

WORD SUMS AND MATHEMATICAL MODELLING

Learning Outcomes and Assessment Standards

Learning Outcome 2: Functions and Algebra

The learner is able to investigate, analyse, describe and represent a wide range of functions and solve related problems.

Assessment Standards

We know this when the learner is able to:

- Use mathematical models to investigate problems that arise in real-life contexts:
 - making conjectures, demonstrating and explaining their validity
 - expressing and justifying mathematical generalisations of situations.



Overview

In this lesson you will :

- Use equations and simultaneous equations to solve various story sums.

Lesson

Different types of problems

Age

If Josh is x years old and Nick is three years older than Josh, he is $x + 3$ years old.

In ten years time Josh will be $(x + 10)$ years old.

Nick will be $(x + 3 + 10)$ years old.

Examples

1. Zinzi is 8 years older than her cousin in Nelson. In 5 years time, Zinzi will be twice as old as Nelson. What are their ages now?

Solution

Let Zinzi be x years old:

Then Nelson is $(x - 8)$ years old.

5 years from now:– Zinzi $\rightarrow (x + 5)$ years old

Nelson $\rightarrow (x + 5 - 8) = (x - 3)$ years old

Then: Zinzi = $2 \times$ Nelson

$$x + 5 = 2(x - 3)$$

$$= 2x - 6$$

$$\therefore -x = -11$$

$$\therefore x = 11$$

So Zinzi is 11 years old and Nelson is 3 years old.



2. Four years ago, a new bakkie cost three times more than a new trailer. Nowadays, a new bakkie costs twice as much as a trailer. How much do each cost?

Solution

Let the trailer cost Rx now. Then the bakkie will cost $R2x$ now. (x is in thousands of rands)

Four years ago: Trailer: $x - 4$ Bakkies $2x - 4$

and Bakkie = $3 \times$ Trailer

$$\therefore 2x - 4 = 3(x - 4)$$

$$\therefore 2x - 4 = 3x - 12 \text{ So Bakkie costs: R16 000}$$

$$\therefore -x = -8 \quad \text{and Trailer costs: R8 000}$$

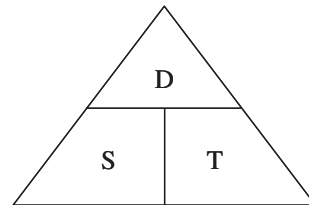
$$\therefore x = 8$$

Speed, Distance and Time

Distance = Speed \times Time

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$



Examples

1. Samantha walks at an average speed of 4 km/h and runs at 6 km/h. She can save $2\frac{1}{2}$ minutes by running to the station instead of walking. How far is the station?

Solution

Let distance to the station be x . Make a table

	S	D	T
Walk	4	x	$\frac{x}{4}$
Run	6	x	$\frac{x}{6}$

Make the equation

Slow – fast = (Time in hours)

$$\frac{x}{4} - \frac{x}{6} = \frac{2,5}{60}$$

$$\frac{x}{4} - \frac{x}{6} = \frac{1}{24}$$

LCD 24

$$6x - 4x = 1$$

$$2x = 1$$



$$x = \frac{1}{2}$$

Distance to the station is 0,5 km.

2. Two light aeroplanes take off at the same instant. Plane A flies from Lanseria to Nelspruit at 215 km/h and Plane B flies from Nelspruit to Lanseria at 320 km/h. At some point on their journey they will fly over one another. How far would Plane B have flown at this instant? (The airports are 396 km apart)

Solution

Assume they fly over one another x hours after their journeys commenced.

	Distance	Speed	Time
A	$215x$	215	x
B	$320x$	320	x

$$\text{Now: } 215x + 320x = 396$$

$$535x = 396$$

$$x = \frac{396}{535}$$

$$x = 0,7383$$

\therefore They will cross $44\frac{1}{2}$ minutes after take off and Plane B would then have flown $320(0,7383) = 236,3$ km

Shopping

Example

Baboostika ordered 400 vases for her flower shop and discovered that x of them had broken during the shipping. She paid R10 000 for the shipment, and decides to sell the remaining vases for R30 more than what she purchased them for. Her profit is then R9 305. How many vases broke?

Solution

She has left: $(400 - x)$

Purchase price per vase: $\frac{10\,000}{400} = \text{R}25$ per vase

Selling price per vase: R55

Income: $(400 - x) \times 55 = 22\,000 - 55x$

\therefore Profit = Income – Expenses

$$9\,305 = 22\,000 - 55x - 10\,000$$

$$19\,305 - 22\,000 = -55x$$

$$55x = 2\,695$$

$$x = 49$$

Thus 49 vases broke.



Different problems

This is the epitaph of Diophantus, the great Greek mathematician.

“Diophantus was a child for a sixth of his life, a youth for a twelfth of his life, a bachelor for another seventh of his life. Five years after his marriage his son was born, who died four years before his father at half the age at which his father died”. At what age did Diophantus die?

Solution

Let him die after x years. (His life was x years long)

$$x = \frac{1}{6}x + \frac{1}{12}x + \frac{1}{7}x + 5 + \text{son's life} + 4$$

$$x = \frac{1}{6}x + \frac{1}{12}x + \frac{1}{7}x + 5 + \frac{1}{2}x + 4$$

$$x = \frac{1}{6}x + \frac{1}{12}x + \frac{1}{7}x + 5 + \frac{1}{2}x + 4$$

LCD 84

$$84x = 14x + 7x + 12x + 420 + 42x + 336$$

$$84x = 75x + 756$$

$$9x = 756$$

$$x = 84$$



Activity 1

Word problems

1. You need to make the required quantity x
2. Tabulate information if you need to. It is often easier to understand.
3. Get the rest of the problem into an algebraic expression.
4. Now solve for the unknown.

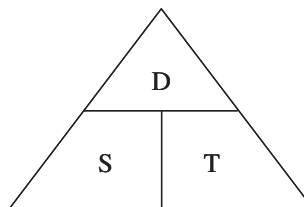
Remember: $S = \frac{D}{T}$

$$D = ST$$

D = distance (km)

S = speed (km·hr⁻¹)

T = time (hr)



1. The sum of four consecutive even numbers is 68. Find the numbers.
2. Divide R50 between two workers so that one gets R6 more than the other.
3. Two numbers differ by 13. Their sum is 29. What are the numbers?
4. There were 579 spectators at a tennis match. Of these, 123 spectators were children and there were twice as many men as women. How many women were there?

5. I bought 10 shirts, some at R4 each and some at R4.50 each. If the total cost was R43.50, how many of each type did I buy?
6. A mother is three times as old as her daughter. In 12 years' time she will be twice as old as her daughter is then. How old is the mother now?
7. A man is 35 years old and his son is 7 years old. In how many years will the father be twice the son's age?
8. John is the star batsman at Beaconwood High School. His batting average, after being dismissed five times, was 52. He made 70 in his next innings. What will he have to make in the last match (one innings) to boost his average to 60?
9. I have twice as many 20 cent coins as 10 cent coins and half the number of 5 cent coins as 10 cent coins. If I have R4.20 altogether, find out how many of each coin denomination I have.
10. At Sandile High School, promotion marks are calculated by adding the percentage gained in each of three cycles to three times the percentage gained in the mid-year examination and six times the percentage gained in the final examination, and then dividing this total by 12. If Marie's marks in the three cycles were 42, 40 and 35 and her mid-year examination mark was 35, what percentage must she get in her final examination to get a promotion mark of 40?
11. Two aeroplanes set off simultaneously, flying towards each other from airports 900 km apart. One flies at 250 km/h and the other at 350 km/h. After how many hours will they pass each other?
12. One motorist covers a journey of 360 km in half an hour less than another. If the slower motorist travelled at 80 km/h, at what speed did the faster motorist travel?
13. Geoff walked to town at 4 km/h and ran home at 9 km/h. If it took him $4\frac{1}{3}$ hours to get to town and back, how far was he from town?
14. Two cyclists, travelling at 20 km/h and 24 km/h, are 198 km apart at 09h00. If they are travelling towards each other, at what time will they meet?

