# ANALYTICAL GEOMETRY (1)

Lesson 27

**Learning Outcomes and Assessment Standards**

**Learning Outcome 3: Space, Shape and Measurement**
**Assessment Standard**
- The gradient and inclination of a straight line
- The equation of a straight line

## Overview

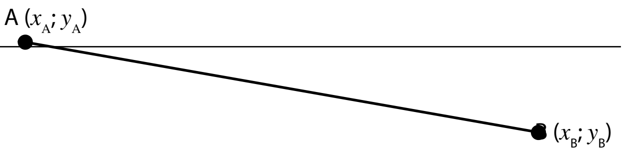
In this lesson you will:

- Review the distance formula, the mid-point and the gradient covered in Grade 10 in order to cope with the progression needed for this section of the curriculum.
- Use analytical geometry and properties of quadrilaterals to solve various problems.
- Use correct formulae, interpret questions and make the necessary equations.

## Lesson

DVD

## The distance formula:




To find the distance between two points A and B, we use the formula:

$$AB = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2}$$

**Examples**

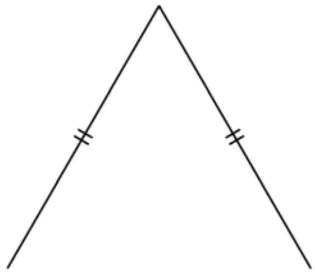
1.The distance between A(–7; $y$) and B(–3; 4) is $4\sqrt{5}$, find $y$

$$AB = 4\sqrt{5} \quad \text{(given)}$$
$$\therefore AB^2 = (4\sqrt{5})^2$$
$$= 16 \cdot 5$$
$$= 80$$
$$\therefore (x_A - x_B)^2 + (y_A - y_B)^2 = 80$$
$$\therefore (-7 + 3)^2 + (y - 4)^2 = 80$$
$$\therefore (y - 4)^2 = 80 - 16$$
$$= 64$$
$$\therefore y - 4 = 8$$
$$\therefore y = 4 \pm 8$$
$$y = 12 \quad \text{or} \quad y = -4$$

2. P(–1; –1) is equidistant from Q(0; 2) and R($x$; –2)

Find the value of $x$

Draw a picture



Form an equation PQ = PR

So $PQ^2 = PR^2$

Distance formula:

$\therefore (x_P - x_Q)^2 + (y_P - y_Q)^2 = (x_P - x_R)^2 + (y_P - y_R)^2$

$\therefore (-1 - 0)^2 + (-1 - 2)^2 = (x + 1)^2 + (-2 + 1)^2$

$\therefore 1 + 9 = (x + 1)^2 + 1$

$\therefore (x + 1)^2 = 9$

$\therefore x + 1 = \pm 3$

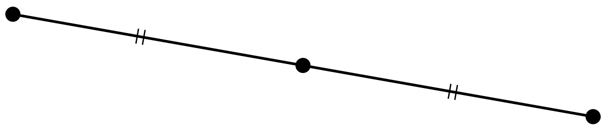
$\therefore x = -1 \pm 3$

$\therefore x = -4$ or $x = 2$

## The mid-point of a line

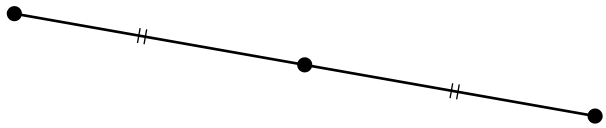
To determine the co-ordinates of the midpoint of a line segment AB:

$x_C = \frac{x_A + x_B}{2}$ $\quad y_C = \frac{y_A + y_B}{2}$



**Example**

B(–1; 3) is the mid-point of AC.



Find the co-ordinates of A if

A($x$; $y$) and C(6; -5)

Draw a picture

Form equations

$-1 = \frac{x + 6}{2}$ and $3 = \frac{y - 5}{2}$

$-2 = x + 6$ $\quad 6 = y - 5$

$x = -8$ $\quad y = 11$

A(–8; 11)

Activity 1a–e

BASELINE ASSESSMENT

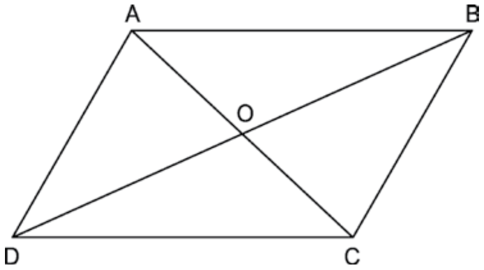
PAIRS

DVD

**Working with the parallelogram**

Diagonals bisect each other

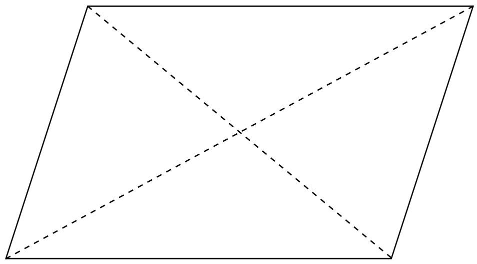
Show that the mid-point of AC is the same as the mid-point of BD.




**Example**

A(2; 3), C(5; –1), B($x$; $y$) and D(3; –3) are the co-ordinates of the parallelogram ACBD, find the co-ordinates of B.

**Gradient**



Since ABCD is a $||^m$, we can use the properties of $||^m$ to help us find the co-ordinates of point B.

**Using the fact that the diagonals bisect one another:**

Midpoint of AC:

$$\left.\begin{aligned} x_E &= \frac{x_A + x_C}{2} = \frac{5+2}{2} = \frac{7}{2} \\ y_E &= \frac{y_A + y_C}{2} = \frac{3-1}{2} = 1 \end{aligned}\right\}$$

Now midpoint BD:

$$x_E = \frac{x_B + x_D}{2} \text{ and } y_E = \frac{y_B + y_D}{2}$$

$$\frac{7}{2} = \frac{x+3}{2} \qquad 1 = \frac{y-3}{2}$$

$$\therefore x = 7 - 3 \qquad \therefore y - 3 = 2$$

$$x = 4 \qquad y = 5$$

$$\therefore \text{E} = (4; 5)$$

**Using the fact that the gradients of sides are equal:**

$m_{AB} = m_{DC}$

$\therefore \triangle y_{AB} = \triangle y_{DC}$ and $\triangle x_{AB} = \triangle x_{DC}$

$\therefore y - 3 = -1 - (-3)$ $\quad$ $\therefore x - 2 = 5 - 3$

$\therefore y - 3 = 2)$ $\quad$ $\therefore x = 2 + 2$

$y = 5$ $\quad$ $x = 4$

$\therefore$ E = (4; 5)

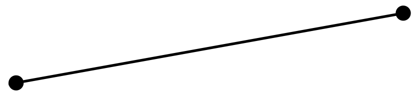
We can use BC = AD, and the equation of BC, to find the co-ordinates of B.

We could also use $m_{BC} = m_{AD}$ in the same way as above.

**The gradient of a line segment**

To determine the gradient (slope) of a line PQ, we use

$m_{PQ} = \frac{\triangle y}{\triangle x} = \frac{y_Q - y_P}{x_Q - x_P}$



If two lines are parallel, then they have equal gradients

If two lines are perpendicular, then the product of their gradients is –1

If a line is horizontal, then $\triangle y = 0$ : So m = 0

If a line is vertical, then $\triangle x = 0$ : So m = $\alpha$

3 points A, B and C are said to be collinear if $m_{AB} = m_{BC}$

**Examples**

If A(1; 4), B(–3; 2), C(–1; –1) and D(*x*; 0) are four points in the Cartesian plane, find the value of *x* if:

(a) AB || CD

(b) AB ⊥ BD

(c) BC and D are collinear.

1. a) AB || CD Make the equation

$m_{AB} = m_{CD}$

$\frac{2-4}{-3-1} = \frac{0+1}{x+1}$

$\frac{-2}{-4} = \frac{1}{x+1}$

$\frac{1}{2} = \frac{1}{x+1}$ cross multiply

$x + 1 = 2$

$x = 1$

b) AB ⊥ BD

$m_{AB} \times m_{BD} = -1$

$\therefore \frac{1}{2} \times \frac{2-0}{-3-x} = -1$

$\frac{1}{2} \times \frac{2}{-3-x} = -1$

$\therefore \frac{1}{x+3} = 1$

$\therefore 1 = 3 + x$

$x = -2$

c) B; C and D are collinear

$m_{BC} = m_{CD}$

$\frac{2+1}{-3+1} = \frac{0+1}{x+1}$

$-\frac{3}{2} = \frac{1}{x+1}$

$3(x + 1) = -2$

$3x + 3 = -2$

$3x = -5$

$x = \frac{-5}{3}$

Activities 2 – 4

FORMATIVE ASSESSMENT

PAIRS

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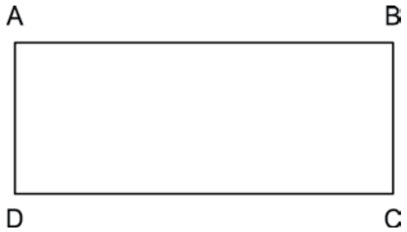
**How to prove a quadrilateral is a rectangle**

1. Prove ABCD is a parallelogram




So mid-point of AC = mid-point BD

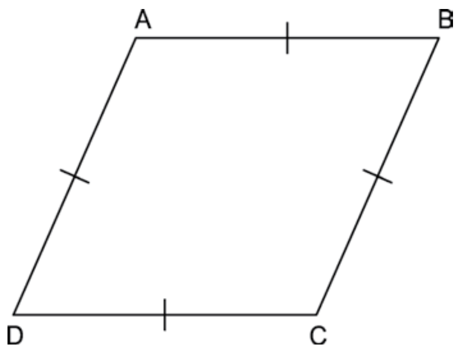
2. Then prove that there is one right angle.

Perhaps $m_{AD} \times m_{AB} = -1$

Then $\hat{A} = 90°$

**How to prove a quadrilateral is a rhombus, or prove that the diagonals bisect each other at 90°**

Prove all four sides are equal.



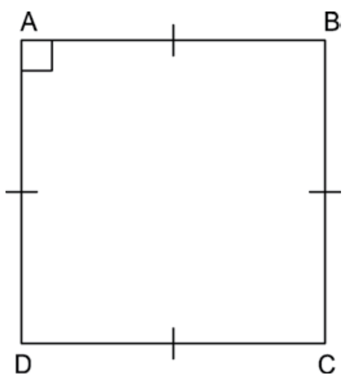

**How to prove a quadrilateral is a square**

Prove all four sides are equal

Prove there is one right angle

perhaps $m_{AB} \times m_{BC} = -1$

so $\hat{B} = 90°$



BASELINE ASSESSMENT

PAIRS

## Activity 5

1. In each case determine the value of $x$ and $y$.
   a) The distance of (2; $y$) from the origin is $\sqrt{40}$.
   b) (2; –3) is equidistant from (–3; 2) and ($x$; –8).
   c) ($x$; $y$) is the mid-point of the line segment joining (–1; 3) and (7; 1).
   d) (1;-2) is the centre of the circle passing through (5; 1) and (–2; $y$).
2. If A(3; 1), B–5; 7), C(11; –5) and D($x$; $y$) are the co-ordinates of parallelogram ABCD, find the co-ordinates of D.
3. Prove that K(4; 4), L(5; –1) and M(–6; 2) are the vertices of a right angled triangle and hence decide which angle is right angled.
4. Find $p$ and $q$ if P(–3; 2) is the midpoint of the line joining A($p$; $q$) and B(–1; 5).
5. The vertices of ΔDEF are D(2; 3), E(–3; –1) and F(6; –2). Show that it is a right angle triangle. If P and Q are the midpoints of DE and EF respectively, prove also that PQ$\parallel$EF and that PQ = $\frac{1}{2}$EF.
6. Prove that the figure bounded by the lines $y = 2x + 1$; $y - 3x = 6$; $2y + 6x = 7$; $y = 3x + 1$ is a trapezium. Find the coordinates of its vertices.