac{72}{12} = 6$

(b) Driver A: Driver B: Driver C: Driver D:

Min: 1 Min: 1 Min: 1 Min: 2

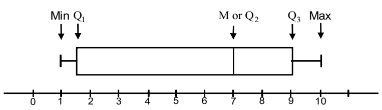
$Q_1 = 1{,}5$ $Q_1 = 7$ $Q_1 = 2$ $Q_1 = 3$

$Q_2 = 7$ $Q_2 = 8$ $Q_2 = 4$ $Q_2 = 6$

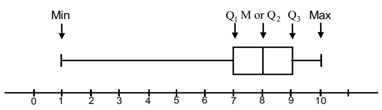
$Q_3 = 9$ $Q_3 = 9$ $Q_3 = 8$ $Q_3 = 9$

Max: 10 Max: 10 Max: 10 Max: 10

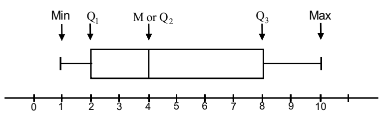
(c) Driver A:



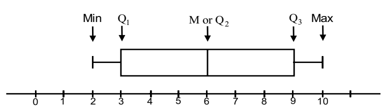
Driver B:



Driver C:



Driver D:



(d) Driver A: Skewed to the left of the median.

Median is to the right of the mean. Data is negatively skewed.

Driver B: Symmetrical with respect to the median.

Median is to the right of the mean. Data is negatively skewed.

Driver C: Skewed to the right of the median.

Median is to the left of the mean. Data is positively skewed.

Driver D: Symmetrical with respect to the median.

Data is symmetrical about the mean.

(e) Driver B performed the best because 50% of his points are 8 or above.

Driver A had the second best performance because 50% of his points are 7 or above.

Driver D had the third best performance because 50% of his points are 6 or above.

Driver A did the worst because 50% of his points are 4 or above.

Lesson 48

Activity 1

Complete this exercise in this workbook.

The following table contains the number of learners who obtained certain marks on a class test out of 30.

Marks	20	21	22	23	24	25	26	27	28	29
No of learners	3	3	4	5	7	10	13	5	4	2

(a) Draw a cumulative frequency table for this data.

Marks	Frequency	Cumulative Frequency
20	3	3
21	3	6
22	4	10
23	5	15
24	7	22
25	10	32
26	13	45
27	5	50
28	4	54
29	2	56
Total	56	

(b) On the set of axes provided on the next page, draw a cumulative frequency graph (ogive curve) for this data.

(c) Determine the lower quartile.

Position of $Q_1 = \frac{1}{4}(57) = 14{,}25$

$\frac{56}{2} = 28$ which is even (Q_1 and Q_3 are not part of data).

$Q1 = \frac{23 + 23}{2} = 23$

(d) Determine the median.

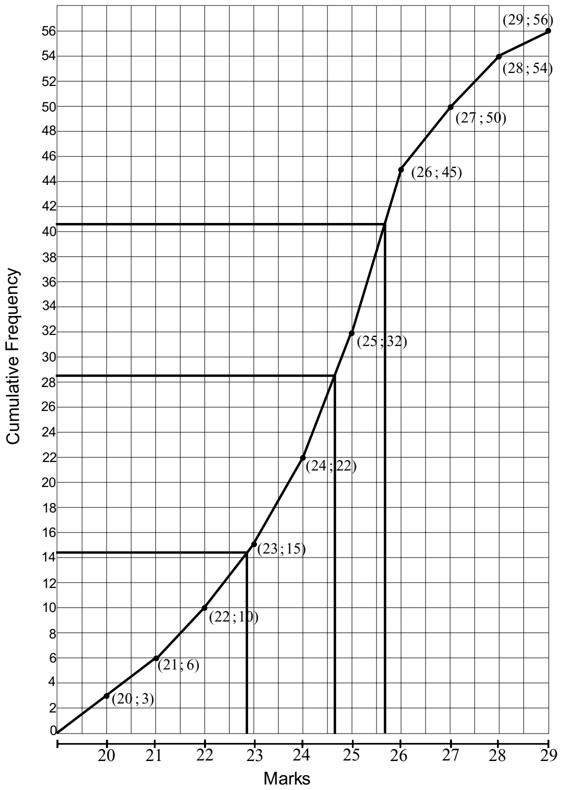
Position of $Q_2 = \frac{1}{2}(57) = 28{,}5\text{th}$

$$Q_2 = \frac{25 + 25}{2} = 25$$

(e) Determine the upper quartile.

Position of $Q_3 = \frac{3}{4}(57) = 42{,}25\text{th}$

$$Q_3 = \frac{26 + 26}{2} = 26$$



Activity 2

1. The following table (grouped frequency distribution) shows the mark obtained by 220 learners in a Science examination.

Percentage	1-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90	91-100
Frequency	2	6	11	22	39	59	45	20	11	5

(a) Complete the cumulative frequency table for this data.

Marks	Frequency	Cumulative Frequency
1-10	2	2
11-20	6	8
21-30	11	19
31-40	22	41
41-50	39	80
51-60	59	139
61-70	45	184
71-80	20	204

81-90	11	215
91-100	5	220
Total	220	

(b) On the set of axes provided on the next page, draw a cumulative frequency graph (ogive curve) for this data.

(c) Determine the lower quartile.

Position of $Q_1 = \frac{1}{4}(221) = 55{,}25\text{th}$

$Q_1 \approx 44$ (read off from graph)

(d) Determine the median.

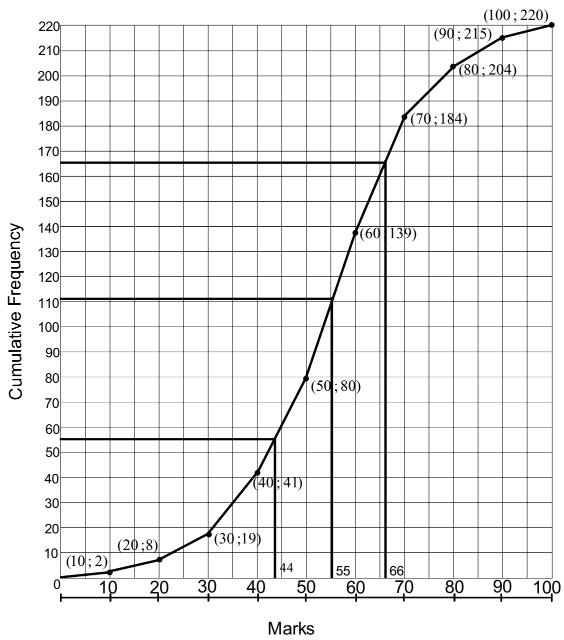
Position of $Q_2 = \frac{1}{2}(221) = 110{,}5\text{th}$

$Q_2 \approx 55$ (read off from graph)

(e) Determine the upper quartile.

Position of $Q_3 = \frac{3}{4}(221) = 165{,}75\text{th}$

$Q_3 \approx 55$ (read off from graph)



2. The table represents the percentage of income spent on recreation by 50 people.

(a) Complete the cumulative frequency table for this data.

Percentage	**Frequency**	**Cumulative Frequency**
$12 < p \leq 18$	8	8
$18 < p \leq 24$	20	28
$24 < p \leq 30$	12	40
$30 < p \leq 36$	8	48
$36 < p \leq 42$	2	50
Total	50	

(b) On the set of axes provided on the next page, draw a cumulative frequency graph (ogive curve) for this data.

(c) Determine the lower quartile.

$\frac{50}{2} = 25$ which is off (Q_1 and Q_3 are part of data)

Position of $Q_1 = \frac{1}{4}(51) = 12{,}75 = 13$th position

$Q_1 \approx 19{,}5$ (read off from graph)

(d) Determine the median.

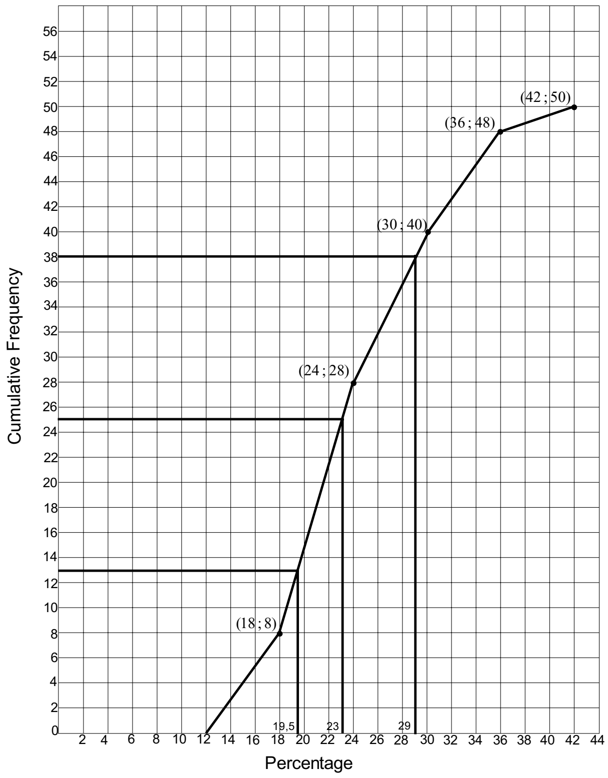
Position of $Q_2 = \frac{1}{2}(51) = 25{,}5$th

$Q_2 \approx 23$ (read off from graph)

(e) Determine the upper quartile.

Position of $Q_3 = \frac{3}{4}(51) = 38{,}25 = 38$th

$Q_3 \approx 29$ (read off from graph)



Lesson 49

Activity 1

1. (a) A: $\bar{x} = \frac{79}{13} = 6{,}1$ B: $\bar{x} = \frac{87}{12} = 7{,}3$

C: $\bar{x} = \frac{52}{11} = 4{,}7$ D: $\bar{x} = \frac{72}{12} = 6$

(b) Driver A:

x	$(x-\bar{x})$	$(x-\bar{x})^2$
1	–5,1	26,01
1	–5,1	26,01
1	–5,1	26,01
2	–4,1	16,81
6	–0,1	0,01
6	–0,1	0,01
8	1,9	3,61
8	1,9	3,61
8	1,9	3,61
8	1,9	3,61
10	3,9	15,21
10	3,9	15,21
10	3,9	15,21
		$\sum(x-\bar{x})^2 = 154,93$

variance $(s^2) = \frac{\sum(x-\bar{x})^2}{n} = \frac{154,93}{13} = 11,91769231$

standard deviation $(s) = \sqrt{\frac{\sum(x-\bar{x})^2}{n}} = 3,45$

Driver B:

x	$(x-\bar{x})$	$(x-\bar{x})^2$
1	–6,3	39,69
2	–6,3	28,09
6	–1,3	1,69
8	0,7	0,49
8	0,7	0,49
8	0,7	0,49
8	0,7	0,49
8	0,7	0,49
8	0,7	0,49
10	2,7	7,29
10	2,7	7,29
10	2,7	7,29
		$\sum(x-\bar{x})^2 = 94,28$

variance $(s^2) = \frac{\sum(x-\bar{x})^2}{n} = \frac{94,28}{12} = 7,856666667$

standard deviation $(s) = \sqrt{\frac{\sum(x-\bar{x})^2}{n}} = 2,8$

Driver C:

x	$(x-\bar{x})$	$(x-\bar{x})^2$
1	–3,7	13,69
1	–3,7	13,69
2	–2,7	7,29
2	–2,7	7,29
4	–0,7	0,49
4	–0,7	0,49
6	1,3	1,69
6	1,3	1,69

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8	3,3	10,89
8	3,3	10,89
10	5,3	28,09
		$\sum(x-\bar{x})^2 = 96,19$

variance $(s^2) = \frac{\sum(x-\bar{x})^2}{n} = \frac{96,19}{11} = 8,744545455$

standard deviation $(s) = \sqrt{\frac{\sum(x-\bar{x})^2}{n}} = 2,96$

Driver D:

x	$(x-\bar{x})$	$(x-\bar{x})^2$
2	–4	16
2	–4	16
2	–4	16
4	–2	4
4	–2	4
6	0	0
6	0	0
8	2	4
8	2	4
10	4	16
10	4	16
10	4	16
		$\sum(x-\bar{x})^2 = 112$

variance $(s^2) = \frac{\sum(x-\bar{x})^2}{n} = \frac{112}{12} = 9,\dot{3}$

standard deviation $(s) = \sqrt{\frac{\sum(x-\bar{x})^2}{n}} = 3,1$

(c) Driver B's performance is clustered closely around its mean of 7,3 in comparison with the other drivers. The mean is the highest for B. Therefore, B performed the best.

2.

Marks x	**Freq f**	$f \times x$	$(x-\bar{x})$	$(x-\bar{x})^2$	$f \times (x-\bar{x})^2$
20	3	60	–4,8	23,04	69,12
21	3	63	–3,8	14,44	43,32
22	4	88	–2,8	7,84	31,36
23	5	115	–1,8	3,24	16,2
24	7	168	–0,8	0,64	4,48
25	10	250	0,2	0,04	0,4
26	13	338	1,2	1,44	18,72
27	5	135	2,2	4,84	24,2
28	4	112	3,2	10,24	40,96
29	2	58	4,2	17,64	35,28
Total	**56**	$\bar{x} = \frac{1387}{56} = 24,8$			284,04

variance $(s^2) = \frac{\sum f(x-\bar{x})^2}{n} = \frac{284}{56} = 5,072142857$

standard deviation $(s) = \sqrt{\frac{\sum(x-\bar{x})^2}{n}} = 2,25$

Activity 2

1. The table represents the percentage of income spent on recreation by 50 people.

   (a) Complete the following table for this data.

Percentage	f	Midp (m)	$f \times m$	$m - \bar{x}$	$(m - \bar{x})^2$	$f \times (m - \bar{x})^2$
$12<p \leq 18$	8	15	120	–9,1	82,81	662,48
$18<p \leq 24$	20	21	420	–3,1	9,61	192,2
$24<p \leq 30$	12	27	324	2,9	8,41	100,92
$30<p \leq 36$	8	33	264	8,9	79,21	633,68
$36<p \leq 42$	2	39	78	14,9	222,01	444,02
Total	50		$\bar{x} = \frac{1206}{50} = 24,1$			2033,3

   (b) Calculate the mean for this data

   $\bar{x} = \frac{1206}{50} = 24,1$

   (c) Now calculate the standard deviation for this data.

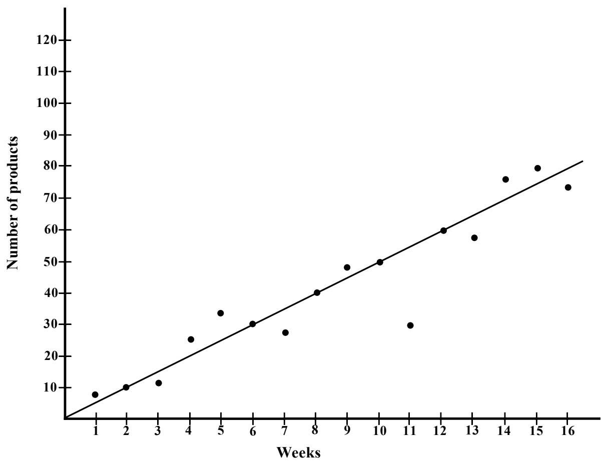
   $$\text{variance } (s^2) = \frac{\sum f(x - \bar{x})^2}{n} = \frac{2033,3}{50} = 40,666$$

   $$\text{standard deviation } (s) = \sqrt{\frac{\sum f(xf - \bar{x})^2}{n}} = 6,4$$

   (d) Now verify your answer by using a calculator.

Lesson 50

1. A company records the number of products sold per week.



   (a) Draw a line of best fit on the diagram above.

   (b) Determine the equation of your line of best fit.

   (2; 10)(6;30)

   gradient $= \frac{30 - 10}{6 - 2} = \frac{20}{4} = 5$ y-intercept is 0

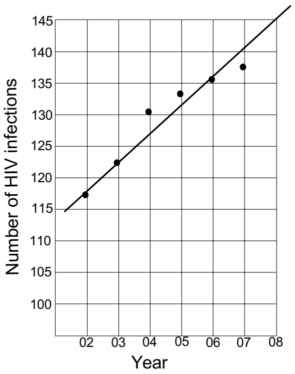
   $\therefore$ $y = 5x$ is the equation of the line of best fit

(c) Predict how many products will be sold after 40 weeks.

$y = 5(40) = 200$

An estimated 200 products might be sold.

2. (a)



(b) There is a line of best fit for this data.

(c) There would probably be an estimated 145 infections in 2008.