Grade 10
Educators Guide
2008

Grade 10 Assessment Guide

Learning Outcomes 1 and 2

2 Learning Outcomes 3 and 4

Grade 10 Assignment: Functions

Marks: 30

	6 – 8	4 – 5	2 – 3	0 - 1
Accuracy of graphs (If technology is used learners must provide printouts of all graphs)	Accurate and Correct Throughout	Almost all Correct	Some correct	Mostly incorrect
Completion of Table		All Entries Correct	Most Entries Correct	Incomplete with Errors
Observations Made	Clear and correct explanations given for all conclusions	Clear explanations given but conclusions incomplete	Did not clearly explain the reasoning	No Attempt /Have given a Vague Description of Incomplete Conclusions
Correctness of expressions for linear, parabolic and rational functions		Accurate and Correct	Almost all Correct	No attempt / Many errors
Punctuality			Deadline met	Deadline not met /Negotiated Deadline met
Presentation				Acceptable / not acceptable

Grade 10 Investigation: Number Patterns — Marks: 100

Option 1

The sequence recurs from the 6th term, provided neither T_1 nor T_2 is zero.
If zero is chosen as one of the first two terms, in calculating subsequent terms we land up with a zero denominator, which makes it impossible to continue.

Proof:

Let $T_1 = a$ and $T_2 = b$

Then $T_3 = \frac{b+1}{a}$ and hence

$$T_4 = \frac{\frac{b+1}{a}+1}{b} = \frac{b+1+a}{a} \times \frac{1}{b} = \frac{b+1+a}{ab}$$

$$T_5 = \frac{\frac{b+1+a}{ab}+1}{\frac{b+1}{a}} = \frac{b+1+a+ab}{ab} \times \frac{a}{b+1} = \frac{(b+1)+a(b+1)}{ab} \times \frac{a}{b+1} = \frac{1+a}{b}$$

$$T_6 = \frac{\frac{1+a}{b}+1}{\frac{b+1+a}{ab}} = \frac{1+a+b}{b} \times \frac{ab}{b+1+a} = a$$

$$T_7 = \frac{a+1}{\frac{1+a}{b}} = \frac{a+1}{1} \times \frac{b}{1+a} = b$$

Then terms 3 to 5 are repeated and so on…

Option 2

Conjectures:

1. If the first and third digits are the same, the difference is zero (and hence the sum of the difference and the reversed difference is also zero).
2. If the first and third digits differ by 1, the difference is 99 and the difference and the sum of the difference and the reversed difference is 99 + 99 = 198
3. For all other cases, the sum of the difference and the reversed difference is 1 089

Note: if the third digit is zero, this will result in a two digit number on reversal, but the conjectures above still hold. If the first digit is zero, the number will, strictly speaking, not be a three digit number and the given conditions will not be satisfied.

Proof:

Let the digits of the number be *x*, *y* and *z*

Then the value of the number is $100x + 10y + z$ $(x > z)$

The reversed number has the value $100z + 10y + x$

The difference is $100(x - z) + (z - x) = 99(x - z)$

We can assume, without a loss of generality that $x > z$

Conjecture 1:

If $x = z$, then Difference = $99 \times 0 = 0$ and the sum of zero and zero is zero!

Conjecture 2:

If $x = z + 1$

Difference = $99 \times 1 = 99$ and the sum = 99 + 99 = 198

Conjecture 3:

If $x = z + 2$, then difference = $99 \times 2 = 198$, reversed difference = 891 and the sum is 1 089.
If $x = z + 3$, then difference = $99 \times 3 = 297$, reversed difference = 792 and the sum is 1 089.
If $x = z + 4$, then difference = $99 \times 4 = 396$, reversed difference = 693 and the sum is 1 089.
If $x = z + 5$, then difference = $99 \times 5 = 495$, reversed difference = 594 and the sum is 1 089.
If $x = z + 6$, then difference = $99 \times 6 = 594$, reversed difference = 495 and the sum is 1 089.
If $x = z + 7$, then difference = $99 \times 7 = 693$, reversed difference = 396 and the sum is 1 089.
If $x = z + 8$, then difference = $99 \times 8 = 792$, reversed difference = 297 and the sum is 1 089.
If $x = z + 9$, then difference = $99 \times 9 = 891$, reversed difference = 198 and the sum is 1 089.
There are no other options.

Grade 10 Test: Products, Factors, Equations, Surds and Number Patterns
Time: 1 hour **Marks: 50**

1.1 $9x^4 - 30x^2y + 25y^2$ ✓✓✓✓

1.2 $6 + 2a - 3b - ab$ ✓✓✓✓

1.3 $p^3 - 8$ ✓✓✓✓

1.4 $2\sqrt{6} - 6$ ✓✓

2.1 $\left(x^2 + 4\right)\left(x^2 - 4\right)$ ✓✓

$= \left(x^2 + 4\right)(x + 2)(x - 2)$ ✓✓

2.2 $(2m - 3)(m - 1)$ ✓✓✓✓

3.1 Hypotenuse $= \sqrt{\left(\sqrt{2} - 1\right)^2 + \left(\sqrt{2} + 1\right)^2}$ ✓✓

$= \sqrt{2 - 2\sqrt{2} + 1 + 2 + 2\sqrt{2} + 1}$

$= \sqrt{6}$ ✓✓

3.2 $(x - y)(x + y - 1)$ ✓✓✓✓

One side $x - y$, other side $x + y - 1$

Hence perimeter = 2x-2y+2x+2y-2

$= 4x - 2$ units ✓✓✓✓

4.1 $E_{10} = 5 + (3 \times 10) = 35$ ✓✓

4.2 $E_n = 5 + 3n$ ✓✓

4.3 $5 + 3k = 116$ ✓✓

$\therefore k = 37$ ✓✓

4.4 When n is any odd multiple of 5 ✓✓

5.1 Any valid set $\frac{x}{p}, \frac{p}{q}$ and $\frac{q}{x}$ ✓✓

Grade 10 Project: Finance — Marks: 75

Section A

1. R101,70 ✓✓
2. More: inflation since 2006, cost of shipping, profit for shop… ✓✓
3. In 2006: exchange rate less: not published in 2004 when rate was even more favourable. ✓✓
4. US$13,46 (to the nearest cent) ✓✓✓
5. Best in 2005, worst in 2002. Any valid explanation. ✓✓✓
6. Best in 2002, worst in 2005. Any valid explanation. ✓✓✓
7. R100 = A$17 ✓✓✓
8. 1A$=R5,88 (to the nearest cent) ✓✓✓
9. R1=US$0,14 (to the nearest cent) ✓✓✓
10. US$1=A$0,83 (to the nearest cent) ✓✓✓
11. R649,52 In RSA ✓✓✓, A$76,36 in Aus.✓✓✓
12. US$1=British Pounds(0,50)(to the nearest penny) or 0,49964…✓✓✓
13. $3,20 = R 22,59 ✓✓
14. More expensive in USA ✓✓
15. R1 = $0,2645… or US$ 1 = R4,845375 Rand is undervalued. ✓✓

Section B

1. R650,27 (toi nearest cent) ✓✓✓✓
2. R684,79 (to nearest cent) ✓✓✓✓✓
3. Monthly payments=R27,79 (to nearest cent) ✓✓✓✓✓
4.1 R375,32 (to nearest cent) ✓✓✓✓✓
4.2 $170\left(1+\frac{0{,}08}{12}\right)^{24}+160\left(1+\frac{0{,}08}{12}\right)^{12}+170\left(1+\frac{0{,}08}{12}\right)=R538{,}45$ (to nearest cent) ✓✓✓✓✓✓✓✓✓
4.3 $890(1{,}067)^2=R1\,013{,}26$ (to nearest cent) ✓✓✓
4.4 Total available =R913,77 Parents' offer is better! ✓✓

Exam A

Grade 10 Mathematics Exam **Paper 1**
Time: 2 hours **Marks: 100**

1.1.1 $(x-1)(x^2+x+1) = x^3+x^2+x-x^2-x-1$ ✓✓
$= x^3-1$ ✓

1.1.2 $\left(\frac{9x^2}{y^4}\right)^{-\frac{1}{2}} = \frac{3^{2x-\frac{1}{2}}.x^{2x-\frac{1}{2}}}{y^{4x-\frac{1}{2}}}$ ✓✓
$= \frac{3^{-1}x^{-1}}{y^{-2}}$ ✓
$= \frac{y^2}{3x}$ ✓

1.2.1 $x^2-10x+24 = (x-6)(x-4)$ ✓✓

1.2.2 $x^2-y^2-2x+2y = (x^2-y^2)-2(x-y)$ ✓
$= (x+y)(x-y)-2(x-y)$ ✓
$= (x-y)(x+y-2)$ ✓✓

2.1.1 $2x^2-x-3=0$
$\therefore (2x-3)(x+1)=0$ ✓
$\therefore x = \frac{3}{2}$ or $x=-1$ ✓

2.1.2 $5^x = 5\sqrt{5}$
$= 5\times 5^{\frac{1}{2}}$ ✓✓
$\therefore x = \frac{3}{2}$ ✓

2.1.3 $-1 \le 2-3x < 8$ ✓✓
$\therefore -3 \le -3x \le 6$ ✓
$\therefore 1 \ge x \ge -2$ ✓

2.2 Let the distance from Cape Town to Stellenbosch be *x km* ✓
Then time taken on trip to Stellenbosch = $\frac{x}{60}$ hours
Time taken on trip from Stellenbosch to Cape Town = $\frac{x}{90}$ hours ✓
$\therefore$ Time taken for return trip = $\left(\frac{x}{60}+\frac{x}{90}\right)$ hours ✓

$= \frac{3x + 2x}{180}$ hours ✓

Distance covered on return trip = $2x\ km$ ✓

Hence average speed is $\frac{2x}{\frac{5x}{180}} = \frac{360}{5} = 72\ km/h$ ✓

3. Let the number of boys be x and the number of girls be y ✓

Then $x + y = 38$ (Eqn 1) ✓

Then total mass = $(35x + 32y)\ kg$ ✓

But total mass = 1 270 kg

$\therefore 35x + 32y = 1\,270$ (Eqn 2) ✓

From eqn. 1: $y = 38 - x$ ✓

Subst. Into eqn. 2: $35x + 32(38 - x) = 1\,270$ ✓

$\therefore 35x + 1\,216 - 32x = 1\,270$

$\therefore 3x = 54$

$\therefore x = 18$ ✓

and $y = 38 - 18 = 20$

Ms Shoko has 18 boys and 20 girls in her class ✓

4.1 The height of the water after 25 minutes is $35 + 8 = 43$ ✓

4.2 Height after 60 minutes = $3 + 12 \times 8 = 99\ cm$ ✓✓

4.3 Rising at $\frac{8}{5} = 1{,}4\ cm / minute$ ✓✓

4.4 Height after $5n$ minutes is $(3 + 8n)\ cm$ ✓✓✓

4.5 Height will be $403\ cm$:when $403 = 3 + 8n$ ✓

$\therefore 400 = 8n$

$\therefore n = 50$ ✓

Time taken = $5 \times 50 = 550$ minutes ✓

4.6 After 5 hours = 300 minutes: height = $3 + 8 \times \frac{300}{5} = 483\ cm < 500\ cm$ ✓✓

The water will not have overflowed ✓

5.1 NZ$ 1 = R 5.1265

$\therefore NZ\$\ 300 = R(300 \times 5{,}1265)$ ✓✓

$= R\,1\,540$ (correct to the nearest R10)

5.2 Money from investments = $R[500(1{,}085)^2 + 500(1{,}085)] = R1\,131{,}1125$ ✓✓✓✓✓

She will need to save a further R $(1\,540 - 1\,131{,}11) = R408{,}89$ (to the nearest cent) ✓

5.3 $US\$\,55 = R(55 \times 6{,}7)$ ✓✓

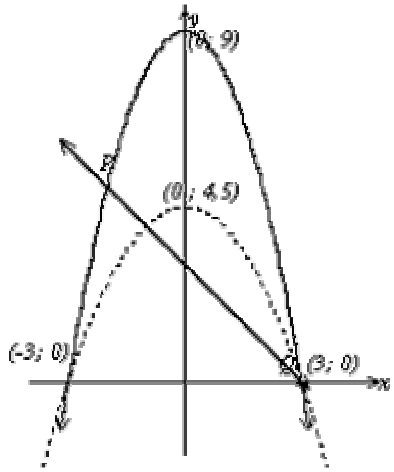
$= NZ\$\left(\frac{55 \times 6{,}7}{5{,}1265}\right)$ ✓✓✓

$= NZ\$\,72$ ✓

6.1 Parabola: intercepts ✓✓✓

shape ✓✓

Straight line: intercepts ✓



6.2 $-x^2 + 9 = -x + 3$ ✓

$\therefore 0 = x^2 - x + 6$

$\therefore 0 = (x - 3)(x + 2)$ ✓

$\therefore x = 3$ or $x = -2$

P is the point $(-2; 5)$ and Q is the point $(3; 0)$ ✓✓

6.3 $f(x) > 0$ between the point $(-3; 0)$ and Q i.e for $-3 < x < 3$ ✓✓

6.4 Dotted line ✓✓✓

7.1 $a.b^0 = 2$ $\therefore a = 2$ ✓✓

$2.b^1 = 6$ $\therefore b = 3$ ✓✓

7.2 $C(1;6)$ lies on the graph $\therefore 6 = \frac{p}{1} + q$ (Eqn. 1) ✓

$B(4;0)$. Also lies on the curve $\therefore 0 = \frac{p}{4} + q$. (Eqn. 2) ✓

From eqn.1 $q = 6 - p$

Subst. Into eqn. 2 $0 = p + 4 \times (6 - p)$

$\therefore 3p = 24$

$\therefore p = 8$ ✓

and $q = -2$ ✓

7.3 The equation of the horizontal asymptote of $g(x)$ is $y = -2$ ✓✓

8.1 $a = -1$; ✓

8.2 period of $f = 180^0$ ✓

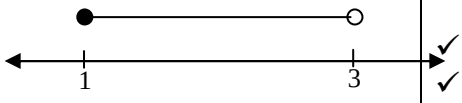
8.3 $b = 2$ and $c = 0$ ✓✓

8.4 range of $g = [-2;2]$ ✓

8.5 For $x \in \left[-180^0;-120^0\right] \cup \left(-90^0;0^0\right] \cup \left[120^0;180^0\right]$ ✓✓✓

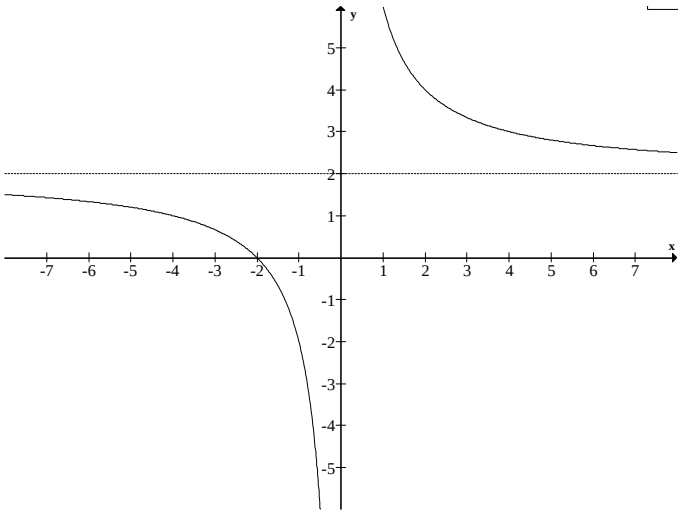
Exam B

Grade 10 Mathematics Exam **Paper 1**
Time: 2 hours **Marks: 100**

No	Solutions	Comments
1.1.1	$(2a-1)(a^2+5a-3)$ $=2a^3+9a^2-11a+3$	✓✓ simplifying ✓ answer (3)
1.1.2	$\frac{2^{a+1}.3^{a-1}}{6^a}$ $=\frac{2^{a+1}.3^{a-1}}{2^a.3^a}$ $=2^{a+1-a}.3^{a-1-a}$ $=\frac{2}{3}$	✓ exponential law (writing as prime bases) ✓ exponential law ✓ answer (3)
1.1.3	$\frac{3x+7}{5}-\frac{x-2}{3}$ $=\frac{3(3x+7)-5(x-2)}{15}$ $=\frac{9x+21-5x+10}{15}$ $=\frac{4x+31}{15}$	✓✓ numerator ✓ denominator ✓ answer (4)
1.2	$M^3 \times N^3 = 1000$ $M \times N = 10$	✓✓ dividing both sides by 5 ✓ answer (3)
1.3.1	$(x+1)(x-2)=4$ $x^2-x-2-4=0$ $x^2-x-6=0$ $(x+2)(x-3)=0$ $x=-2 \quad or \quad x=3$	✓ multiplying ✓ standard form ✓ factors ✓ answer (4)
1.3.2	$3^x = 50$ $3^3 = 27$ and $3^4 = 81$ $\therefore\ 3 < x < 4$ $\therefore\ x \approx 3{,}56$	✓✓ critical values ✓ answer (3)
1.4	$-2 \le 2x-4 < 2$ $2 \le 2x < 6$ $1 \le x < 3$ 	✓✓ critical values ✓✓ graph (4
1.5.1		✓ correct step

1.5.2	Step 2: $-2m(3k+2) \neq 2m(3k+2)$	✓ explanation (2)
	$5(2+3k) - 2m(3k+2)$ ..... (step 1) $= 5(2+3k) - 2m(2+3k)$..... (step 2) $= (5-2m)(2+3k)$ .....(step 3)	✓ ✓ (2)
1.6	No: the numberline is dense everywhere so the number of rational numbers on any interval is infinite	✓✓ explanation (2) [30]
2	Let the number of km @ 4 km/h = x ∴ the number of km @ 5 km/h = $22 - x$ ∴ $\frac{x}{4}+\frac{22-x}{5}=5$ ∴ $5x+88-4x=100$ ∴ $x=12$ ∴ 12 km @ 4 km/h and 10 @ 5 km/h	✓ ✓✓ setting up equation ✓ simplifying ✓ answer [5]
3.1	17 cm	✓✓
3.2	$T_n = 33-2n$	✓✓✓
3.3	$33-2n=0$ $n = 16\frac{1}{2}$ hours	✓✓ ✓ [8]
4.1	$T_n = n^2+n-1$	✓✓✓
4.2	$T_{50} = 50^2+50-1$ $= 2549$	✓ subst. ✓ answer [5]
5.1.1	848,5 Swiss Francs	✓✓ (2)
5.1.2	R311, 91	✓✓ (2)
5.1.3	R4761, 90	✓✓ (2)
5.1.4	In Japan computer will cost R6 000 In South Africa computer will cost R8 500 ∴ import provided cost of importing is less than R2 500	✓ ✓ ✓ (3)
5.2.1	$A = P(1+i)^n$ $A = 7000(1.1)^5$ $A = R11\,273{,}57$	✓ formula ✓✓ subst ✓ answer

5.2.2	$A = P(1+ni)$ $A = 7000[1+5(0.12)]$ $A = R11\,200$ $\therefore$ compound interest wi ll be more profitable	(4) ✓ formula ✓ subst ✓ answer (3) [16]
6.1	$y = a^x + q$ $9 = a^3 + 1$ $a^3 = 8$ $a = 2$ $q = 1$	 ✓✓ value of a ✓✓ value of q (4)
6.2	$y = 1$	✓✓ equation (2)
6.3.1	$h(x) = \left(\frac{1}{2}\right)^x + 1$ or $h(x) = 2^{-x} + 1$	✓✓ equation (2)
6.3.2	$h(x) = -2^x - 1$	✓✓ equation (2)
6.4	$2^x + 1 = \frac{17}{16}$ $2^x = 2^{-4}$ $\therefore \quad x = -4$	✓✓ setting up equation ✓ answer (3) [13]

7.1	$f(x) = \frac{7}{2}x - 7$	✓✓ equation (2)
7.2		✓ shape of graph ✓ asymptote ✓ x - intercept (3)
7.3.1	P(0 ; 9) R(3) $PR^2 = 9^2 + 3^2$ $= 90$ $\therefore PR = 9{,}49$	✓ coordinates of P and Q ✓ ✓ ✓ (4)
7.3.2	$-3 < x < 3$	✓✓ (2)
7.3.3	The graph will be narrower	✓✓ (2) [13]
8.1	$a = 2$; $q = 1$	✓ ✓
8.2	$x = 0^o$; 180; 360^o	✓✓
8.3	180^o	✓ [5]
9.1	Motorist M_1 travels at 150 km/h Motorist M_2 travels at 100 km/h	✓ ✓
9.2	M_1 : $y = 150x$	✓✓ formula
9.3	The co-efficient of x (the gradient)	✓ [5]

Grade 10 Assignment: Shape, Space and Measurement Marks: 75

NOTES FOR TEACHERS:

1. Learners require compasses, ruler, eraser, pencil and at least two unlined sheets of paper.
2. Where the question requires learners to plot points on a grid, they should either use the grid provided on the question paper or should be provided with grid paper. The grid must be square otherwise learners cannot make conjectures based on visual perceptions.
3. It is suggested that learners discuss their definitions of polygons with the teacher before proceeding with 4.1 – 4.8.
4. Where learners are asked to prove conjectures, any valid proof is acceptable. It does not have to match the proof in the memorandum.

1. Construction AB = 9cm and DC = 9cm; ✓✓ AB//DC ✓ (2)

   1.1. parallelogram ✓ (1)

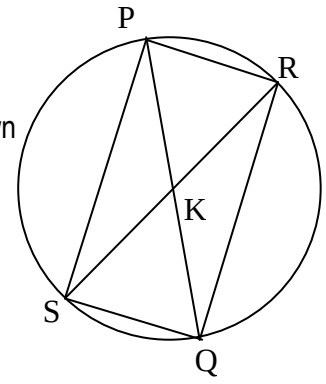
   1.2. AD=BC, $\hat{A} = \hat{C}$, $\hat{B} = \hat{D}$ (4)

   Let diagonals intersect at O, then AO = OC, DO = OB (any four) ✓✓✓✓

   (Note: conjectures about alternate angles are not conjectures about ABCD)

   1.3. 1 mark for each equal pair measured accurately✓✓✓✓✓

2. Construction of circle, radius 5 cm ✓ (1)



   2.1. 1 mark for each diameter. NB the diameter MUST be drawn accurately through circle centre. ✓✓ (2)

   2.2. PQ = 10cm.✓ Diameter is twice radius. ✓ (2)

   2.3. RK, SK, QK.✓✓ All radii of circle and radii equal. ✓ (2)

   2.4. Isosceles.✓✓ PK = RK (from 2.3)✓ (2)

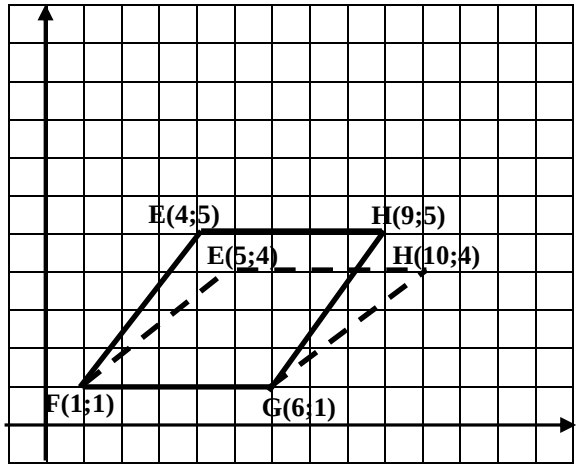
   2.5. Isosceles.✓ PK = KS (from 2,3)✓ (2)

   2.6. $K\hat{S}P = K\hat{P}S = x$ ..... base angles isos triangle ✓
   $K\hat{P}R = K\hat{R}P = y$ ..... base angles isos triangle ✓
   $\therefore 2x + 2y = 180°$.....angles of Δ ✓
   $\therefore x + y = 90°$ ✓ (4)
   $\therefore S\hat{P}R = 90°$

2.7. $P\hat{S}Q,\ S\hat{Q}R,\ Q\hat{R}S$ ✓✓ (2)

2.8. The proof is not ✓ complete, but the fact that $P\hat{S}Q,\ S\hat{Q}R,\ Q\hat{R}S$ are all right angles can be proved in the same way that it was proved that $S\hat{P}R = 90°$, in which case it would have been proved that PRQS is a rectangle (all angles $= 90°$). ✓ (2)

3. Plot F and G. ✓ (1)



Note: Two possible solutions in given quadrant.

3.1. 5 units ✓ (1)

3.2. Plotting E correctly ✓
Explanation: 3,4,5 Pythagorean triangle used. ✓✓ (3)

3.3. H plotted correctly. ✓ (1)

3.4. All sides equal.✓ (1)

3.5. $\text{Midpoint of EG} = \left(\frac{4+6}{2};\frac{5+1}{2}\right)$ $\text{Midpoint of EG} = \left(\frac{5+6}{2};\frac{4+1}{2}\right)$ ✓

$= (5;2)$ $= (5{,}5;2{,}5)$

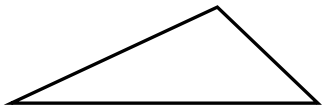
$\text{Midpoint of FH} = \left(\frac{1+9}{2};\frac{1+5}{2}\right)$ $\text{Midpoint of FH} = \left(\frac{1+10}{2};\frac{1+4}{2}\right)$ ✓

$= (5;2)$ $= (5{,}5;2{,}5)$

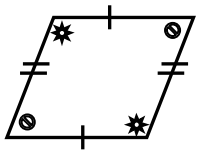
The diagonals have the same midpoint and therefore they bisect each other. ✓ (3)

4. Three or more sides; all sides equal; all angles equal.

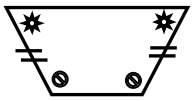
4.1. No. ✓ No equal sides. No equal angles.✓ (2)



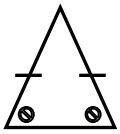
4.2. No. ✓ Equal sides. Opposite angles equal.✓ (2)



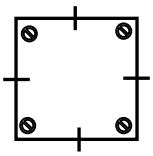
4.3. No. ✓ One pair of opposite sides equal.
Two pairs of equal angles.✓ (2)



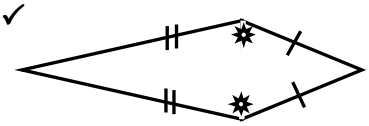
4.4. No.✓ Two equal sides. Two equal angles.✓ (2)



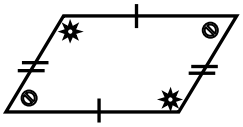
4.5. Yes.✓ All sides equal. All angles = 90°✓ (2)



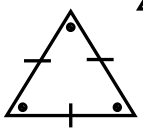
4.6. No.✓ Two pairs of equal sides. 1 pair of equal angles.✓ (2)



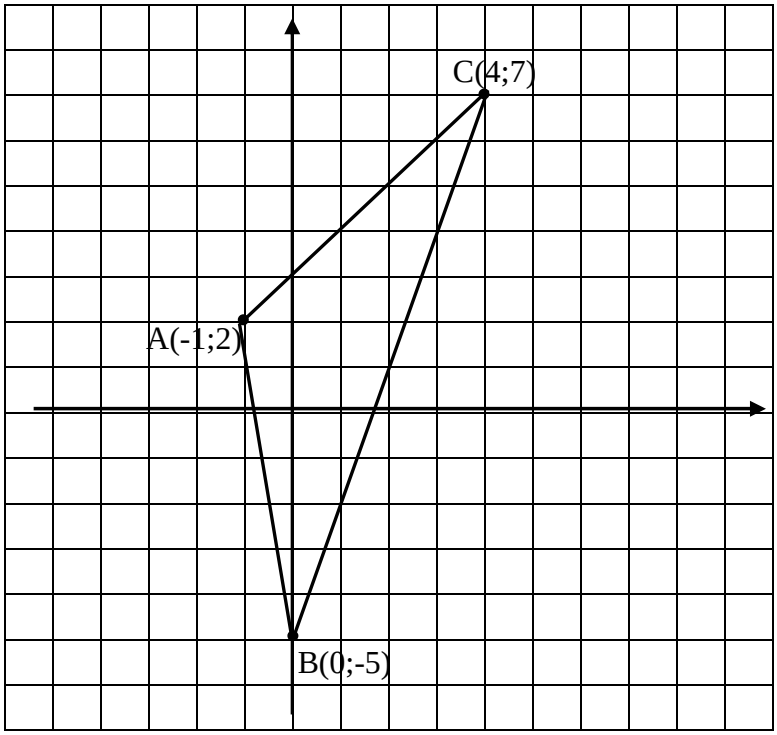
4.7. No.✓ Opposite sides equal. Opposite angles equal.✓ (2)



4.8. Yes.✓ All sides equal. All angles equal.✓ (2)



5.



5.1. Isosceles (1)

5.2.

$$AB = \sqrt{(-1-0)^2 + (2-(-5))^2} \quad \checkmark$$

$$= \sqrt{50}$$

$$AC = \sqrt{(-1-4)^2 + (2-7)^2} \quad \checkmark$$

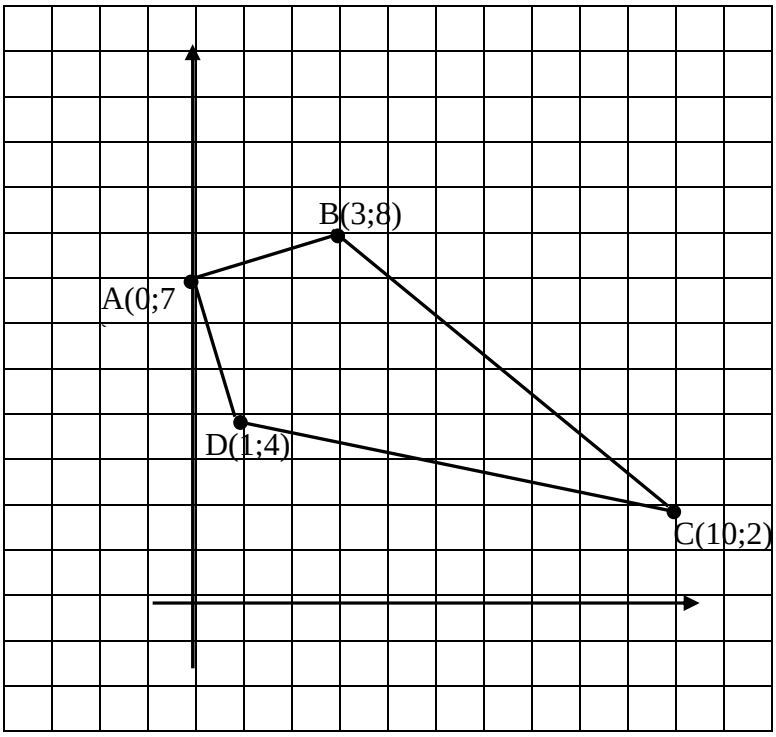
$$= \sqrt{50}$$

$$BC = \sqrt{(0-4)^2 + (-5-7)^2}$$

$$= \sqrt{160}$$

AB=AC therefore Δ ABC is isosceles. (5)

5.3.



One solution above. Drawing: 1 pair equal sides; 2 pairs equal sides (4)

5.4. Marks awarded as follows: AD AB BC DC
Both pairs of adjacent sides =, therefore a ABCD a kite (5)

Grade 10 Investigation: Trigonometry

Marks: 85

1.1 $\Delta ABC,\ \Delta DAC,\ \Delta EDC,\ \Delta EAC,\ \Delta DBA$ ✓✓✓(−1 for each error or omission)

1.2 $AB = 4\sqrt{6},\ AC = 4\sqrt{3},\ AE = \frac{8\sqrt{3}}{3},\ EC = \frac{4\sqrt{3}}{3},\ DE = \frac{4\sqrt{6}}{3}$ ✓✓✓✓✓

1.3 $\sin\alpha = \frac{BA}{BC} = \frac{AD}{AC} = \frac{DE}{DC} = \frac{AE}{AD} = \frac{BD}{AB}$ ✓✓✓✓✓

$\cos\alpha = \frac{AC}{BC} = \frac{DC}{AC} = \frac{EC}{DC} = \frac{DE}{AD} = \frac{AD}{AB}$ ✓✓✓✓✓

$\tan\alpha = \frac{AB}{AC} = \frac{AD}{DC} = \frac{DE}{EC} = \frac{AE}{AD} = \frac{BD}{AD}$ ✓✓✓✓✓

1.4 $\sin\beta = \frac{AC}{BC} = \frac{AD}{AB} = \frac{DE}{AD} = \frac{DC}{AC} = \frac{EC}{DC}$ ✓✓✓✓✓

$\cos\beta = \frac{AB}{BC} = \frac{BD}{AB} = \frac{AD}{AC} = \frac{AE}{AD} = \frac{ED}{DC}$ ✓✓✓✓✓

$\tan\beta = \frac{AC}{AB} = \frac{AD}{BD} = \frac{DC}{AD} = \frac{DE}{AE} = \frac{EC}{ED}$ ✓✓✓✓✓

2.1

	***x* co-ordinate**	***y* co-ordinate**	$\frac{x}{r}$	$\frac{y}{r}$	$\frac{y}{x}$
A_1	1,6	1,2	0,8	0,6	0,75
A_2	2,4	1,8	0,8	0,6	0,75
A_3	4	3	0,8	0,6	0,75
B_1	−1,2	1,6	−0,6	0,8	$-1,\dot{3}$
B_2	−1,2 −1,8	2,4	−0,6	0,8	$-1,\dot{3}$
B_3	-3	4	−0,6	0,8	$-1,\dot{3}$
C_1	−1,4	−1,4	−0,7	−0,7	1
C_2	−2,1	−2,1	−0,7	−0,7	1
C_3	−3,5	−3,5	−0,7	−0,7	1

✓✓✓✓✓✓✓✓✓✓

2.2 Each ratio is the same for the three points on the terminal ray of the same angle.
$\sin B\hat{O}D > 0$, $\cos B\hat{O}D < 0$ and $\tan B\hat{O}D < 0$ etc... ✓✓✓✓✓✓✓✓✓

3.1 $A\hat{O}B = 38^0$, $B\hat{O}D = 126^0$, $C\hat{O}D = 225^0$ ✓✓✓✓✓✓

3.2 They are the same ✓✓

3.3.1 $-1 \leq \sin\theta \leq 1$, $-1 \leq \cos\theta \leq 1$, no minimum or maximum for $\tan\theta$

✓✓✓✓✓✓✓✓✓✓✓

3.3.2 any correct values ✓✓

3.3.3 Any correct values ✓✓✓✓

3.4 Signs of the three ratios in the three quads

$\sin^2\theta + \cos^2\theta = 1$ for all θ and $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ for all $\theta \neq 90^0$, 270^0...

✓✓✓✓

Grade 10 Test: Trigonometry, Measurement and Coordinate Geometry
Time: 1 hour **Marks: 50**

1.

1.1. $\sin\theta = \frac{8}{15}$ ✓
$\theta = 32.23°$ ✓ (2)

1.2. $\cos\theta = \frac{2}{\sqrt{29}}$ ✓
$\theta = 68{,}20°$ ✓ (2)

1.3. $\tan\theta = \frac{\sqrt{5}}{3}$ ✓
$\theta = 36{,}70°$ ✓ (2)

2. $\cos 40° = \frac{24}{PQ}$ ✓
$PQ = \frac{24}{\cos 40°}$ ✓✓ (4)
$= 31{,}33m$ ✓

3.

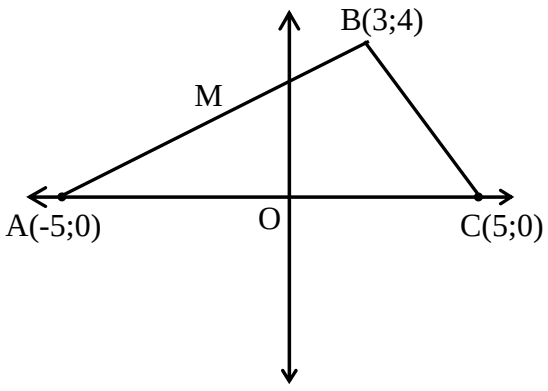
3.1. $XY = 66 \times 100$ ✓
$= 6600mm$ ✓ (2)

3.2. $XZ = 2900 - 2400$ (1)
$= 500mm$ ✓

3.3. $\tan X\hat{Y}Z = \frac{500}{6600}$ ✓ (2)
$X\hat{Y}Z = 4{,}33°$ ✓

4.

4.1. $M = \left(\frac{-5+3}{2}; \frac{4+0}{2}\right)$ ✓
$= (-1;2)$ ✓ (2)



4.2. $gradient\ of\ MO = \frac{2}{-1}$

$= -2$ ✓ (3)

$Equation\ of\ MO: y = -2x$ ✓✓

4.3. $gradient\ of\ BC = \frac{4-0}{3-5}$

$= -2$ ✓

$subst.(5;0)\ into\ y = -2x + c$

$0 = -10 + c$ ✓

$c = 10$ ✓ (4)

$\therefore eqn\ BC: y = -2x + 10$ ✓

4.4. $BC \,//\, MO \ldots equal\ gradients$ ✓ ✓ (2)

5. $AB = \sqrt{(12-3)^2 + (21-6)^2}$ ✓

$= \sqrt{306}$

$= 3\sqrt{34}$ ✓

$\therefore \frac{1}{3}AB = \sqrt{34}$ ✓

$AC = \sqrt{(5-3)^2 + (9-6)^2}$ ✓

$= \sqrt{13}$ (5)

$\therefore AC \neq \frac{1}{3}AB$ ✓

6.

6.1. $\frac{1.25}{1.5} = 0.83$ ✓

$\frac{1.0}{1.25} = 0.8$ ✓

The scale factors are similar, but not exactly equal. ✓ (3)

6.2. $1.5l = 1500ml = 1500cm^3$ ✓

$1.25l = 1250ml = 1250cm^3$ ✓

$1.0l = 1000ml = 1000cm^3$ ✓ (3)

6.3. $v = l \times b \times h$ ✓

$\therefore 1500 = 15 \times 12 \times h$ ✓

$\therefore h = \frac{1500}{15 \times 12}$

$\therefore h = 8{,}3cm$ ✓ (3)

6.4. length = 14cm; breadth = 11cm ✓ ✓ (2)

6.5. $Area = 13\times10 + 2\times13\times7{,}7 + 2\times10\times7{,}7$ ✓✓ (3)

$= 484.2cm^2$ ✓

7. Cube

$V_1 = (x)(x)(x)$

$= x^3$ ✓

$V_2 = (2x)(2x)(x)$

$= 4x^3$ ✓

Cylinder

$V_1 = \pi(\frac{y}{2})^2(y)$

$= \frac{\pi y^3}{4}$ ✓

$V_2 = \pi(\frac{2y}{2})^2(y)$

$= \pi y^3$ ✓ (5)

✓

In each case, the volume has increased by a factor of 4, so volumes will be equal.

Grade 10 Project: Introduction to Data Handling Marks: 55

Note to teachers

Before the learners embark on this project, it may be worthwhile doing a class work exercise where they decide on the standard questions they will ask and develop a questionnaire. They will also need to warn their sample group in advance about their intentions so that they can measure their distances.

Part 1

5	4	3	2	1	0
Logical method of recording, grouped well, clear distinction between subjects	Clear records, good, accurate information collected, well presented	Good records made	Records are present but randomly presented	Disorganized recording, messy, incomplete	No attempt

Part 2

- Modal transport ✓✓
  Mean Time✓
  Median Time✓
  Mean Distance✓
  Median Dist✓ (6)

- Not numeric, any reasonable explanation ✓✓

- Data very varied, any reasonable explanation ✓✓

- Suitable intervals ✓✓
  Correct groupings ✓✓
  Title of histogram ✓
  Labelling of axes ✓
  Correct representation of data ✓✓✓✓✓✓

- Correct answer read off histogram ✓✓

- Good explanation, discussing possibly availability of public transport, wealth of learners, school environment, degree to which school is attended by those in the community etc ✓✓✓✓ (28)

Part 3

Scatter plot: Title ✓
Axes ✓✓
Plotting of points ✓✓✓✓✓✓✓
Discussion: Correctly interpreting scatter plot, making mention of type of correlation, if any and good explanation for the reason ✓✓✓✓ (10)

Part 4

4	3	2	1	0
Well presented, accurate calcultions with accurate chart	Correct calcultions, correct chart	Correct calculations, inaccurate chart	Inaccurate, incomplete	No attempt

Part 5

4	3	2	1	0
Good conclusion, tying up different findings well, clearly understood	Good explanations, using some findings	Discussion without using findings	Attempted, but inconclusive	No attempt

Exam A

Grade 10 Mathematics Exam **Paper 2**
Time: 2 hours **Marks: 100**

Question 1

1.1 $m_{PQ} = \frac{9-6}{-5-4}$ ✓
$= \frac{3}{-9}$ ✓
$= -\frac{1}{3}$ ✓ (3)

1.2 $PQ \perp PS \therefore m_{PS} = 3$ ✓
$\therefore \frac{9-3}{-5-x} = 3$ ✓✓
$\therefore 6 = 3(-5-x)$
$\therefore 6 = -15-3x$ ✓
$\therefore x = -7$ ✓ (5)

1.31 $m_{SR} = \frac{3-1}{-7+1} = -\frac{1}{3}$ ✓✓ (2)

1.3.2 $SR \| PQ$ ✓, same gradient ✓ (2)

1.4.1 $SP = \sqrt{(-7+5)^2 + (3-9)^2} = \sqrt{40}$ ✓
$SR = \sqrt{(-7+1)^2 + (3-1)^2} = \sqrt{40}$ ✓
$SP \perp SR$ $(proven\ PQ \| SR)$ ✓
ΔPRS is a right angled, isosceles triangle ✓✓ (5)

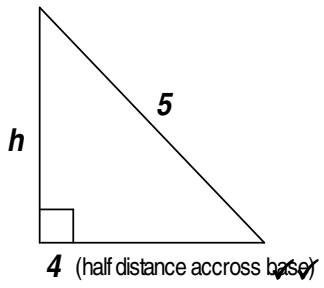
1.4.2 $Area \Delta PRS = \frac{1}{2} \cdot PS \cdot SR$
$= \frac{1}{2} \cdot \sqrt{40} \cdot \sqrt{40}$ ✓✓
$= 20$ ✓ (3)

Question 2

2.1 $Area\ base = 8 \times 8 = 64cm^2$ ✓✓ (2)

2.2 $Area\ \Delta = \frac{1}{2} \times 8 \times 5 = 20$ ✓✓
✓✓
$\therefore TSA = 4 \times 20 + 64 = 144cm^2$ ✓ (5)

2.3



$\therefore h = \sqrt{25-16} = 3cm$ ✓✓ (4)

2.4 $V = \frac{Area\ base \times height}{3}$ ✓
$= \frac{64 \times 3}{3} = 64cm^3$ ✓✓ (3)

Question 3

3.1 L ✓ 3.2 C ✓ 3.3 G ✓✓ 3.4 D ✓✓ (6)

✓ ✓ ✓ ✓

3.5 reflected about the x-axis followed by a reflection about the y-axis (or other way round) or rotate through the origin by 180° (4)

3.6 $(x;y) \rightarrow (-x;y)$ ✓✓ (2)

✓ ✓

3.7 reflection about $y = x$ followed by reflection in the y-axis ✓✓ or rotate through the origin 90° anti-clockwise (4)

Question 4

4.1.1 $\sin\theta = \frac{5}{13}$ ✓ (1)

4.1.2 $AC = 12$ (pythag) ✓

4.1.3 $\cos^2\theta = \left(\frac{12}{13}\right)^2 = \frac{144}{169}$ ✓✓ (2)

$\therefore 2\tan\theta = 2 \cdot \frac{5}{12} = \frac{5}{6}$ ✓✓ (3)

4.2.1 $\frac{BC}{3} = \sin 30$ ✓✓

$\therefore BC = 3\sin 30 = 1{,}5\ units$ ✓✓ (4)

4.2.2 $\frac{CD}{3} = \cos 30$ ✓

$\therefore CD = 3\cos 30 = 2{,}60\ units$ ✓✓ (3)

4.2.3 $BE = 2{,}60$ (rect BCDE) ✓

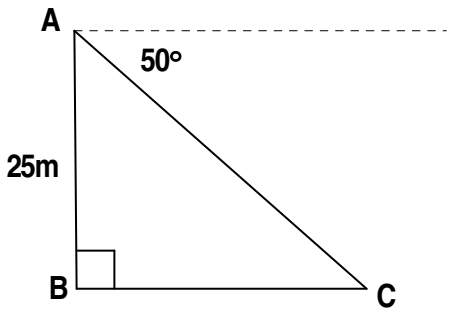
$ED = 1{,}5$ (rect BCDE) ✓

$\frac{AE}{2.60} = \tan 20$ ✓

$\therefore AE = 2{,}60 \cdot \tan 20 = 0{,}95$ ✓

$\therefore AD = 2{,}45\ units$ ✓ (5)

Question 5



$A\hat{C}B = 50^\circ$ (alt angles) ✓

$\frac{25}{BC} = \tan 50^\circ$ ✓ $\therefore 25 = BC \cdot \tan 50^\circ$ ✓

$\therefore \frac{25}{\tan 50^\circ} = BC$ ✓

$\therefore BC = 20{,}98m$ ✓

✓

$\therefore$ shark is 15,98 m from swimmer

Question 6

6.1 $range = 287 - 70 = 217$ ✓✓ (2)

6.2.1 $\bar{x} = \frac{287 + 268 + 283 + 283 + 270 + 70 + 283 + 275}{8}$ ✓✓

$= 252{,}375$ ✓ (3)

6.2.2 70, 268, 270, 275, 283, 283, 283, 287

$\therefore median = \frac{275 + 283}{2} = 279$ ✓✓ (2)

6.2.3 $\text{mod}\, e = 283$ ✓ (1)

6.3 mean has greatest increase ✓ mode unaffected and median only slightly ✓ although sum drops by 70, divisor drops too and therefore answer significantly changes ✓ (3)

Question 7

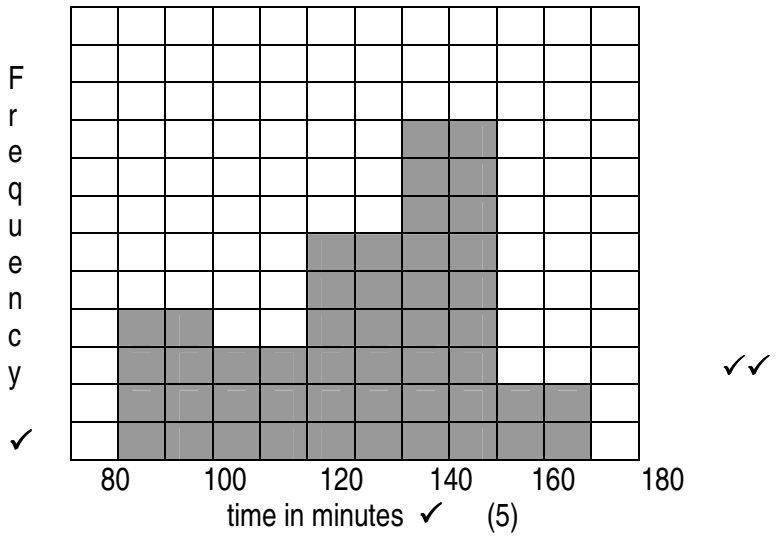
7.1

Time in minutes	Frequency
80 - 99	4
100 - 119	3
120 - 139	6
140 - 159	9
160 - 179	2

✓✓✓✓✓ (5)

7.2

Cell phone usage over one week ✓



✓ ✓✓ ✓ (5)

7.3 modal class: 140 – 159 ✓ (1)

7.4 $\frac{17}{24} \times 100 = 70.83\,\%$ ✓✓ (2)

7.5 any acceptable comment based on data displayed ✓✓ (2)

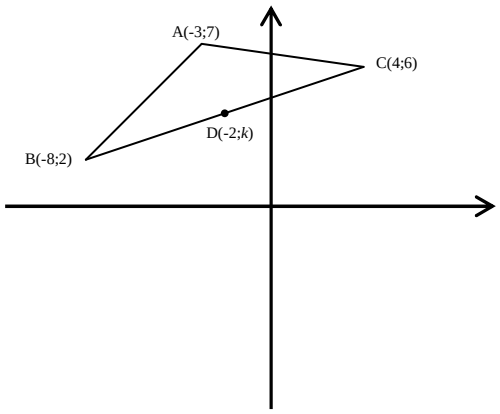
Grade 10 Mathematics Exam **Paper 2**
Time: 2 hours **Marks: 100**

Question 1

1.1 $Gradient\ of\ BC = \frac{6-2}{4-(-8)}$ ✓

$= \frac{4}{12}$ ✓

$= \frac{1}{3}$ ✓



(3)

1.2 $mBC \times mDA = -1$ ✓

$\frac{1}{3} \times \frac{k-7}{-2-(-3)} = -1$ ✓

$\frac{k-7}{3} = -1$ ✓

$k - 7 = -3$ ✓

$k = 4$

(4)

1.3 $Length\ BC = \sqrt{(4-(-8)^2 + (6-2)^2}$ ✓✓

$= \sqrt{160}$

$= 12{,}65$ ✓

(3)

1.4 $Length\ \ height\ AD = \sqrt{(-3-(-2))^2 + (7-4)^2}$ ✓

$= \sqrt{10}$

$= 3{,}16$ ✓

$Area\ \Delta ABC = 0{,}5 \times \sqrt{160} \times \sqrt{10}$ ✓ ✓

$= 20\ units^2$ ✓

(5)

✓✓

1.5 $E = (-8+7;\ 2-1) = (-1;1)$ (2)

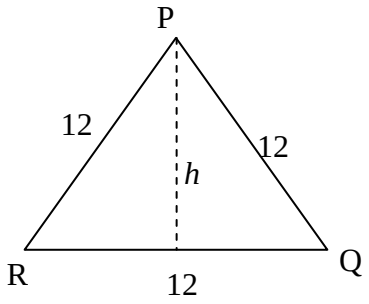
1.6 $AC \parallel BE$ ✓, translation results in gradients ✓ $\left(\frac{change\ in\ y}{change\ in\ x}\right)$ being equal. (2)

1.7 Transformation from A to B: down five units, left five units ✓
Transformation from C to E: down five units, left five units. ✓ (2)

1.8 ACEB is a parallelogram because both pairs of opp. sides are parallel. ✓✓ (2)

Question 2

2.1



2.1.1 $\frac{h}{12} = \sin 60°$ ✓ (3)

$h = 12\sin 60°$ ✓

$= 10.39cm$ ✓

or

ΔPQR *is equilateral*, $\therefore h$ *is* $\perp$ *bisector of* RQ ✓

Using Pythagoras :

$h^2 = 12^2 - 6^2$

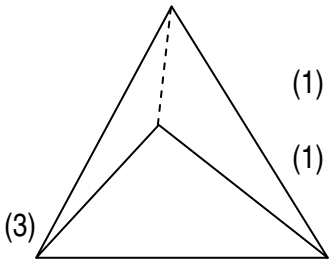
$= 108$

$h = 10{,}39$ ✓

2.1.2 $Area\ \Delta = 0{,}5 \times 12 \times 10{,}39$ ✓ (2)

$= 62{,}34\ units^2$ ✓

2.2



2.2.1 $Perimeter\ of\ base = 36\ units$ ✓ (1)

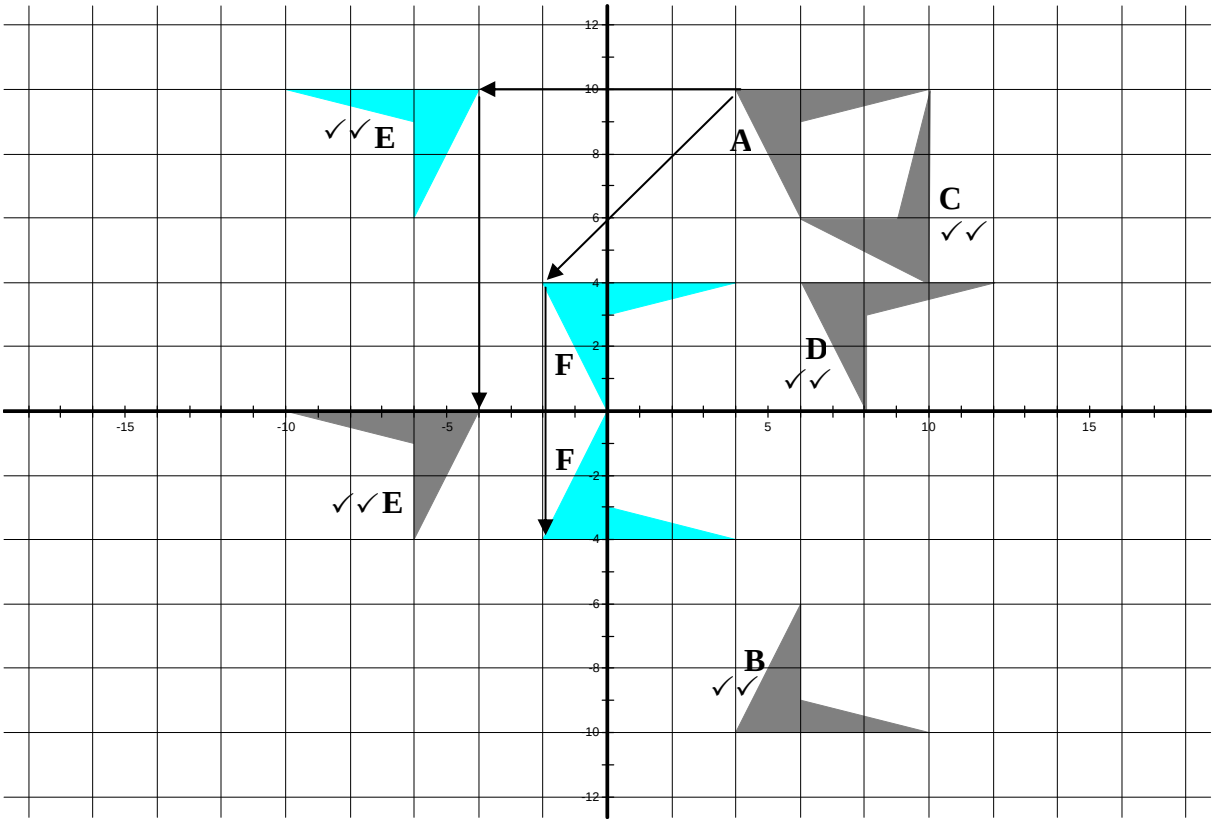
2.2.2 *The slanted triangles are congruent to the base.* ✓ (1)

2.2.3 $Total\ surface\ area = 4 \times Area\ \Delta$ ✓ (3)

$= 4 \times 62{,}34$

$= 249{,}36\ units^2$ ✓

Question 3

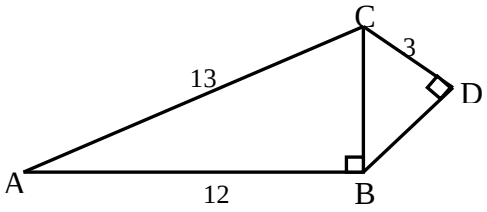


3.1.1 On diagram (2)
3.1.2 On diagram (2)
3.1.3 On diagram (2)
3.1.4 On diagram (4)
3.1.5 A is moved six units right and six units down and then reflected about the x-axis. (2)

Question 4

4.1

$$CB = \sqrt{13^2 - 12^2}$$
$$= 5\ units$$
$$BD = \sqrt{25 - 9}$$
$$= 4\ units$$



(4)

4.2

4.2.1 $\tan A = \frac{5}{12}$ (1)

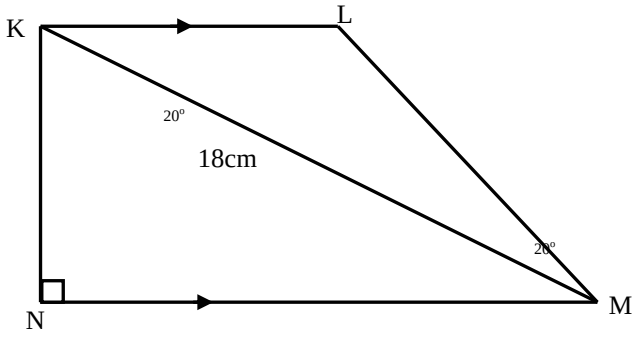
4.2.2 $\sin C\hat{B}D = \frac{3}{5}$ (1)

4.2.3 $\tan A\hat{C}B + \cos D\hat{B}C = \frac{12}{5} + \frac{4}{5}$
$= \frac{16}{5}$ (3)

4.3 $A = \cos^{-1}\left(\frac{12}{13}\right)$
$= 22{,}6°$ (2)

Question 5

5.1



5.1.1 $NMK = 20° \ (alt.\angle s \parallel lines)$ ✓

$\frac{KN}{18} = \sin 20°$ ✓ (3)

$KN = 18\sin 20°$

$KN = 6{,}16cm$ ✓

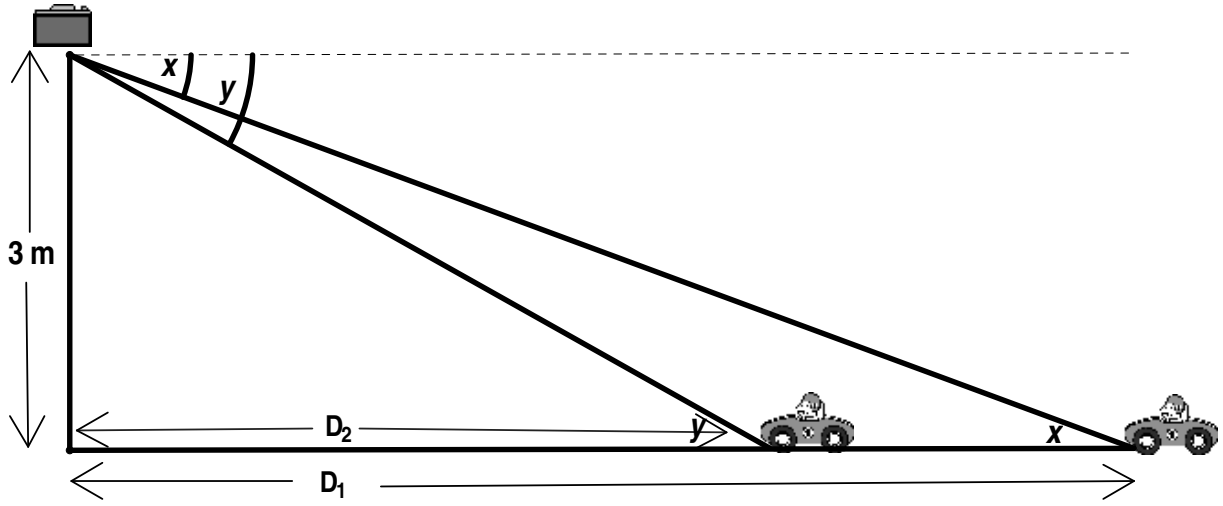
5.1.2 $NM = 18\cos 20°$

$= 16{,}91cm$ ✓

$Area\ KLMN = 0{,}5\times(16{,}91+14)\times 6{,}16$ ✓ (3)

$= 95{,}2\ units^2$ ✓

Question 6



6.1 $\tan x = \frac{3}{D_1}$ ✓ $\tan y = \frac{3}{D_2}$ ✓

$D_1 = \frac{3}{\tan x}$ ✓ $D_2 = \frac{3}{\tan y}$ (3)

6.2 *Distance covered in 1 second:*

$$Dist = \frac{3}{\tan 3{,}5^\circ} - \frac{3}{\tan 6^\circ} \quad \checkmark \quad \checkmark$$
$$= 20{,}51m \quad \checkmark$$
(3)

6.3 $21.51m/\sec = 0{,}02151Km/\sec \quad \checkmark$
$= 0{,}02151 \times 3600Km/hr$
$= 77{,}43Km/hr$ (2)

Question 7

7.1

	AB	Ashwell
	15	5
	25	60
	30	70
	40	5
	30	5
	10	10
	30	60
Sum	**180**	**215**
N	**7**	**7**
Mean	**25.71**	**30.71**
Mode	**30**	**5**

7.1.1 $AB's\ mean = \frac{180}{7}$ ✓ $= 25{,}71\ runs$ ✓ $\quad Ashwell's\ mean = \frac{215}{7}$ ✓ $= 30{,}71\ runs$ ✓ (4)

7.1.2 *AB's mode = 30* ✓ *Ashwell's mode = 5* ✓ (2)

7.1.3 Would advise coach to select AB. Although his average is lower, his modal number of runs is higher, indicating that he is more reliable. Ashwell's mean number of runs has been affected by a few high scores, but he is not consistent. ✓ ✓ (2)

Question 8

8.1 128 men and 243 women took part in the survey. ✓ ✓ (2)

8.2 Multiply the midpoint of each class by the class frequency. Sum these numbers and divide by the number of men surveyed (128). ✓ ✓ ✓ (3)

8.3 90 – 119 ✓ (1)

8.4 $\frac{59}{243} \times 100 = 24{,}3\%$ ✓ ✓ (2)

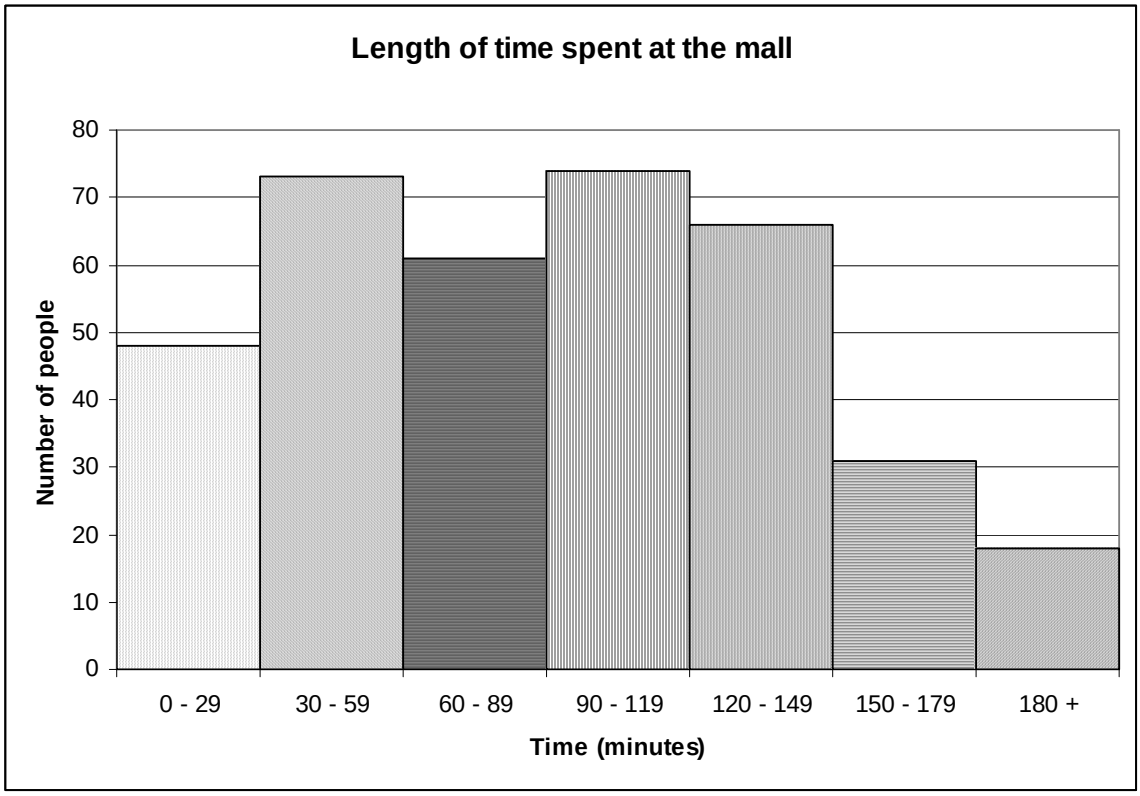
8.5 90 – 119 ✓ (2)

8.6 The statement does not take into account the fact that significantly more women than men visit the mall overall. If the frequency for the class 60 – 89 minutes is converted to a percentage for both men and women, then the figures can be compared directly. The percentages are similar, as shown below. ✓ (3)

$$\%\ women = \frac{41}{243} \times 100 = 16{,}87\%$$ ✓

$$\%\ men = \frac{20}{128} \times 100 = 15{,}62\%$$

8.7



8.8 Accept any valid and substantiated observations, for example:

- The most number of people spend either 30 – 59 minutes or 90 – 119 minutes at the mall (the data is effectively bi-modal).
- Very few people spend more than 180 minutes at the mall.
- There is not a significant difference in the number of people visiting the mall in the time range from 30 minutes to149 minutes.