Grade 12

Educators Guide

2008

Grade 12 Assessment Exemplars

1 Learning Outcomes 1 and 2

2 Learning Outcomes 3 and 4

Grade 12 Assignment: Functions **Marks:**

1 In each case for each value of *x* there is only one associated output value

2 a) 2 b) 1 c) 3

3 Those points where the function values are zero which means they are on the *x*- axis/ The points where the function cuts the *x*-axis

4 The possible number of *x*-intercepts are determined by the degree of *x* in the function
1st degree: one *x*-intercept
2nd degree: two possible *x*-intercepts
3rd degree: three possible *x*-intercepts
In the case of 2nd and 3rd degree functions two of the roots may be the same or non-real.

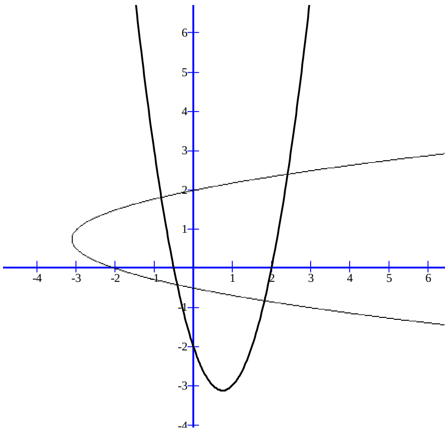
5 *y* is of the first degree in each case hence only one *y*-intercept

6 a) min. value b) no min. or max. value c) local min. and local max. values.

The points where a function has a min or max value are also called stationary points. These are determined where the gradient of a tangent to the function (derivative) equals zero). To determine whether the point is a min. or max. one has investigate the derivative/gradient on either side of the stationary points. Min or max. can also be determined by the value of the 2nd derivative.

7

7.1 and 7.2



7.3 $f^{-1}(x)$ not a function – one-to-many mapping

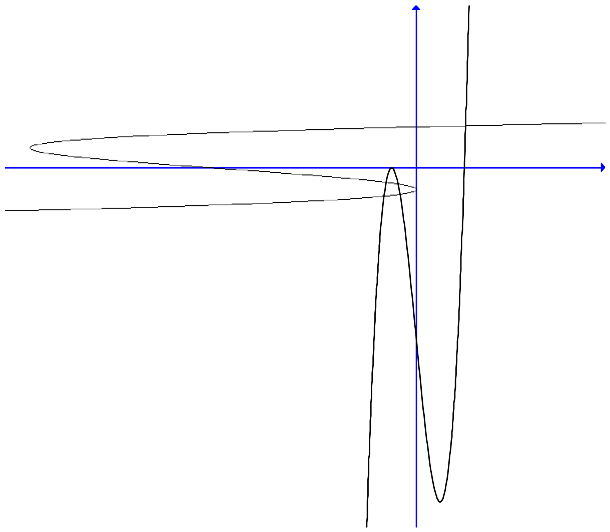
7.4 domain needs to be restricted to either

$$x \leq \frac{3}{4} \text{ or } x \geq \frac{3}{4}$$

7.1 and 7.2

7.3 $g^{-1}(x)$ is a function – one-to-one mapping

7.1 and 7.2



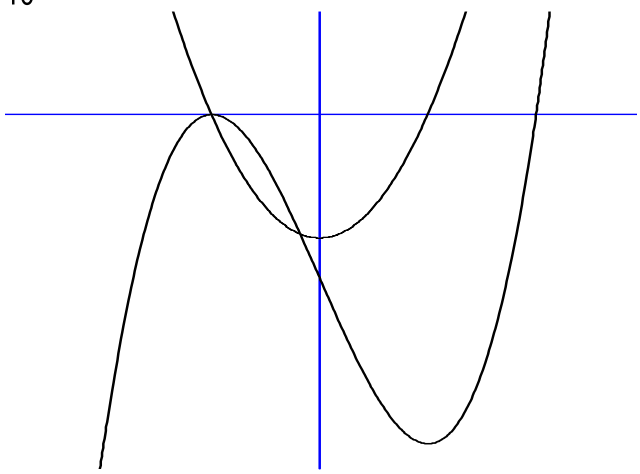
7.3 $f^{-1}(x)$ not a function – one-to-many mapping

7.4 domain needs to be restricted to $x \leq -2$ or
$-2 \leq x \leq 0$ or $x \geq 0$

8 *h*(2) and g(2) gives the function value when *x* = 2. *h′*(2) and g′ (2) gives value of the gradient (derivative) when *x* = 2.

9 0

10



11 If $f(x) = 2x^2 - 3x - 2$ then $f^{/}(x) = 4x - 3$ and $f^{//}(x) = 4 \neq 0$ thus no points of inflection.
If $g(x) = -5x + 10$ then $g^{/}(x) = -5$ thus constant gradient so no points of inflection.
If $h(x) = x^3 - 12x - 16$ then $h^{/}(x) = 3x^2 - 12$ and $h^{//}(x) = 6x$. $h^{//}(x) = 0$ when *x* = 0 thus *h*(*x*) has a point of inflection when *x* = 0.

12 12.1 *k*(*x*) is the reflection of *k*(*x*) in the *x*-axis
12.2 *k*(– *x*) is the reflection of *k*(*x*) in the *y*-axis
12.3 *k*(*y*) is the reflection of *k*(*x*) in the line *x* = *y*

Grade 12 Investigation: The Koch Snowflake **Marks: 100**

Section A

4^{th} stage snowflake on dotty paper (20)

Section B

1. 20 marks for first 5 rows (in any form) -2 for each error or omission (4 X 5)

Snowflake	A:Length of 1 side	B: Number of sides	C:Perimeter
1	1	3	3
2	3^{-1}	4 x 3	$3\left(\frac{4}{3}\right)$
3	3^{-2}	4^2x3	$3\left(\frac{4}{3}\right)^2$
4	3^{-3}	4^3x3	$3\left(\frac{4}{3}\right)^3$
5	3^{-4}	4^4x3	$3\left(\frac{4}{3}\right)^4$
n	3^{1-n}	$4^{n-1}\times 3$	$3\left(\frac{4}{3}\right)^{n-1}$

2. Last row of table above (5 X 3)

3. Column A: geometric sequence: $a=1$, $r=\frac{1}{3}$

   Column B: geometric sequence: $a=3,\quad r=4$

   Column C: geometric sequence $a=3,\quad r=\frac{4}{3}$ any valid observations (10)

4. The perimeter at each stage is a term of a diverging geometric sequence since $r=\frac{4}{3}$.

   Hence the perimeter increases without limit.

   Correct answer and valid explanation (5)

Section 3

1. 20 marks for first 5 rows (20)

Stage	A: Area of each added triangle	B: number of triangles added	C: Increase in area	D: Total area
1				1
2	$\frac{1}{9}=\left(\frac{1}{3}\right)^2$	3	$3\left(\frac{1}{9}\right)=\frac{1}{3}$	$1+\frac{1}{3}$
3	$\frac{1}{81}=\left(\frac{1}{3}\right)^4$	4×3	$4\times 3\times\frac{1}{81}=\frac{1}{3}\left(\frac{4}{9}\right)$	$1+\frac{1}{3}+\frac{4}{27}$
4	$\left(\frac{1}{3}\right)^6$	$4^2\times 3$	$\frac{1}{3}\left(\frac{4}{9}\right)^2$	$1+\frac{1}{3}+\frac{4}{27}+\frac{16}{81}$
5	$\left(\frac{1}{3}\right)^8$	$4^3\times 3$	$\frac{1}{3}\left(\frac{4}{9}\right)^3$	$1+\frac{1}{3}+\ldots+\frac{64}{729}$
n	$\left(\frac{1}{3}\right)^{2(n-1)}$	$4^{n-2}\times 3$	$\frac{1}{3}\left(\frac{4}{9}\right)^{n-2}$	$1+\sum_{n=1}^{\infty}\frac{1}{3}\left(\frac{4}{9}\right)^{n-1}$

2. $S_n=\dfrac{\frac{1}{3}\left(1-\left(\frac{4}{9}\right)^n\right)}{1-\frac{4}{9}}=\frac{3}{5}\left(1-\left(\frac{4}{9}\right)^n\right)$ ie. where $T_1=a=\frac{1}{3}$ and $r=\frac{4}{9}$ (5)

3. As $n\to\infty$, $S_n\to\frac{3}{5}$ and the total area $\to 1+\frac{3}{5}=1{,}6$ (2)

4. As the number of stages gets very large, the perimeter of the snowflake increases without limit, but the area approaches a fixed limit. (3)

Grade 12 Test: Number Patterns, Finance and Functions

Time: 1 hour **Marks: 50**

1.

1.1. $f(x) = a^x$

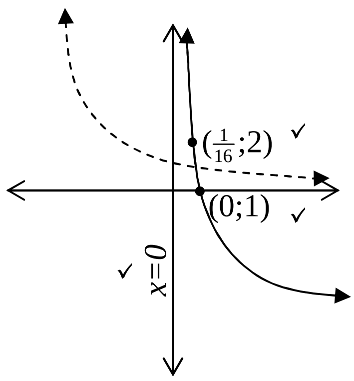
$substitute\ (2;\tfrac{1}{16})$

$\therefore \tfrac{1}{16} = a^2$ ✓

$\therefore (\tfrac{1}{4})^2 = a^2$

$\therefore a = \tfrac{1}{4}$ ✓ (2)

1.2.



(3)

1.3. $p = -2;\ q = -1$ ✓ ✓ (2)

[7]

2.

2.1. The area of the rectangle depends on the ✓ value of *x*. For each value ✓ of *x* there is only one value for the area.

2.2. $35 + 2x - x^2 = 20$ ✓

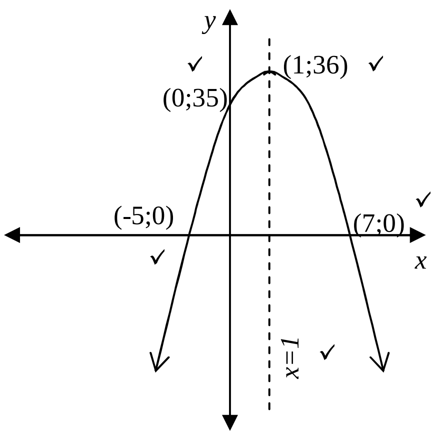
$\therefore x^2 - 2x - 15 = 0$

$\therefore (x-5)(x+3) = 0$ ✓

$\therefore x = 5\ \ (x = -3\ invalid)$ ✓ (3)

2.3. *x*-intercepts: (-5;0) and (7;0)
y-intercept: (0;35)
turning point: (1;36)
axis of symmetry: *x* = 1

Note: full marks if Quad I only.



(5)

2.4.

2.4.1. $0 < x < 7$ ✓✓ (2)

2.4.2. $x > 1$ ✓✓ (2)

2.4.3. 36 square units ✓✓ (2)

[17]

3.

3.1. $a = 6; \quad r = 3$ (2)

3.2. $6.(3)^{n-1} = 1458$

$3^{n-1} = 243$ (4)

$3^{n-1} = 3^5$ **[6]**

$n = 6$

4.

	Stage 1	Stage 2	Stage 3	Stage 4	Stage *n*
Number of patterned tiles	3	5	7	9	$2n+1$
Number of black tiles	1	4	9	16	n^2
Number of white tiles	2	6	12	20	n^2+n
Total number of tiles	6	15	28	45	$2n^2+3n+1$

$$Patterned\ tiles: \ T_n = 3+(n-1)2 \ \checkmark$$
$$T_n = 2n+1 \ \checkmark$$
$$Black\ tiles: \ T_n = n^2 \ \ (by\ inspection) \ \checkmark$$ (5)
$$Total\ number\ of\ tiles: \ T_n = 2n+1+n^2+n^2+n \ \checkmark$$
$$= 2n^2+3n+1 \ \checkmark$$

4.1.

$2n+1 = 21$ ✓

$2n = 20$ ✓ (3)

$n = 10$ ✓

10th stage will require 21 patterned tiles.

4.2. $Area = 0.25(2n^2+3n+1)\ \text{m}^2$ ✓✓ (2)

[10]

5.

5.1. $A = P(1+i)^n$

$= 42\,310(1+.07)^5$ ✓✓ (4)

$= R59\,340$ ✓✓

5.2. $59\,340 = x\left(1+\frac{0,095}{12}\right)+x\left(1+\frac{0,095}{12}\right)^2+x\left(1+\frac{0,095}{12}\right)^3+....to\ 60\ terms$ ✓ ✓✓ (6)

$$\frac{x\left(1+\frac{0,095}{12}\right)\left[\left(1+\frac{0,095}{12}\right)^{60}-1\right]}{\frac{0,095}{12}} \ \checkmark$$

$x = R770.38$ ✓✓

Grade 12 Project: Finance **Marks: 50**

Notes to Educator

1. The data sheet for this project should be updated to reflect current vehicle prices, interest rates and rate of inflation.

2. Current and historical data on the fixed and linked lending rates of banks can be obtained from the South African Reserve Bank internet site:
   http://www.reservebank.co.za/economics/netqb2/NetQbsSearch.asp

   The required document codes are: KBP1181M AND KBP1182M

3. Before learners commence the project, the following concepts and terminology should be discussed in class:
   loan, interest, compound interest, annuity, present value, future value, outstanding balance, depreciation, sinking fund, inflation and rate of inflation.

Assessment Guidelines

1. repayment calculation for fixed- and linked rate options:
   substitution✓✓ simplification ✓✓ payment✓✓ total✓✓ (8)

2. balance calculation: formula✓ substitution✓ simplification✓ answer ✓
   repayment calculation: substitution✓simplification✓payment ✓total ✓
   comment✓ (9)

3. calculation of balloon payment✓
   calculation of loan amount✓
   repayment calculation: substitution✓ simplification✓ payment✓ total✓ (6)

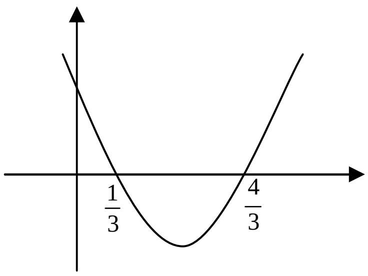
4. calculation of difference between payments✓✓
   annuity calculation: substitution✓ simplification✓ value of *n*✓✓ (6)

5.
   5.1. substitution✓ answer✓ (2)
   5.2. substitution✓ answer✓ (2)
   5.3. F_v of sinking fund✓ substitution✓ simplification✓ answer✓ (4)

6. Method used: (5)
   Deducts F_v of T_{12}, T_{24}, T_{36}, T_{48}✓✓✓ simplification✓ answers✓
   Any other method that has mathematical merit and correct calculations, but does not consider the F_v of the holiday payments that are missed✓✓✓✓

7. Valid comment✓ substantiation using calculated figures from above✓ (2 x 4) (8)
   in respect of each of the following:
   fixed rate versus linked rate
   balloon payment
   payment holiday
   creation of a sinking fund

Grade 12 Mathematics Exam **Paper 1**
Time: 3 hours **Marks: 150**

No.	Solutions	Comments
1.1.1	$(x-4)(x-3)=2$ $x^2-7x+12-2=0$ $x^2-7x+10=0$ $(x-2)(x-5)=0$ $\therefore x=2$ or 5	✓ multiplying ✓ standard form ✓ factors ✓ x values (4)
1.1.2	$3x^2+2x+6=10$ $3x^2+2x-4=0$ $x=\frac{-2\pm\sqrt{4-4(3)(-4)}}{6}$ $=\frac{-2\pm\sqrt{52}}{6}$ $x=0{,}87$ or $-1{,}54$	✓ standard form ✓ formula ✓ subst ✓ simplification ✓ x values (5)
1.1.3	$(3x-2)^2>3x$ $9x^2-12x+4-3x>0$ $9x^2-15x+4>0$ $(3x-1)(3x-4)>0$ $x<\frac{1}{3}$ or $x>\frac{4}{3}$  	✓ simplification ✓ standard form ✓ factors ✓ (graph or any other method used) ✓✓ critical values/inequality (6)
1.2	$4y+3x=50$ and $x^2+y^2=100$ $y=\frac{50-3x}{4}$ $\therefore x^2+\left(\frac{50-3x}{4}\right)^2=100$ $\therefore \frac{16x^2+2500-300x+9x^2}{16}=100$ $\therefore 25x^2-300x+900=0$ $\therefore x^2-12x+36=0$ $(x-6)^2=0$ $x=6$ $y=8$	✓ making y the subject ✓ subst ✓ simplification ✓ standard form ✓ factors ✓ x value ✓ y value (7) [22]

2.1	$A = P(1+i)^n$ $320470 = 150000\left(1+\frac{r}{12}\right)^{72}$ $r = 12{,}72\%$ $1+i = \left(1+\frac{12{,}72}{12}\right)^{12}$ $i = 13{,}49\ \%$	✓✓ for calculating nominal interest rate ✓✓✓ for calculating effective interest rate [5]
2.2.1	Loan amount = R114 800 Let monthly instalment be x $x = \dfrac{114800\left(1+\frac{0{,}13}{12}\right)^{60}\left[\left(1+\frac{0{,}13}{12}\right)-1\right]}{\left(1+\frac{0{,}13}{12}\right)^{60}-1}$ $= R2612{,}05$ *OR* Equating the current value of the loan with the current value of repayments we get: $x = \dfrac{11\,480\times\frac{0{,}13}{12}}{1-\left(1+\frac{0{,}13}{12}\right)^{-60}} = R2\,612{,}05$ (to nearest cent)	✓ ✓✓✓ for numerator ✓✓ for denominator ✓✓ answer [8]
2.2.2	After 2 years balance: $114800\left(1+\frac{0{,}13}{12}\right)^{24} - \dfrac{2612.05\left[\left(1+\frac{0{,}13}{12}\right)^{24}\right]}{\left(1+\frac{0{,}13}{12}\right)-1} = \text{R } 77522{,}97$ OR Calculating the current value of the sum of the outstanding payments: $2\,612.05\left(1+\frac{0{,}13}{12}\right)^{-1} + 2\,612{,}02\left(1+\frac{0{,}13}{12}\right)^{-2} + ... + 2\,612{,}05\left(1+\frac{0{,}13}{12}\right)^{-36}$	✓✓ for balance

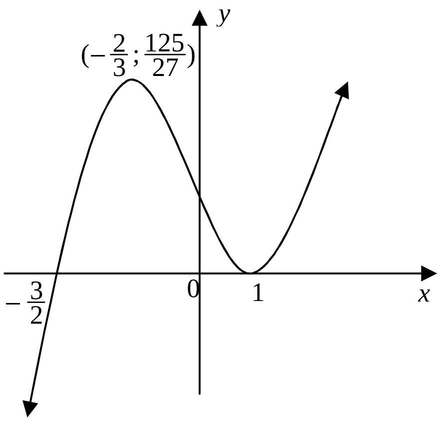
	$$=\frac{2\,612.05\left[1-\left(1+\frac{0{,}13}{12}\right)^{-36}\right]}{\frac{0{,}13}{12}}=R77\,522{,}81$$ $$\text{R }77522{,}97\left(1+\frac{0{,}13}{12}\right)^{n}=\frac{3500\left[\left(1+\frac{0{,}13}{12}\right)^{n}-1\right]}{\frac{0{,}13}{12}}$$ $$=323076{,}9231\left[\left(1+\frac{0{,}13}{12}\right)^{n}-1\right]$$ $$\frac{\left(1+\frac{0{,}13}{12}\right)^{n}}{\left(1+\frac{0{,}13}{12}\right)^{n}-1}=4{,}167499298$$ $$\left(1+\frac{0{,}13}{12}\right)^{n}=4{,}167499298\left(1+\frac{0{,}13}{12}\right)^{n}-4{,}167499298$$ $$\left(1+\frac{0{,}13}{12}\right)^{n}=1{,}315706463$$ $$n=\frac{\log 1{,}315706463}{\log\left(1+\frac{0{,}13}{12}\right)}$$ $$n\approx 25{,}5\text{ months}$$ OR $$77\,522{,}81=\frac{3\,500\left[1-\left(1+\frac{0{,}13}{12}\right)^{-n}\right]}{\frac{0{,}13}{12}}$$ $$\text{Then } n=\frac{\log\left[1-\frac{77\,522{,}81}{3\,500}\times\frac{0{,}13}{12}\right]}{-\log\left(1+\frac{0{,}13}{12}\right)}=25{,}46\ldots$$ As above there are 25 payments of R3 500 and a final lesser payment.	✓✓ ✓✓ ✓✓ [8]
3.1.1	The number of HIV positive people increase by 1,2 mill people per year. H = 1,2 t + 2,7	✓✓ ✓✓
3.1.2	H = 1,2 (16) + 2,7 = 21,9 million	✓✓

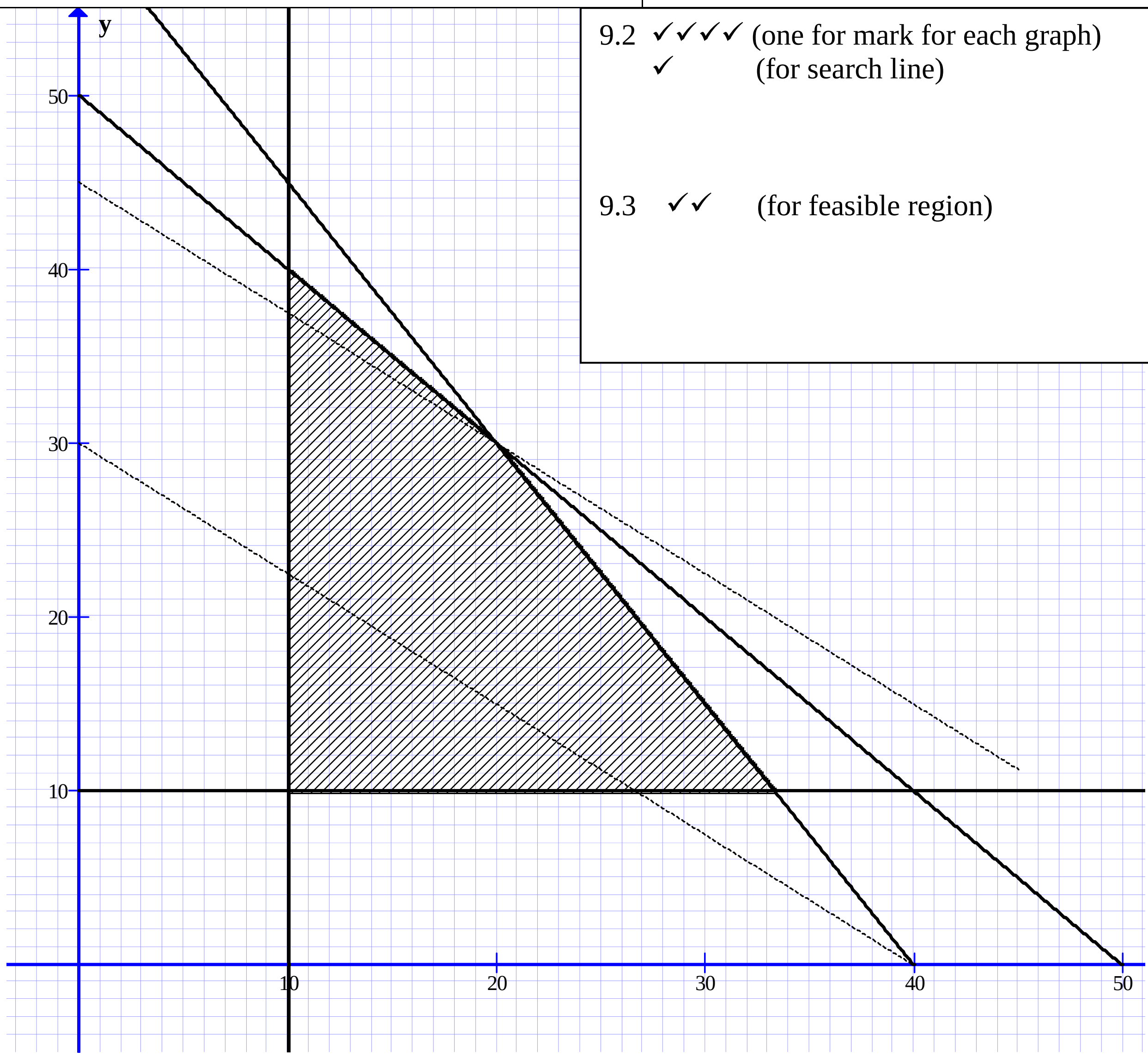
3.1.3	For a series to converge $-1 < r < -1$; $r = \frac{1}{3}$ thus series converges.	(6) ✓✓
3.2.1	$S_\infty = \frac{a}{1-r} = \frac{5(3)^4}{1-\frac{1}{3}} = 607{,}5$	(2) ✓ formula ✓ subst
3.2.2	$S_9 == \frac{5(3)^4\left[1-\left(\frac{1}{3}\right)^9\right]}{1-\frac{1}{3}} = 607{,}47$ 607,5 – 607,47 = 0,03	✓ answer (3) ✓ formula ✓✓ subst
	46 ; 60	✓ answer (4)
3.3.3	Second common difference $\therefore T_n = an^2 + bn + c$ Second common difference = 2 $\therefore$ $a = 1$ $\therefore T_n = n^2 + bn + c$	✓✓ answer (2) ✓✓ (2)
3.3.4	$n = 1:$ $T_1 = 1 + b + c = 6$ ………….. (i) $n = 2:$ $T_1 = 4 + 2b + c = 10$ ………….. (ii) (ii) – (i) 3 + b = 4	✓
3.3.1	b = 1 c = 4	
3.3.2	$\therefore T_n = n^2 + n + 4$ $n^2 + n + 4 = 1264$ $n^2 + n + 1260 = 0$ $(n-35)(n+36) = 0$ $n = 35$	✓ ✓ ✓ ✓ (5) ✓ equation ✓ standard form ✓ factors ✓ value of n

3.3.3		(4)

4.1	$q = 1$ $5 = k + 1$ $k = 4$	✓ value of q ✓✓ value of k
4.2	$f(x) = a(x-3)^2$ $2 = a(0-3)^2$ $a = \frac{2}{9}$	✓ $f(x)$ ✓ subst ✓ value of a
4.3	g is a one-to-many relation	✓✓
4.4	$x \in [3;\infty)$ $x \in (-\infty;3]$	✓ ✓
4.5	$f^{-1}: \quad x = k^y + 1$ $x - 1 = k^y$ $\log_k(x-1) = y$	✓✓
4.6	$h(x) = -\frac{2}{9}(x-3)^2$	✓✓ [14]
5.1.1	$x = 2$ $y = -4$	✓ ✓
5.1.2	$y/c: \quad y = \frac{5}{0-2} - 4 = -\frac{13}{2}$ $x/c: \quad \frac{5}{x-2} - 4 = 0$ $x = 3\frac{1}{4}$	✓ ✓ ✓ ✓
5.2.1	120°	
5.2.2	$f(x) = \sin x$ or $f(x) = \cos(x - 90^{0})$	✓✓
5.2.3	$x = -30^{\circ};\ 60^{\circ}$	✓✓ [10]

6.1	$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{-5(x+h)-(-5x^2)}{h}$ $=\lim_{h\to 0}\frac{-5x^2-10xh-5h^2+5x^2}{h}$ $=\lim_{h\to 0}\frac{-10xh-5h^2}{h}$ $\quad h\neq 0$ $=\lim_{h\to 0}\ [-10x-5h]$ $=-10x$	✓ ✓ ✓ ✓ ✓ (5)
6.2	$\frac{d}{dx}\left[4x^{\frac{1}{2}}-8x^{-\frac{1}{2}}\right]=2x^{-\frac{1}{2}}+4x^{-\frac{3}{2}}$ or $\frac{2}{\sqrt{x}}+\frac{4}{\sqrt{x^3}}$	✓ power form ✓✓ derivatives (3)
6.3	Tangent: $y = x + 4$ (– 1 ; 3) is on f: $3 = a(-1)^3 + b(-1)$ $3 = -a - b$ $f'(x)=3ax^2+b$ $f'(-1)=3a+b$ $1=3a+b$ $3=-a-b$ $2a = 4$ $a = 2$ $b = -5$	✓ ✓ ✓ ✓ ✓ ✓ ✓ (7)

7.1	$4xh = 540$ $h = \dfrac{540}{4x}$	✓ ✓ (2)
7.2	$A = 8x + 8h + 2xh$ $= 8x + 8\left(\dfrac{540}{4x}\right) + 2x\left(\dfrac{540}{4x}\right)$ $= 8x + 1080x^{-1} + 270$	✓ ✓✓ (3)
7.3	For min A: $A' = 0$ $8 - 1080x^{-2} = 0$ $1080x^{-2} = 8$ $x^2 = 135$ $x = 3\sqrt{15}$ $x = 12cm$ (to nearest cm)	✓ $A' = 0$ ✓ ✓ ✓ (4)
8.1	$f(x) = 2x^3 - x^2 - 4x + 3$ $f(1) = 2 - 1 - 4 + 3 = 0$ $\therefore$ x -1 is a factor of $f(x)$	✓ $f(1)$ ✓ $f(1) = 0$
8.2	$2x^3 - x^2 - 4x + 3$ $= (x-1)(2x^2 + x - 3)$ $= (x-1)(x-1)(2x+3)$	✓ quadratic factor ✓ linear factors
8.3	$f'(x) = 6x^2 - 2x - 2 = 0$ $6x^2 - 2x - 2 = 0$ $(3x+2)(x-1) = 0$ $x = -\dfrac{2}{3}$ or $x = 1$ 	✓ derivative ✓ factors ✓ x-values ✓ y-values Graph: ✓ y-intercept ✓✓ turning points ✓ shape
8.5	$f''(x) = 12x - 2 = 0$ $x = \dfrac{1}{6}$	✓✓ f'' ✓ $f'' = 0$ ✓ answer [16]

9.1	$6x + 4y \leq 240$ $\quad \therefore\ 3x + 2y \leq 120$ $5x + 5y \leq 250$ $\quad \therefore\ x + y \leq 50$ $x \geq 10$ $y \geq 10$	✓ (one mark for each constraint) ✓ ✓ ✓
9.2		9.2 ✓✓✓✓ (one for mark for each graph) ✓ (for search line) 9.3 ✓✓ (for feasible region)
9.4	P = 3000*x* + 4000*y*	✓✓
9.5	10 Plasma's and 40 LCD's	✓✓ [15]

Exam B

Grade 12 Mathematics Exam **Paper 1**
Time: 3 hours **Marks: 150**

QUESTION 1

1.1

1.1.1 $(x-4)^2 = 49$

$x-4 = \pm 7$

$x = 11 \text{ or } x = -3$

$\sqrt{\ }$ both sides ✓
$\pm$ ✓
Solution ✓

Multiply & simplify ✓
Factorise ✓
Solution ✓

(3)

1.1.2 $(x^2-2)(x+3) = x^3+6x$

$x^3+3x^2-2x-6 = x^3+6x$ ✓

$3x^2-8x-6 = 0$ ✓

$x = \dfrac{8 \pm \sqrt{136}}{2}$ ✓ ✓

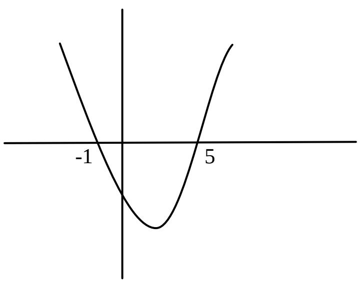
$x = 3{,}28 \text{ or } x = 0{,}61$ ✓

(5)

1.2

1.2.1 $x = -1$ ✓ or $x = 5$ ✓ (2)

1.2.2



Shape ✓
Intercepts ✓

(2)

1.2.3 $-1 \le x \le 5$ ✓ (1)

1.3 (5)

$sub \ \ y = x-3 \ \ in \ \ x^2 - x = 6 + y$

$x^2 - x = 6 + x - 3$ ✓

$x^2 - x - 6 - x + 3 = 0$

$x^2 - 2x - 3 = 0$ ✓

$(x-3)(x+1) = 0$ ✓

$x = 3 \ \ or \ \ x = -1$ ✓

$and \ \ y = 0 \ \ or \ \ y = -4$ ✓

1.4

1.4.1 $x = -\frac{2}{3}$ *and* $y = 4$

Values ✓
And ✓

(2)

1.4.2 $x = -\frac{2}{3}$ *or* $y = 4$

or ✓

(1)

[19]

QUESTION 2

2.1

2.1.1 $T_6 = 1500(1{,}015)^5$

$= R1615{,}92$

Formula ✓
Subst ✓
Answer ✓

(3)

2.1.2 $S_6 = \frac{1500(1{,}015^6 - 1)}{1{,}015 - 1}$ ✓✓

$= R9344{,}33$ ✓

Earnings for year :

$R9344{,}33 \times 2 = R18688{,}65$ ✓ ✓

(5)

2.2

2.2.1 $T_n = 162000(1 - 0{,}22)^{n-1}$ ✓

$T_6 = 162000(0{,}78)^6$ ✓ ✓

$= R36482.34$ ✓

(4)

2.2.2 70% *of book value* $= .7 \times 36482.34$ ✓

$= R25537{,}64$ ✓

Less than book value

Will not be written of f ✓

(3)

2.2.3 $F = \frac{x(1 + \frac{0{,}75}{12})^{84} - 1}{\frac{0{,}75}{12}}$

$80000 = (110{,}0318...)(x)$

$x = R727{,}06$

Formula ✓
Subst ✓
Calculation ✓

(4)

[19]

QUESTION 3

3.1

3.1.1 17; 19; 20 ✓ ✓ ✓ (3)

3.1.2 *Numbers from 1 to 150 not divisible by 3* ✓ ✓ (2)

3.1.3 *Subtract number of terms in following two sequences:*
1; 2; 3; … 150 *and* 3; 6; 9; … 150

$150 = 3 + (n-1)3$ ✓

$3n = 150$ ✓

$n = 50$ ✓ (5)

$\textit{Number of terms in sequence} = 150 - 50$ ✓

$= 100$ ✓

3.2 $$\sum_{k=1}^{4} \frac{2k-1}{2^k} = \frac{1}{2} + \frac{3}{4} + \frac{5}{8} + \frac{7}{16}$$ ✓ ✓ (4)

$$= \frac{37}{16}$$ ✓ ✓

3.3

3.3.1 ✓ ✓ ✓ (3)

	T_1	T_2	T_3	T_4	T_n
Side	8	4	2	1	$8(\frac{1}{2})^{n-1}$
Perimeter	24	12	6	3	$3\times 8(\frac{1}{2})^{n-1}$

3.3.2 $\textit{Sum of perimeter of inner triangles} = 12 + 6 + 3 ......$ ✓

$$= \frac{12}{1-\frac{1}{2}}$$ ✓

$$= 24cm$$ ✓

Thus the limit of the sum of the inner triangles is 24cm and the sum will not exceed 24cm, which is the perimeter of the outer triangle. ✓ (4)

[21]

QUESTION 4

4.1 *Because the given function is a many-to-one function, its inverse is one-to-many and hence is not a function.* ✓✓

4.2 $y = f^{-1}(x) = -\sqrt{-\frac{x}{2}}$ ✓✓✓

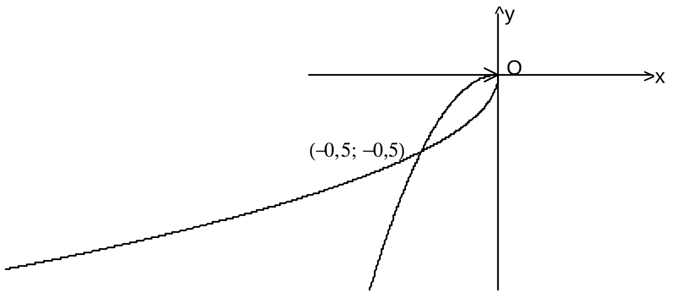
4.3 $(-\infty;0]$ ✓

4.4 *At points of intersection*: $-\sqrt{-\frac{x}{2}} = -2x^2$

$\therefore -\frac{x}{2} = 4x^4$

$\therefore x(8x^3+1) = 0$

Hence $x = 0$ or $x = -\frac{1}{2}$



✓✓✓✓

4.5 $f^{-1}[f(-3)] = f^{-1}(-18) = -\sqrt{-\frac{-18}{2}} = -\sqrt{9} = -3$ ✓✓

And $f\left[f^{-1}(-3)\right] = f\left(-\sqrt{\frac{3}{2}}\right) = -2\left(-\sqrt{\frac{3}{2}}\right)^2 = -3$ ✓✓

[14]

QUESTION 5

5.1 $m=-\frac{4}{5}$ *and* $c=400$ (1)

Therefore equation is:

$h(x)=-\frac{4}{5}x+400$ ✓

5.2 (5)

$$\frac{400}{x+1}=-\frac{4}{5}x+400 \checkmark$$

$$2000=-4x(x+1)+400(5)(x+1) \checkmark$$

$$2000+4x^2+4x-2000x-2000=0$$

$$4x^2-1996x=0 \checkmark$$

$$4x(x-499)=0 \checkmark$$

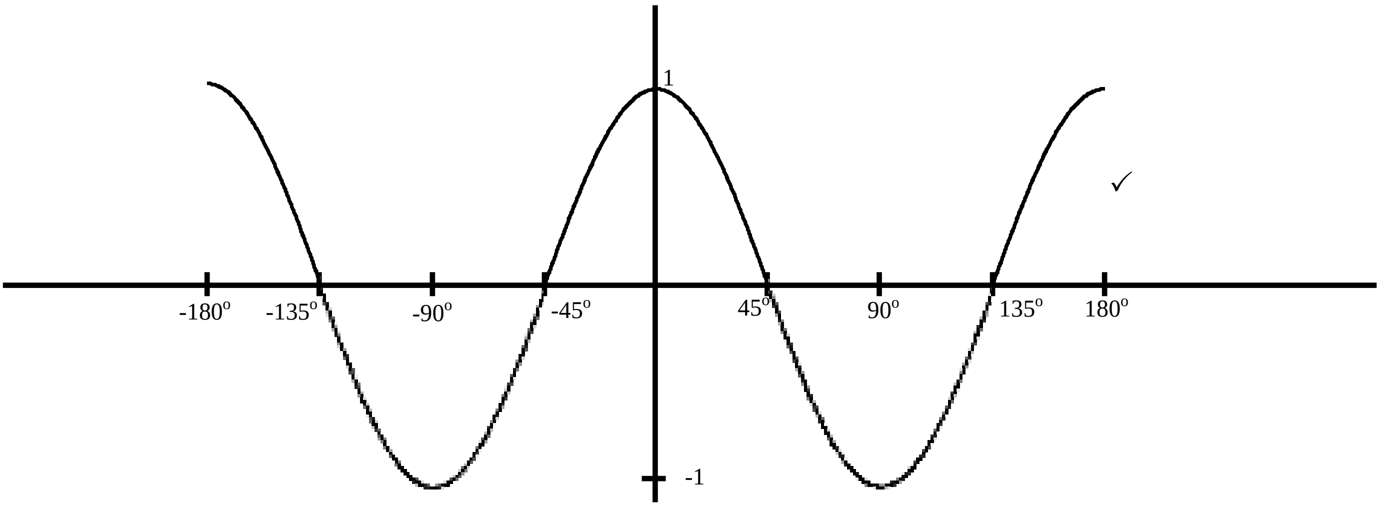
$$x=0 \quad or \quad x=499$$

Cable touches ground again at $499m$ ✓

[6]

QUESTION 6

6.1 (1)



6.2 *The graph repeats itself every 180 degrees.* ✓ ✓ (2)

6.3 *Each x-intercept would be 30 degrees more than it currently is.* ✓ (1)

6.4 0 ✓ (1)

[5]

QUESTION 7

7.1 (5)

$$f'(x) = \lim_{h\to 0}\left[\frac{f(x+h)-f(x)}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{\frac{-2}{x+h}-\frac{-2}{x}}{h}\right] \quad \checkmark$$

$$= \lim_{h\to 0}\left[\frac{-2x+2x+2h}{(x+h)x}\times\frac{1}{h}\right] \quad \checkmark$$

$$= \lim_{h\to 0}\left[\frac{2h}{xh(x+h)}\right] \quad \checkmark$$

$$= \lim_{h\to 0}\left[\frac{2}{x(x+h)}\right] \quad \checkmark$$

$$= \frac{2}{x^2} \quad \checkmark$$

7.2

7.2.1 (3)

$$f(x) = 7x^3 - 4x + 6$$

$$f'(x) = \underset{\checkmark}{21x^2} - \underset{\checkmark}{4}$$

7.2.2

$$f(x) = 3\sqrt{x} - \frac{1}{3x}$$

$$f(x) = \underset{\checkmark}{3x^{\frac{1}{2}}} - \frac{1}{3}\underset{\checkmark}{x^{-1}}$$

$$f'(x) = \underset{\checkmark}{\frac{3}{2}x^{-\frac{3}{2}}} + \underset{\checkmark}{\frac{1}{3}x^{-2}}$$

[12]

QUESTION 8

8.1 (2)

$$f(x) = x^3 + x^2 - 5x + 3$$

$$f(1) = 1 + 1 - 5 + 3 \quad \checkmark$$

$$= 0 \quad \checkmark$$

$$\therefore x - 1 \textit{ is a factor of } f(x)$$

8.2 (3)

$$x^3 + x^2 - 5x + 3 = (\overset{\checkmark}{x}-1)(x^2 + \overset{\checkmark}{2}x - 3)$$

$$= (x-1)(x-1)(x+3) \quad \checkmark$$

8.3 *x-intercepts:* x = 1 *or* x = -3 ✓
y-intercept: y = 3 ✓ (2)

8.4 *Stationery points at* :

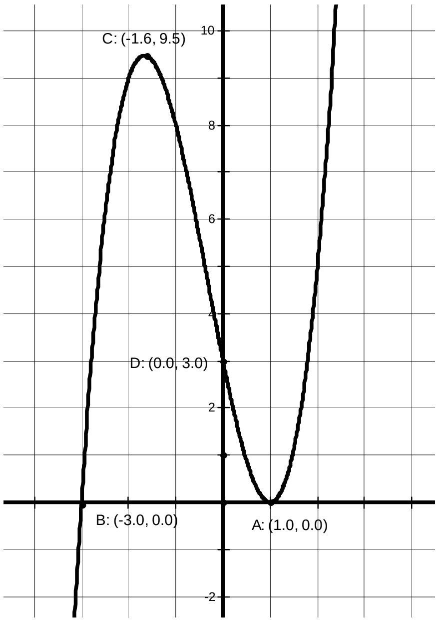
$$f'(x) = 0 \quad \checkmark$$
$$3x^2 + 2x - 5 = 0$$
$$(3x+5)(x-1) = 0 \quad \checkmark$$
$$x = -\frac{5}{3} \quad or \quad x = 1 \quad \checkmark$$

f(x)	$-\frac{5}{3}$	1
$f''(x) = 6x + 2$	-8	8
	Local max	*Local min*

Turning points: $(-1;6;\ 9,5)$ *or* $(1;\ 0)$ ✓ (4)

8.5 *Point of inflection at 6x+2=0* (1)

$$x = -\frac{1}{3} \quad \checkmark$$



8.6 Sketch ✓✓ (2)

8.7 $x < -1{,}6 \quad or \quad x > 1$ ✓ ✓ (2)

8.8 *Shift graph more than 3 units to right.* ✓✓ (2)

Or: Shift graph down more than 1,6 units and right more than one unit.

8.9
$$-g(x) = x^3 + x^2 - 5x + 3 \quad \checkmark\checkmark$$
$$g(x) = -x^3 - x^2 + 5x - 3 \quad \checkmark$$
(3)

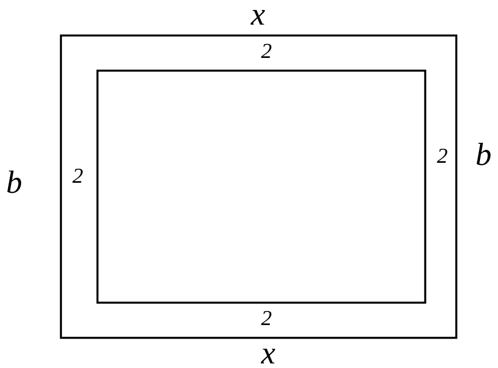
8.10
$$Avg\ rate\ of\ change = \frac{\Delta f(x)}{\Delta x}$$
$$= \frac{3-0}{0-1} \quad \checkmark$$
$$= -3 \quad \checkmark$$
(2)

8.11
$$f'(-2) = 3(-2)^2 + (-2) + 5$$
$$= 9 \quad \checkmark$$
$$f(-2) = 9 \quad \checkmark$$
Eqn of line :
$$y - 9 = 3(x - (-2)) \quad \checkmark$$
$$y = 3x + 15 \quad \checkmark$$
(4)

8.12
$$x^3 + x^2 - 5x + 3 = 3x + 15 \quad \checkmark$$
$$x^3 + x^2 - 8x - 12 = 0$$
$$(x+2)(x^2 - x - 6) = 0 \quad \checkmark$$
$$(x+2)(x+2)(x-3) = 0 \quad \checkmark$$
$\therefore$ *Tangent cuts curve at* :
$$x = -2 \quad or \quad x = 3 \quad \checkmark$$
(4)

[31]

QUESTION 9



(1)

9.1 $2x + 2b = 72$

$b = \frac{72 - 2x}{2}$

$b = 36 - x$ ✓

9.2 *length = x - 4* ✓

breadth = 36 – x – 4 = 32 – x ✓ (2)

9.3 $A(x) = (x - 4)(32 - x)$ ✓ (2)

$= -x^2 + 36x - 128\ cm^2$ ✓

9.4 $A'(x) = -2x + 36$ ✓

Max at $A'(x) = 0$ ✓

$-2x + 36 = 0$ ✓

$x = 9$ ✓

x	*x*<9	9	*x*>9
A'(x)	+	0	-
		Local max	

(4)

[9]

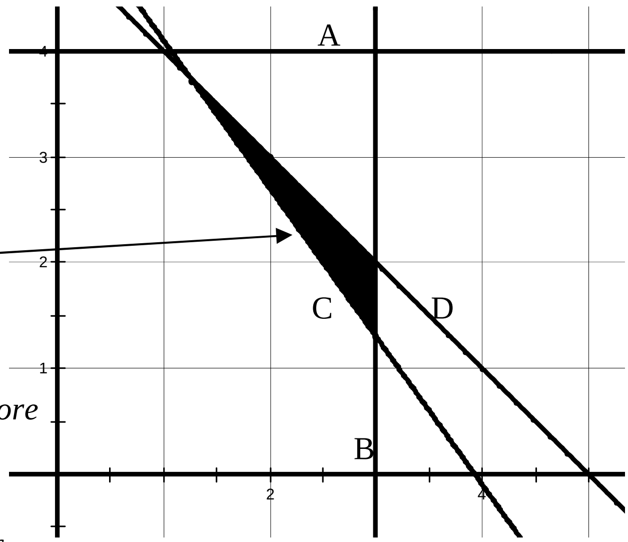
QUESTION 10

10.1 $A:\ y \le 4$ ✓

$B:\ x \le 3$ ✓

$C:\ 14x + 10y \ge 55$ ✓

$D:\ x + y \le 5$ ✓ (4)



10.2 on diagram ✓ (1)

10.3 $A:\ y \le 4$ ✓ (1)

10.4 *Must be integer solutions, therefore feasible solutions are* (2; 3) *and* (3;2) ✓ ✓ (2)

10.5 *More ball skills coaches. Therefore choose* (2; 3) ✓ (1)

10.6 $Cost = 14 \times 2 \times 15 + 10 \times 3 \times 10$ ✓ (3)

$= R720$

[12]

Grade 12 Assignment: Recap of Grade 11 Data Handling **Marks: 50**

Question 1

1.1.1 $\bar{x} = 14736,19$ ✓✓

1.1.2 $\sigma = 2089,60$ ✓✓

1.2

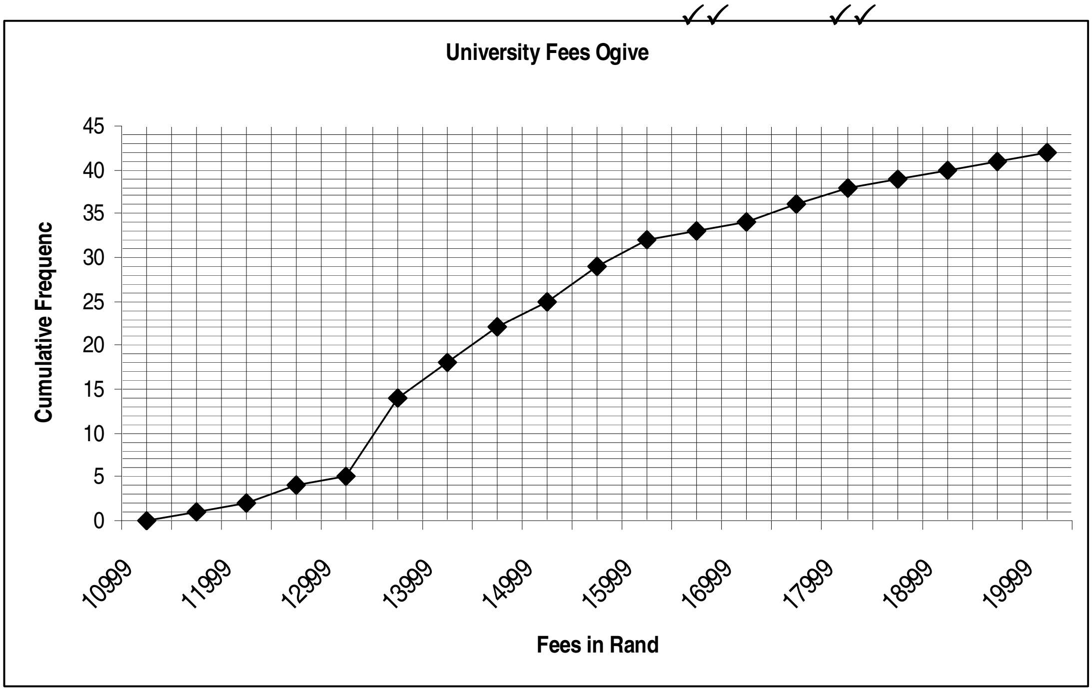
Class Interval	Tally	Frequency	Cumulative Frequency
less than 10 999			
11 000 - 11 499	I	1	1
11 500 - 11 999	I	1	2
12 000 - 12 499	II	2	4
12 500 - 12 999	I	1	5
13 000 - 13 499	IIIII IIII	9	14
13 500 - 13 999	IIII	4	18
14 000 - 14 499	IIII	4	22
14 500 - 14 999	III	3	25
15 000 - 15 499	IIII	4	29
15 500 - 15 999	III	3	32
16 000 - 16 499	I	1	33
16 500 - 16 999	I	1	34
17 000 - 17 499	II	2	36
17 500 - 17 999	II	2	38
18 000 - 18 499	I	1	39
18 500 - 18 999	I	1	40
19 000 - 19 499	I	1	41
19 500 - 19 999	I	1	42

✓✓ ✓✓

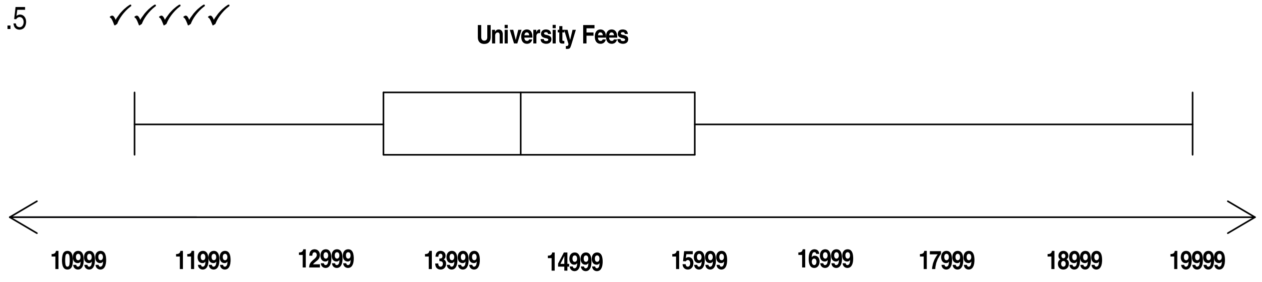
1.3 14 000 - 14 499 ✓

1.4

✓✓✓
✓✓✓



1.5 ✓✓✓✓✓



1.6 Skewed to the right, more clustered to left of median than to the right, large range, smaller interquartile range, a few high figures skewing the data (any reasonable explanation) ✓✓✓

Question 2

2.1

Values	$x-\bar{x}$	$(x-\bar{x})^2$
40	-23	529
56	-7	49
59	-4	16
60	-3	9
60	-3	9
62	-1	1
65	2	4
69	6	36
75	12	144
84	21	441
$\bar{x}=63$	$\sum(x-\bar{x})^2$	**1238**
	Var	**123,8**

✓✓
✓✓

2.2 $\sigma = 11{,}3$ ✓✓

2.3 How far from the mean most figures are, the average deviation from the norm ✓✓

2.4 large range, most learners in excess of 10% away from the mean, couple of high marks skewing data ✓✓

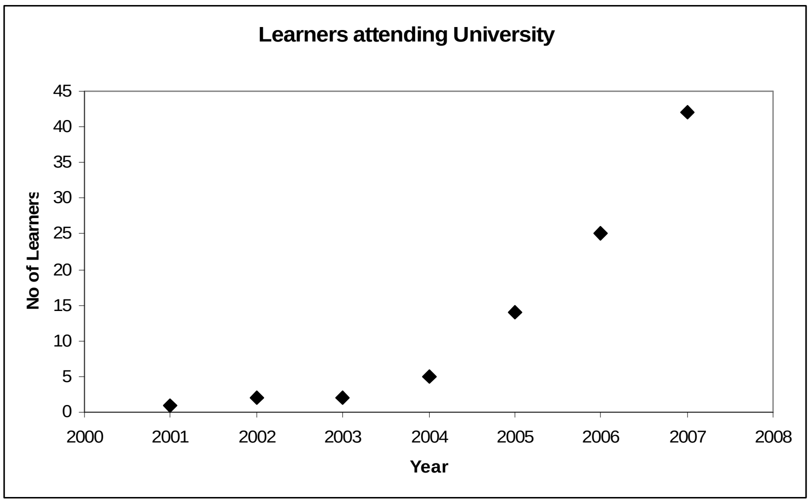
Question 3

3.1 Boys: $\bar{x} = 1249 \div 20 = 62{,}45$ ✓✓
$m = 61$ ✓
Girls: $\bar{x} = 1361 \div 22 = 61{,}86$ ✓✓
$m = 60$ ✓

3.2 Boys have large range, some high and some low, very little bundling in the middle. Girls marks don't start as low but have fewer girls scoring as high as the boys. Mean and median very similar but marks very different. Low scores of boys are offset by high scores. (any reasonable explanation) ✓✓✓✓✓

Question 4

4.1 ✓✓✓



4.2 Exponential ✓✓

Grade 12 Investigation: Polygons with 12 Matches — Marks: 100

Some answers and Marking Rubric

Triangles

1 Isosceles triangle with sides 2, 5, 5

$$\begin{aligned} Area &= 0{,}5 \times base \times ht \\ &= 0{,}5 \times 2 \times \sqrt{24} \\ &= 2\sqrt{6} \end{aligned}$$

2 Right angled triangle with sides 3, 4, 5

$$\begin{aligned} Area &= 0{,}5 \times 3 \times 4 \\ &= 6 \end{aligned}$$

3 Equilateral triangle with sides 4, 4, 4

$$\begin{aligned} Area &= 0{,}5 \times 4 \times 4 \times \sin 60^0 \\ &= 8 \times \frac{\sqrt{3}}{2} \\ &= 4\sqrt{3} \end{aligned}$$

$$4\sqrt{3} = \sqrt{48} > 6 = \sqrt{36} > 2\sqrt{6} = \sqrt{24}$$

So the equilateral triangle has the greatest area.

There are no other triangles because the longest side must be shorter than the sum of the other two sides, otherwise there is no triangle!
So longest side < 6
If longest side = 5, then the sum of the other sides is 7. They can be 5 and 2 or 3 and 4.
If the 'longest' side is 4, then the sum of the other sides is 8 and the only possibility is the equilateral triangle 4, 4, 4.

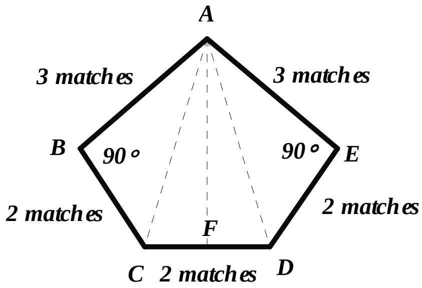
Quadrilaterals with equal angles

If all angles are equal, each is 90^0 and the quadrilaterals will be rectangles
The possible rectangles are 1, 1, 5, 5
2, 2, 4, 4
3, 3, 3, 3

The 3 by 3 square has the greatest area. $9 > 8 > 5$

Pentagons

Without breaking the matches, it is not possible to create a regular pentagon (or a regular polygon with *n* sides where *n* is not a factor of 12).
It is also not possible (as can be proved using the sine and cosine rules) to create a pentagon in which all angles are 108^0 and the perimeter 12 matches.



This pentagon has an area of $6+2\sqrt{3}$ which can be calculated as follows:
Let AF be the altitude from the vertex A to the opposite side, CD.
In ΔABC: $AC=\sqrt{2^2+3^2}=\sqrt{13}$ (Th. of Pythagoras)

And in ΔACF $AF^2=\left(\sqrt{13}\right)^2-1^2$ (Th. of Pythagoras)

Hence altitude AF = $\sqrt{12}=2\sqrt{3}$

Area of pentagon = $\Delta ABC+\Delta ACD+\Delta ADE=3+2\sqrt{3}+3\approx 9{,}464...$ (bigger than the square).

Hexagons

A regular hexagon has sides each 2 matches long and angles of 120^0. This can be broken down into 6 equilateral triangles with all sides 2 matches long.

Area = $6\times 0{,}5\times 2\times 2\times \sin 60^0=\dfrac{12\sqrt{3}}{2}=6\sqrt{3}\approx 10{,}392...$ getting bigger!

Other hexagons have areas less than this. There are endless possibilities.

Twelve sides

Each angle in the regular 12 sided polygon is $\dfrac{10\times 180^0}{12}=150^0$

This polygon can be broken into twelve isosceles triangles, each having equal sides of $r=\sqrt{\dfrac{1}{2-\sqrt{3}}}$ (calculated using the cosine rule) with the included angle being 30^0. Hence the area is

$12\times 0{,}5\times r^2\sin 30^0=\dfrac{3}{2-\sqrt{3}}\approx 11{,}196...$

Again other 12 sided polygons can be constructed but none has an area as large as this.

The circle with a perimeter of 12 has a radius $r=\dfrac{12}{2\pi}=\dfrac{6}{\pi}$ and the area is $\pi\left(\dfrac{6}{\pi}\right)^2=\dfrac{36}{\pi}\approx 11{,}459...$

It should be clear that the biggest possible area that can be enclosed with *n* matches is a regular *n* sided polygon: as close to a circle as possible.

Rubric

Communication	Special cases	Generalisation and justification	Presentation	Total
32-40: Clear, coherent, logical explanations, supported by appropriate sketches and tables of results.All main ideas covered: the more sides, the greater the possible area; regular polygons larger than others with the same number of sides; the limiting area is that of the circle with a perimeter of 12 sides. Some logical extension.	28 to 35: Interesting, comprehensive set of special cases. All calculations accurate.	16 to 20: Quality arguments to support all claims.	5: Neat, striking visual impact	
28 to 31: Clear, logical explanations, supported by appropriate sketches and tables. One main idea might be excluded or no extension	25 to 27: Areas and sketches of the three triangles, the square, the regular hexagon and the regular 12 sided polygon correctly shown and some 'other' examples to support the 'regular is biggest' theory.	14 to15: Quality arguments to support almost all claims.	4: Neat pleasing impact.	
24 to 27: Clear logical explanations, supported by appropriate sketches and tables. One main idea missing and no extension.	21 to 24: Areas and sketches of the three triangles, the square, the regular hexagon and the regular 12 sided polygon correctly shown.	12 to 13: Quality arguments to support most claims.	3: Neat	
20 to 23: Satisfactory explanations with appropriate sketches and tables. More than one main idea missing.	18 to 20: Areas and sketches provided of the three triangles, the square, the regular hexagon and the regular 12 sided polygon. Accurate calculations with at most one minor error.	10 to 11 Some valid attempts at explanations and justifications.	3: Neat	
16 to 19: Adequate explanations with some sketches and tables. At least one main idea satisfactorily covered.	14 to 17: Areas and sketches provided of the three triangles, the square, the regular hexagon and the regular 12 sided polygon. Accurate calculations with at most two minor errors.	8 to 9: Attempts at justifications but flawed or inadequate.	2: Rather untidy.	
12 to 15 Some logical discussion with at least some sketches and tables.	11 to 13: Areas and sketches provided of the three triangles, the square, the regular hexagon and the regular 12 sided polygon. Accurate calculations with at most three minor errors or one major error or omission.	6 to 7: no valid explanations or justifications for claims.	1: Very untidy.	
0 to 11: No main ideas satisfactorily covered. Sketches, tables omitted, flawed or inadequate.	0 to 10: Major errors and/or omissions.	0 to 5	0: A mess: clearly no effort made.	

Grade 12 Test: Geometry and Trigonometry

Time: 1 hour **Marks: 50**

1.1 $x^2 + y^2 = (-5)^2 + (12)^2 = 169$ ✓✓

1.2 $(13)^2 + (0)^2 = 169$ ✓

1.3 $\frac{y-12}{x+5} = \frac{0-12}{13+5} = -\frac{2}{3}$ ✓✓

$\therefore 3y - 36 = -2x - 10$

$\therefore 3y + 2x = 26$ ✓✓

1.4 Line through the origin, which is perpendicular to AB will cut the circle at the points of contact of the required tangents.

This line is $y = \frac{3x}{2}$ ✓✓

Points of intersection are at points where $x^2 + \left(\frac{3x}{2}\right)^2 = 169$

ie $13x^2 = 4 \times 169$

$\therefore x = \pm 3\sqrt{13}$

$\therefore y = \pm 2\sqrt{13}$ ✓✓

One tangent is: $\frac{y - 2\sqrt{13}}{x - 3\sqrt{13}} = -\frac{2}{3}$ the other is $\frac{y + 2\sqrt{13}}{x + 2\sqrt{13}} = -\frac{2}{3}$

$\therefore 3y + 2x = 12\sqrt{13}$ $\therefore 3y + 2x = -12\sqrt{13}$ ✓✓

2.1 Reflection in the *y*-axis ✓✓

2.2 Glide (1 to the right) and reflection about the *x*-axis ✓✓

2.3 Rotation of □□about the origin ✓✓

2.4 Anti-clockwise rotation of 60^0 about the origin ✓✓

2.5 Rotation through 180^0 (because $k < 0$) ✓

Enlargement by a factor of $-k$ (or reduction by a factor of $-\frac{1}{k}$) ✓✓

Reflection about the line $y = x$ ✓✓

3.1 $\sin 2x = \cos(x - 15^0)$

$\therefore \cos(90^0 - 2x) = \cos(x - 15^0)$ ✓✓

$\therefore 90^0 - 2x = \pm(x - 15^0) + k.360^0; k \in Z$ √√

$\therefore 3x = 105^0 + k.360^0$ or $x = 75^0 + k.360^0$ ✓

$\therefore x = 35^0 + k.120^0$ or $x = 75^0 + k.360^0$ $k \in Z$ ✓

3.2 Yes, Mvuyo is correct: $155^0 = 35^0 + 1 \times 120^0$ (or substitute) ✓✓

4.1 $\tan\left(2x+45^0\right)=\dfrac{\tan 2x+\tan 45^0}{1-\tan 2x\tan 45^0}$ ✓✓

$=\dfrac{\dfrac{2\tan x}{1-\tan^2 x}+1}{1-\dfrac{2\tan x}{1-\tan^2 x}.1}$ ✓✓

$=\dfrac{2\tan x+1-\tan^2 x}{1-\tan^2 x}\times\dfrac{1-\tan^2 x}{1-\tan^2 x-2\tan x}$

$=\dfrac{2\tan x+1-\tan^2 x}{1-\tan^2 x-2\tan x}$ ✓✓

4.2 Identity not valid for $2x+45^0=90^0+k.180^0;\ k\in Z$ ✓
ie for $x=22{,}5^0+k.90^0$ as $\tan\left(2x+45^0\right)$ is not defined for these values of θ
✓

Also not defined for $x=90^0+k.90^0$ since $\tan x$ is not defined for these values of θ ✓

5.1 In ΔABC $BC^2=x^2+x^2-2x.x.\cos 30^0$ ✓✓

$=2x^2-\sqrt{3}x^2$ ✓

$\therefore BC=x\sqrt{2-\sqrt{3}}$ ✓

5.2 Area $\Delta ABC=$ Area $\Delta ACD=$ Area $\Delta ADB=0{,}5\times x^2\sin 30^0$ ✓

$=\dfrac{x^2}{4}$ ✓

Area $\Delta BCD=0{,}5\left(x\sqrt{2-\sqrt{3}}\right)^2.\sin 60^0$ ✓

$=0{,}5\times\left(x\sqrt{2-\sqrt{3}}\right)^2\times\dfrac{\sqrt{3}}{2}$

$=\dfrac{x^2\left(2\sqrt{3}-3\right)}{4}$ ✓

$\therefore$ Total surface area $=\dfrac{3x^2}{4}+\dfrac{x^2\left(2\sqrt{3}-3\right)}{4}$

$=\dfrac{\sqrt{3}}{2}x^2$ square units ✓

Grade 12 Project: Escher and Transformation Geometry **Marks: 100**

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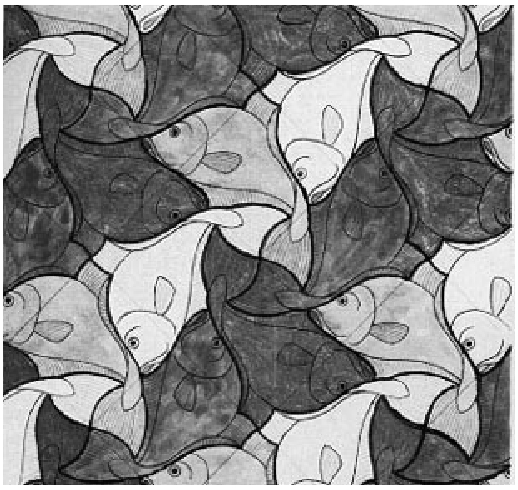
Some Guidelines

The object of this project is to show that transformation geometry is not just a lot of rules and formulae, but that the concepts are used in the fascinating work of the Dutch artist, M.C. Escher.
It is suggested that a rubric is drawn up, with agreement from the learners about the need to show a clear understanding of the transformations learnt in the course of their years at school. It is hoped that the learners will enjoy investigating the various patterns.

The solutions below should be helpful to the marker:

Task 1



This is Escher's version of filling the plane with this fish. Learners may come up with other options. Measure each on its merit.

1. Mark the point of the tail (not the extreme point, the one about 15*mm* from the extreme point), A.
2. Place A at the origin so that the fish faces upwards. Trace the fish and call it 1.
3. Rotate the fish anti-clockwise, about A, through 90^0. Trace the fish again and call it 2.
4. Rotate again through 90^0 around A and call this fish 3.
5. Rotate for a third time and call the resulting fish 4.
6. Place your template fish on fish 1 and then slide (translate) it down and slightly to the right to fit between fish 3 and fish 4. Trace the fish again and call it 5.
7. Repeat steps 3 to 5 and call the resulting fish 6, 7 and 8.
8. Place the template on fish 2. Translate it to the right and slightly up until it lies between fishes 1 and 4. Call this fish 9.

   And so on…

Task 2

- Three shapes are used: a fish, a bird and a lizard/gecko.
- The fish meet at the mouth and each is a rotation of another through 120^0 about the point where the mouths meet.
- The birds fit between the fish and are also 120^0 rotations of each other.
- The geckos fit into the tails of the fish and the birds.
- The geckos are reflections of each other about a vertical line through the centre of the fish.
- There is also an axis of reflection at an angle of 60^0 to the vertical.
- The complete motifs of all three creatures can be translated horizontally and vertically so that the geckos fit on to each other.

Task 3

- The birds and fish are repeated by a series of glide reflections.
- The axis of reflection is a horizontal line through the middle of the frieze.
- The glide is a translation about 2*cm* to the right.

Task 4

- Start with a 160*cm* by 160*cm* square.
- Place the first large gecko in the bottom right hand corner as shown in the given art work.
- Rotate this motif three times, either clockwise or anti-clockwise through the central point (80*cm* from the bottom and 80*cm* from the left) and trace copies in the other three corners.
- The other eight boundary geckos are a reduction of the biggest geckos by a factor of three, but they first need to be flipped over.
- Place a small gecko horizontally at the nose of the bottom left big gecko and then translate it horizontally to form the other.
- Repeat this process (either vertically or horizontally) to complete the outer ring.
- 25*cm* from the bottom, to the right of the central vertical line, is a reduction of the largest gecko by a factor of $\frac{15}{7}$. This gecko is then rotated seven times, about the central point, though an angle of 45^0 each time.
- 52*cm* from the bottom is another ring of geckos, this time reductions of the largest gecko by a factor of 5. This gecko is also rotated seven times through 45^0 to form the ring.
- The next ring of black geckos is formed in the same way.
- In between the rings of black geckos, the space is filled by a collection of reductions (and in some cases also reflections) of the original. Each one is rotated through 90^0 three times to create its identical partners.

Grade 12 Mathematics Exam **Paper 2**
Time: 3 hours **Marks: 150**

1.1 $AC = \sqrt{8^2 + 4^2}$ ✓

$= 4\sqrt{5}$ ✓

1.2 $\frac{y-1}{x+3} = \frac{-3-1}{5+3} = -\frac{1}{2}$ ✓✓

Equation is $2y + x + 1 = 0$ ✓

1.3 At point of intersection of AC and BD: $2(2x-8) + x + 1 = 0$ ✓✓

$\therefore 4x - 16 + x + 1 = 0$

$\therefore 5x = 15$

$\therefore x = 3$

and $y = -2$ ✓✓

But the mid-point of BD is $\left(\frac{4+2}{2}; \frac{0-4}{2}\right) = (3;-2)$ ✓✓

And gradient of AC × gradient of BD = $-\frac{1}{2} \times 2 = -1$ ✓✓

Hence AC is the perpendicular bisector of BD

1.4 Area of kite ABCD = 2× area $\triangle ACD$ ✓

$= 2 \times 0{,}5 \times 4\sqrt{5} \times \sqrt{(4-3)^2 + (0+2)^2}$ ✓✓

= 20 square units ✓

1.5 The inclination of AB = $\tan^{-1}\left(\frac{-4-1}{2+3}\right)$ ✓

$= 135^0$ ✓

1.6 The inclination of AD = $\tan^{-1}\left(\frac{1}{-7}\right) = 171{,}9$ (correct to 1 decimal place) ✓

$B\hat{A}D = 171{,}9^0 - 135^0 = 37^0$ (correct to nearest degree) ✓✓

2.1 The equation of the circle is $(x-2)^2 + (y+3)^2 = (6-2)^2 + (-1+3)^2$ ✓✓✓

=20

or $x^2 - 4x + y^2 + 6y = 7$ ✓✓

2.2 The mid-point of PQ is $(3;-4)$ ✓✓

The gradient of PQ = $\frac{6}{6} = 1$ ✓

Hence the perpendicular bisector has the equation $\frac{y+4}{x-3} = -1$ ✓

Substituting the co-ordinates of the centre into this equation:

$LHS = \frac{2+4}{-3-3} = -1 = RHS$ ✓✓

Hence the perpendicular bisector of PQ passes through the centre of the circle.

2.3 The radius of the given circle is $\sqrt{20}$ or $2\sqrt{5}$

The distance between R $(-1;2)$ and $(2;-3)$ is $\sqrt{(-1-2)^2 + (2+3)^2} = \sqrt{34}$ ✓✓

Since $\sqrt{34} > \sqrt{20}$, the point R lies outside the circle. ✓

2.4 The co-ordinates of M' are $\left(3;-\frac{9}{2}\right)$ ✓

and the co-ordinates of P' are $\left(9;-\frac{3}{2}\right)$ ✓

2.5 Area of original circle $= \pi\left(\sqrt{20}\right)^2$

Area of enlarged circle centre M' is $\pi\left(\sqrt{(9-3)^2+\left(-\frac{3}{2}+\frac{9}{2}\right)^2}\right)^2 = 45\pi$ ✓✓

Area original circle : area of enlarged circle = $20\pi : 45\pi = 4:9$ ✓✓

3.1 D is $(2;4)$ ✓✓

$\triangle OAB$ has been reflected about the line $y = x$ ✓✓

3.2 F is the point $\left(\frac{\sqrt{2}}{2}\times 4-\frac{\sqrt{2}}{2}\times 2;\frac{\sqrt{2}}{2}\times 2+\frac{\sqrt{2}}{2}\times 4\right)$

$= \left(\sqrt{2};3\sqrt{2}\right)$ ✓✓

$\triangle OAB$ has been rotated anticlockwise about the origin through an angle of 45^0 ✓✓

3.3 H is $\left(-\sqrt{2};3\sqrt{2}\right)$ ✓✓

$\triangle OEF$ has been reflected about the y axis ie. $\triangle OAB$ has been rotated anticlockwise about the origin through an angle of 45^0 and then reflected about the y axis. ✓✓

3.4 J is the point $(-2;4)$ ✓✓

$\triangle OAB$ is reflected first about $y = x$ and then about the y axis. ✓✓

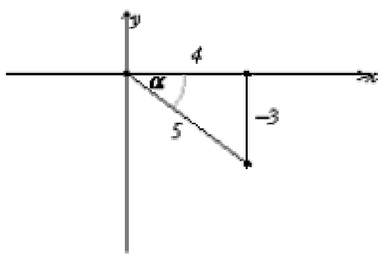
4.1 $\sin\left(90^0+x\right).\cos(-x)-\tan\left(180^0-x\right).\cos\left(180^0+x\right).\sin\left(-x-720^0\right)$

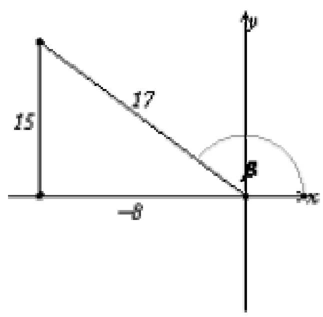
$=\cos x.\cos x-(-\tan x)(-\cos x)(-\sin x)$ ✓✓✓

$= \cos^2 x+\frac{\sin x}{\cos x}.\cos x.\sin x$ ✓✓

$=1$ ✓

4.2.1





✓✓✓

$\tan(\alpha+\beta) = \dfrac{-\frac{3}{4}-\frac{15}{8}}{1-\left(\frac{-3}{4}\right)\left(\frac{15}{-8}\right)}$ ✓✓

$= \frac{-21}{8}\times\frac{32}{-13}$

$= \frac{84}{13}$ ✓

4.2.2 $\sin 2\alpha = 2\sin\alpha\cos\alpha$ ✓

$= 2\left(-\frac{3}{5}\right)\left(\frac{4}{5}\right) = -\frac{24}{25}$ ✓

4.2.3 $2\cos^2\frac{\beta}{2} = \cos\beta + 1$ ✓

$= \left(-\frac{8}{17}\right) + 1$ ✓

$\therefore \cos^2\frac{\beta}{2} = \frac{9}{34}$ ✓

$\therefore \cos\beta = \frac{3}{\sqrt{34}}$ ✓

4.3 $1 + \sin 2\theta - 4\sin^2\theta = 0$

$\therefore \cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta - 4\sin^2\theta = 0$ ✓✓

$\therefore (\cos\theta + 3\sin\theta)(\cos\theta - \sin\theta) = 0$ ✓

$\therefore \tan\theta = -\frac{1}{3}$ ✓ or $\tan\theta = 1$ ✓

$\therefore \theta = \tan^{-1}\left(-\frac{1}{3}\right) + k.180^0;\quad k \in Z$ or $\theta = 45^0 + k.180^0;\quad k \in Z$

$\therefore \theta \in \{-18{,}4^0; 45^0\} + k.180^0;\quad k \in Z$ (correct to 1 decimal place) ✓✓✓

5.1 Bearing = 60^0 ✓

5.2 $AB^2 = 300^2 + 500^2 - 2.300.500.\cos 120^0$ ✓✓✓

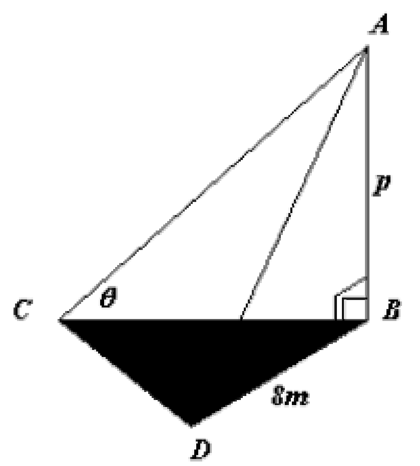
$= 90\,000 + 250\,000 - 300\,000\left(-\frac{1}{2}\right)$ ✓✓

$= 490\,000$

$\therefore AB = 700$ metres

Race distance = $300 + 500 + 700 = 2\,200$ metres ✓

6.



6.1 $C\hat{D}B = 180^0 - [30^0 + \theta] = 150^0 - \theta$ ✓

6.2 $\frac{p}{BC} = \tan\theta$ ✓

$\therefore BC = \frac{p}{\tan\theta}$ ✓

6.3 In $\triangle BCD$, $\frac{BC}{\sin\left(150^0 - \theta\right)} = \frac{8}{\sin\theta}$ ✓✓

$\therefore BC = \frac{8\sin\left(150^0 - \theta\right)}{\sin\theta}$

In $\triangle ACB$, $\frac{p}{BC} = \tan\theta$ ✓

$\therefore p = BC\tan\theta$

$= \frac{8\sin\left(150^0 - \theta\right)}{\sin\theta}.\tan\theta$

$= \frac{8\left(\sin 150^0\cos\theta - \cos 150^0\sin\ \beta\right)}{\sin\theta}.\frac{\sin\theta}{\cos\theta}$ ✓

$= \frac{8\left(\frac{1}{2}\cos\theta - \left(-\frac{\sqrt{3}}{2}\right)\sin\theta\right)}{\cos\theta}$ ✓✓

$= 4\frac{\cos\theta}{\cos\theta} + 4\sqrt{3}\frac{\sin\theta}{\cos\theta}$ ✓

$= 4\left(1 + \sqrt{3}\tan\theta\right)$ ✓

7.1 $\cos 2x = \sin\left(x + 30^0\right)$

$\therefore \cos 2x = \cos\left(90^0 - \left(x = 30^0\right)\right)$ ✓✓

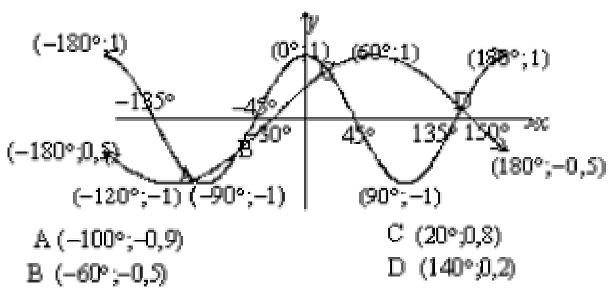
$\therefore 2x = \pm\left(60^0 - x\right) \quad + k.360^0, \quad k \in Z$ ✓✓

$\therefore 3x = 60^0 + k.360^0$ or $x = -60^0 + k.360^°$

$\therefore x = 20^0 + k.120^0$ or $x = -60^0 + k.360^°$; $k \in Z$

For $x \in \left[-180^0;180^0\right]$ $x \in \left\{-100^0;-60^0;20^0;140^0\right\}$ ✓✓

7.2 Function values correct to 1 decimal place where applicable.



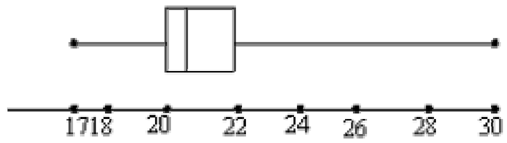
4 marks for each graph

7.3 60^0 ✓

8.1 3 marks for Boys and 3 marks for Girls (need not split the 20s into 20-24 and 25-29)

Boys		Girls
	1	7 9
3 3 2 1 0 0	2	0 0 0 1 1 2 3
8 8 7 6 5 5 5 5	2	
2	3	0

8.2 Box and whisker plot of Grade 2 Girls' Masses ✓✓✓✓



8.3 Standard deviation $= \sqrt{\frac{(x-\mu)^2}{15}}$ = 3,3 (correct to 1 decimal place) ✓✓✓✓

8.4 Modal mass=25 kg ✓

8.5 Interquartile range = $25 - 22 = 5kg$ ✓✓✓✓

8.6 90% of 15 is 13,5. On the ogive, the weight that corresponds to 13,5 is approximately 27,5 kg ✓✓

9.1 True: 50% of the data items are within the inter-quartile range compared to 68% within one standard deviation. ✓✓

9.2 True: the data is spread more to the left of the median than to the right. ✓✓

9.3 False: the greater the time, the lower the water level, so the correlation is negative. ✓✓

9.4 False: the scatter plot will be a straight line: $h = \frac{C - xt}{\pi r^2}$ is the height. C is the original capacity of the dam, t the number of hours and r the radius of the dam. All values except h and t are constants. h is a linear function of t with a negative gradient. ✓✓

Exam B

Grade 12 Mathematics Exam **Paper 2**
Time: 3 hours **Marks: 150**

1.1 $AC = \sqrt{(5-0)^2 + (0-3)^2} = \sqrt{34}$ ✓✓

$AB = \sqrt{(-3-0)^2 + (-2-3)^2} = \sqrt{34}$ ✓

$BC = \sqrt{(5+3)^2 + (0+2)^2} = \sqrt{68}$ ✓

Hence AB=AC and the triangle is isosceles ✓

And $BC^2 = AB^2 + AC^2$ and hence the triangle is right angled at A ✓

Right angle can also be proved by showing that gradient AB $\times$ gradient AC = -1

1.2 Area $\Delta ABC = \frac{1}{2} \times \sqrt{34} \times \sqrt{34} = 17$ square units ✓✓

1.3 M, the mid-point of BC is $\left(\frac{-3+5}{2}; \frac{-2+0}{2} \right) = (1;-1)$ ✓✓

1.4 Circle is $(x-1)^2 + (x+1)^2 = (1+3)^2 + (-1+2)^2 = 17$ ✓✓✓

1.5 Gradient $BC = \frac{0+2}{5=3} = \frac{1}{4}$ ✓✓

$\therefore$ gradient of tangent = -4 ✓

$\therefore$ equation of tangent is $\frac{y-0}{x-5} = -4$ ✓✓

i.e. $y = -4x + 20$ ✓

Gradient $MA = \frac{-1-3}{1-0} = -4$ ✓

Gradient of tangent = gradient MA hance these lines are parallel ✓

2.1 Radius = $\sqrt{10}$ ✓

2.2 Equation of AB is $\frac{y-1}{x+3} = \frac{3-1}{1+3}$ ✓✓✓

$\therefore 4y - 4 = 2x + 6$ i.e. $2y = x + 5$ ✓✓

2.3 At points if intersection: $x^2 + (-2x)^2 = 10$ ✓✓

$\therefore 5x^2 = 10$

$\therefore x = \pm\sqrt{2}$ ✓✓

and $y^2 = 8$

$\therefore y = \pm 2\sqrt{2}$ ✓✓

So the points of intersection are $(-\sqrt{2}; 2\sqrt{2})$ and $(\sqrt{2}; -2\sqrt{2})$ ✓✓

2.4 $A\hat{C}B =$ inclination $AC -$ inclination BC ✓✓

$= \tan^{-1} \dfrac{0-1}{\sqrt{10}+3} - \tan^{-1} \dfrac{0-3}{\sqrt{10}-1}$ ✓✓

$= -170,78...^0 - 125,78...^0$ ✓✓

$= 45^0$ ✓

Hence $A\hat{O}B = 2 \times A\hat{C}B$

3.1.1 $P'(5;-1)$ ✓✓

3.1.2 $P'(1;-5)$ ✓✓

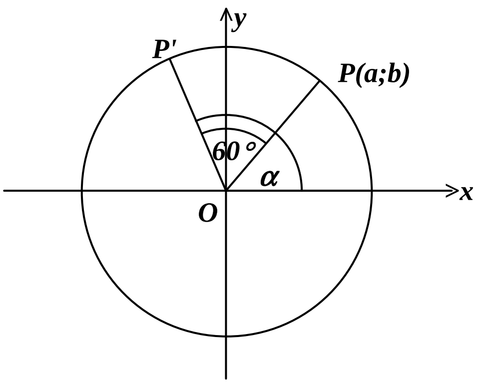
3.2.1 The co-ordinates of the vertices of the images are reduced by a factor of 2:
i.e. they are half the distance of the original vertices from the origin. ✓✓

The area of the transformed triangle is $\frac{1}{4}$ the area of the original triangle. ✓✓

3.2.2 The vertices are reflected about the *x*-axis ✓✓
and translated 2 units to the right. ✓✓

This transformation is a glide reflection which is rigid and hence the area of the transformed triangle is equal to the area of the original triangle. ✓✓

3.3



$a = OP\cos\alpha$ and $b = OP\sin\alpha$ and hence
the x co-ordinate of $P' = OP'\cos(\alpha + 60^0)$ ✓

$= OP[\cos\alpha\cos 60^0 - \sin\alpha\sin 60^0)$ ✓

$= OP\cos\alpha.\dfrac{1}{2} - OP\sin\alpha.\dfrac{\sqrt{3}}{2} = \dfrac{a}{2} - \dfrac{\sqrt{3}b}{2}$ ✓

and the y co-ordinate of $P' = OP'[\sin(\alpha + 60^0)$ ✓

$$= OP'\left[\sin\alpha.\frac{1}{2} + \cos\alpha.\frac{\sqrt{3}}{2}\right]$$ ✓

$$= \frac{b}{2} + \frac{\sqrt{3}a}{2}$$ ✓

4.1 $\dfrac{\sin(-\alpha)\cos(90^0 - \alpha)}{\cos\alpha\cos(180^0 + \alpha)} = \dfrac{(-\sin\alpha)(\sin\alpha)}{\cos\alpha(-\cos\alpha)}$ ✓✓✓

$$= \frac{-\sin^2\alpha}{-\cos^2\alpha}$$ ✓

$$= \tan^2\alpha$$ ✓

4.2.1 $\cos 27^0 = \sqrt{1 - \sin^2 27^0} = \sqrt{1 - t^2}$ ✓✓

4.2.2 $\tan 153^0 = -\tan 27^0$ ✓

$$= -\frac{t}{\sqrt{1 - t^2}}$$ ✓

4.2.3 $\cos 243^0 = -\cos 63^0$ ✓

$$= -\sin 27^0 = -t$$ ✓

4.2.4 $\cos 54^0 = \cos(2 \times 27^0)$ ✓

$$= 1 - 2\sin^2 27^0 = 1 - 2t^2$$ ✓✓

4.3 $\tan(3x + 75^0) = -1$

$\therefore 3x + 75^0 = -45^0 + k.180^0; \quad k \in Z$ ✓✓

$\therefore 3x = -120^0 + k.180^0$ ✓

$\therefore x = -40^0 + k.60^0; \quad k \in Z$ ✓✓

4.4 $\dfrac{\sin 15^0}{2} + \dfrac{\sqrt{3}\cos 195^0}{2} = \dfrac{\sin 15^0}{2} + \dfrac{\sqrt{3}\cos(180^0 + 15^0)}{2}$ ✓

$$= \frac{1}{2}.\sin 15^0 + \frac{\sqrt{3}}{2}(-\cos 15^0)$$

$$= \sin 30^0 \sin 15^0 - \cos 30^0 \cos 15^0$$ ✓✓

$$= -\cos(30^0 + 15^0)$$ ✓

$$= -\frac{\sqrt{2}}{2}$$ ✓

5.1 In ΔADC: $\cos\theta = \dfrac{r^2 + r^2 - k^2}{2r.r}$ ✓

$= \dfrac{2r^2 - k^2}{2r^2}$ ✓

5.2 In ΔABD: $\cos(180^0 - \theta) = \dfrac{r^2 + (2r)^2 - (2k)^2}{2.r.2r}$ ✓

$\therefore -\cos\theta = \dfrac{5r^2 - 4k^2}{4r^2}$ ✓

$\therefore \cos\theta = \dfrac{4k^2 - 5r^2}{4r^2}$ ✓

5.3 Hence $\dfrac{2r^2 - k^2}{2r^2} = \dfrac{4k^2 - 5r^2}{4r^2}$ ✓

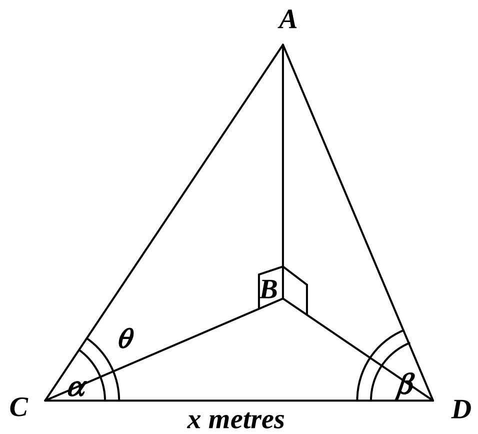
$\therefore 4r^2 - 2k^2 = 4k^2 - 5r^2$

$\therefore 9r^2 = 6k^2$

and $k^2 = \dfrac{3}{2}r^2$ ✓

So $\cos\theta = \dfrac{2r^2 - \frac{3}{2}r^2}{2r^2} = \dfrac{1}{4}$ ✓

6.



6.1 In ΔACD $\dfrac{AC}{\sin\beta} = \dfrac{x}{\sin[180^0 - (\alpha + \beta)]}$ ✓✓

$\therefore AC = \dfrac{x\sin\beta}{\sin(\alpha + \beta)}$ ✓✓

In ΔABC: $\dfrac{AB}{AC} = \sin\theta$ ✓

$\therefore AB = \dfrac{x\sin\beta}{\sin(\alpha + \beta)}.\sin\theta$ ✓

6.2 Height = $\dfrac{40.\sin 70^0.\sin 15^0}{\sin 120^0}$ ✓

= 11,23 *metres* (correct to 2 decimal places) ✓

or 11 metres (correct to the nearest *metre*)

6.3 Area $\Delta ACD = \frac{1}{2} AC.CD.\sin\alpha$ ✓

$= \frac{1}{2} \times \frac{x \sin\beta}{\sin(\alpha+\beta)}.x.\sin\alpha$ ✓

$= 664{,}97\ m^2$ (correct to 2 decimal places ✓

or 665 m^2 (correct to the nearest m^2)

7.1 At A $\sin 2x = 0$ ✓

$\therefore x = k.90^0; \quad k \in Z$

$A(-90^0;0)$ ✓

B is $(60^0;2)$ ✓

At C $x = 0$ so $y = 2\cos(-60^0) = 1$ ✓

C is $(0;1)$ ✓

7.2 $DE = 2\cos(120^0 - 60^0) - \sin(2 \times 120^0)$ ✓✓

$= 2\cos 60^0 + \sin 60^0$ ✓

$= 2 \times \frac{1}{2} + \frac{\sqrt{3}}{2} = 1 + \frac{\sqrt{3}}{2}$ ✓

7.3 At G: $x = -30^0$ ✓

At H: $y = \sin(2 \times -30^0) = -\sin 60^0$ ✓

$= -\frac{\sqrt{3}}{2}$ ✓

Hence the equation of the new curve is $y = 2\cos(x - 60^0) - \frac{\sqrt{3}}{2}$ ✓✓

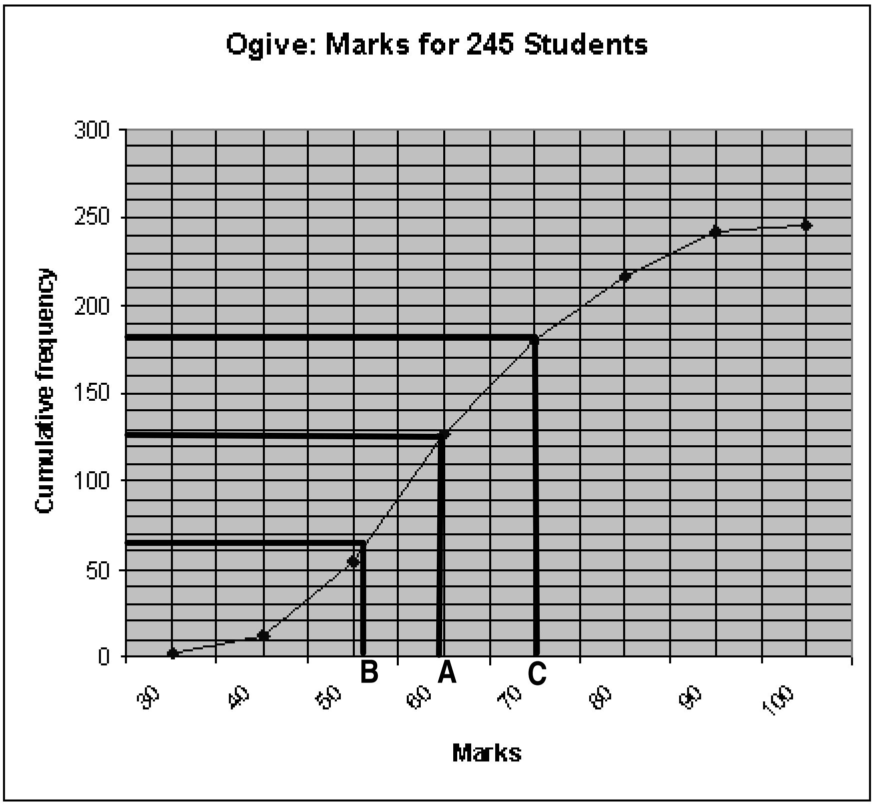
8.1 $\bar{x} = 185\ g$ ✓

$\sigma = \sqrt{\sum_{i=1}^{8} \frac{(x_i - 185)^2}{8}}$ =25,98 g (correct to 2 decimal places) ✓

or 26 g (correct to the nearest g ✓✓

8.2 Standard deviation is a measure of dispersion: the larger the standard deviation, the wider the spread of data items. ✓✓

9.1



✓✓✓✓
✓✓✓✓

9.2 Median $\approx 59\%$ (read at A) ✓✓
Lower quartile ≈ 51 (read at B) ✓✓
Upper quartile $\approx 71\%$ (read at C) ✓✓

9.3 Mean = $\dfrac{2\times15+10\times35+43\times45+72\times55+53\times65+37\times75+25\times85+3\times95}{245}$

✓✓✓✓

$= 60{,}84\%$ (correct to 2 decimal places0 ✓✓

9.4 Mean $\approx 61\%$, median $\approx 59\%$ and mode 55%. Because all these values are approximate, and the differences are not significant, we can say that the distribution is fairly symmetric (the data is not skewed) ✓✓✓