Lesson 30

ANALYTICAL GEOMETRY (4)

(Continuation of Lesson 29)

Learning Outcomes and Assessment Standards

Learning Outcome 3: Space, Shape and Measurement
Assessment Standard
- The gradient and inclination of a straight line
- The equation of a straight line

Overview

In this lesson *you* will:

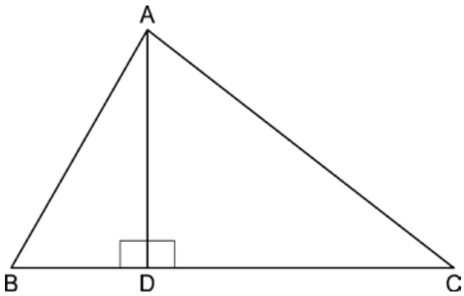
- Use a formula to find the equation of a straight line.
- Find equations of parallel and perpendicular lines.
- Find the co-ordinates of the point of intersection of two lines.

DVD

Lesson

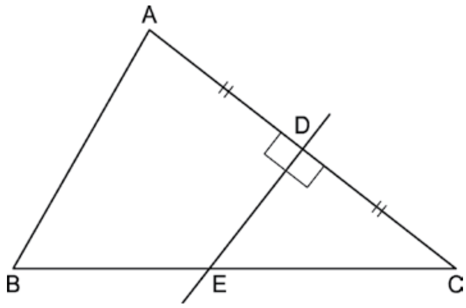
Important Lines

Altitude



AD is an altitude of △ABC. From the angle perpendicular to the opposite side.

Perpendicular Bisector

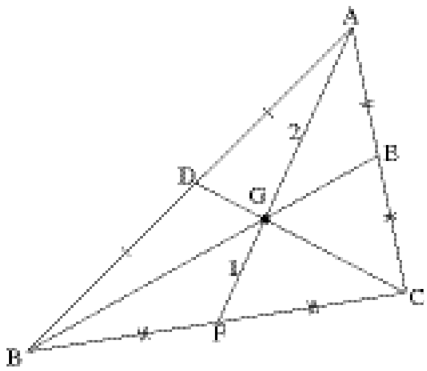


DE is the perpendicular bisector of AC

Medians and the Centroid

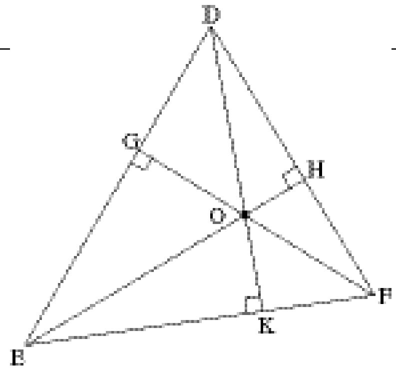
AF, BE and CD are the mediums of △ABC.

The medians run from vertex to the midpoint of the opposite side. Point G is called the **centroid** of △ABC, and this is also the point of concurrency of the medians.

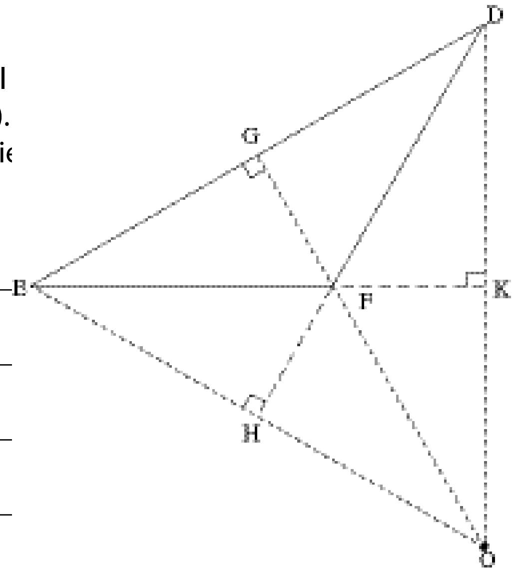


The Orthocentre and the Altitudes

The altitudes DK, FG and EH are drawn from the vertex perpendicular to the opposite side. (They don't necessary go through the midpoint of the side). The point of concurrency of these altitudes is called the **orthocentre**.

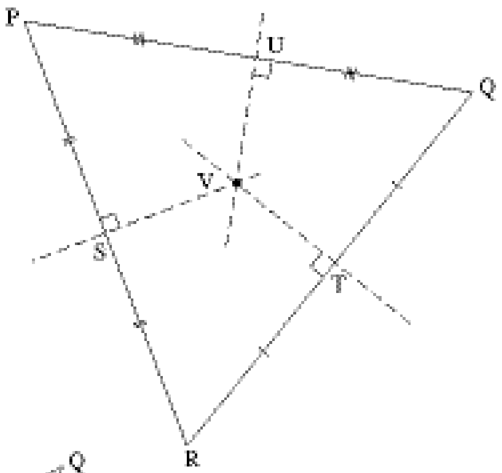


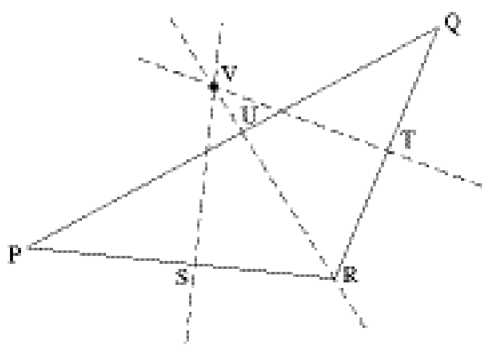
The orthocentre lies inside the triangle. All angles are smaller than 90° (acute angled). obtuse angled triangles, the orthocentre lie outside the triangle.



The Circumcentre and the Perpendicular bisectors of the sides

△PQR has a circumference at V, where the perpendicular bisectors of the sides are concurrent. O is the centre of the circle that can be drawn through P, Q and R. In acute angled triangles the circumcentre lies inside the triangle. In obtuse angled triangles the circumcentre lies outside the triangle. Note that perpendicular bisectors of the sides, does not necessary pass through the vertices of the triangle.



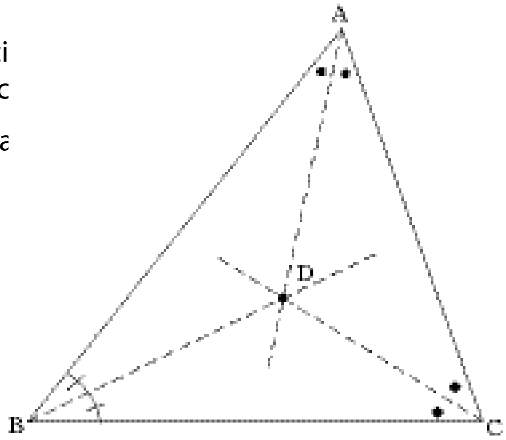


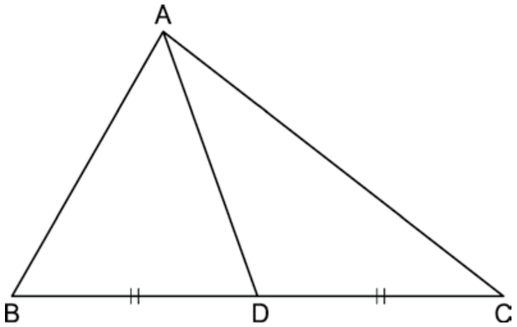
The Incentre and the Bisectors of the interior angles of a triangle

The bisectors of the angles A, B and C are concurrent at the incentre D of the triangle.

The incentre gives us the midpoint of the ci that can be drawn inside the triangle (incirc

Incentres are located inside the triangle at a times.





AD is a median

From the angle to the middle of the opposite side

INVESTIGATION FOR ALL OF YOU.
When will the altitude, perpendicular bisector and median be the same line.

Example 1

A(–3 ; 3)   B(1 ; –1) and C(–5 ; –3) are the co–ordinates of △ABC

a)   Find the equation of the altitude from A to BC

Solution

Plot the points roughly.

We want the equation of altitude AD.

We need a point and the gradient.

Point A(–3 ; 3)   Gradient?

AD ⊥ CB

m_{CB} is $\frac{-1+3}{1+5} = \frac{2}{6} = \frac{1}{3}$

So m_{AD} –3

A(–3 ; +3)

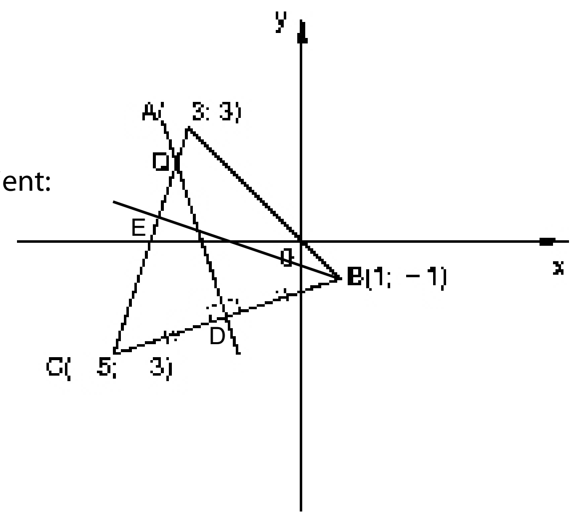
Equation is   So point gradient:

$\frac{y - y_A}{x - x_A} = m$

$y - 3 = -3(x + 3)$

$y - 3 = -3x - 9$

$y = -3x - 6$



b) Find the equation of the median from B to AC.

Solution

To find the median, we need the midpoint of AC

$E = \left(\frac{-3-5}{2}; \frac{3-3}{2}\right) = (-4 ; 0)$

So equation of median: (2 points given)

$\frac{y-y_B}{x-x_B} = \frac{y_B-y_E}{x_B-x_E}$

$\therefore \frac{y+1}{x-1} = \frac{-1}{1+4} = -\frac{1}{5}$

$\therefore 5y + 5 = -x + 1$

$5y = -x - 4$

c) Find the equation of the perpendicular bisector of BC

Solution

We need the gradient of BC:

$m_{BC} = +\frac{1}{3} \rightarrow m_{\perp} = -3$

We need the midpoint of BC:

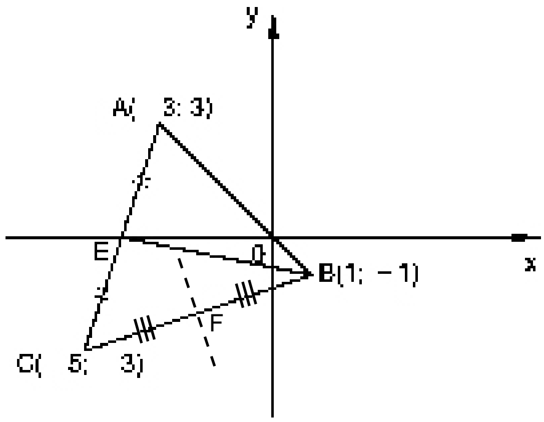
$F\frac{-5+1}{2}; \frac{-3-1}{2} = F(-2; -2)$ pt$(-2 ; -2)$

Equation of bisector

$\frac{y+2}{x+2}$

$\therefore y + 2 = -3x - 6$

$\therefore y = -3x - 8$



Activities 1 – 4

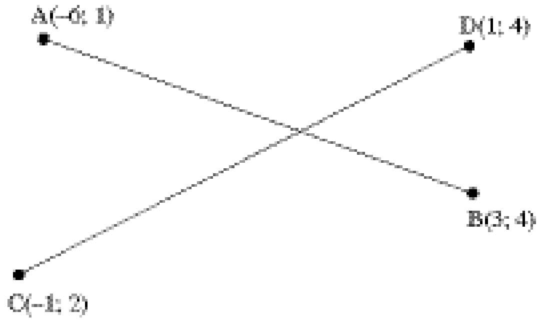
Points of Intersection

Example

A(–6 ; 1) B(3 ; 4) C(–1 ; 2) and D(1 ; 4) are four points.

Find the co–ordinates of the point of intersection of lines AB and CD.

Solution



Plan: Equation of AB
Equation of CD

$m_{AB} = \frac{3}{9} = \frac{1}{3}$

$Eq_{AB}: \frac{y-4}{x-3} = \frac{1}{3}$

$3y - 12 = x - 3$

$3y = x + 9$

$m_{CD} = \frac{2}{2} = 1$

$Eq_{CD}: \frac{y-4}{x-1} = 1$

$y - 4 = x - 1$

$\therefore y = x + 3$

Solve simultaneously

Substitute 2 into 1

$3(x + 3) = x + 9$

$3x + 9 = x + 9$

$2x = 0$

$x = 0$

$y = 3$

Point of interception (0 ; 3)

How do we know if a point is on a line?

Is the point (9 ; –3) on the line $2y + x = 3$?

Plan: Make $x = 0$ and $y = -3$ and see whether the equation is true

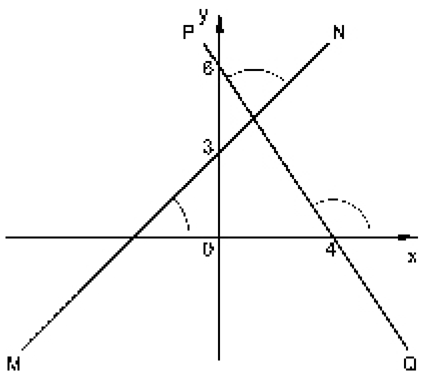
LHS $2y + x = 2(-3) + 9 = 3 =$ RHS

INDIVIDUAL

SUMMATIVE ASSESSMENT

Activities 5–7

1.



Determine the size of

a) α b) β c) θ

2. M is a point t units from the origin O(0 ; 0).

   Calculate the co-ordinates of M in terms of t if the line $x + \sqrt{3}y = 2t$ passes through M.

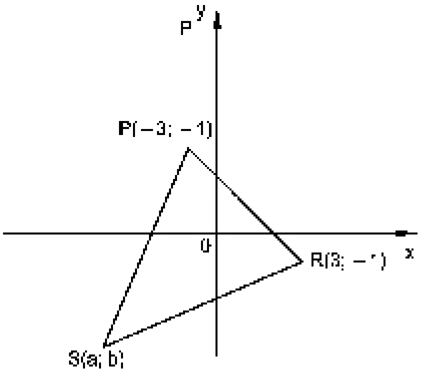
3. The vertices of $\triangle$ABE are A(0 ; 4) : B(5 ; 3) and E(2 ; 1)

   a) Prove that $\hat{E} = 90°$

   b) If ABCD is a rhombus with diagonals AC and BD intersecting at E, determine the co-ordinates of C and D.

   c) Prove ABCD is a square.

4. △PRS has vertices P(−1 ; 3)   R(3 ; −1)   5(a ; b)



a) Find T, the mid–point of PR.

b) If the perpendicular bisector of PR passes through S, show that $a =$
b.

c) If the area of △PRS is $12u^2$, find the co–ordinates of S.

d) If Q is (4 ; 4) show that PQRS is a rhombus.

5. a) M(a ; 2) is the mid point of A(−2 ; 5) and B(8 ; −1).
Find the value of a.

b) Find e if the distance between the point (0 ; −4) and (e ; 0) is 5.

c) Given the point A(1 ; 3) B(3 ; 2) and C(−1 ; −1) find

i) the equation of the straight line through C parallel to AB.

ii) the equation of the straight line through B perpendicular to
AB.

iii) The point of intersection of these lines.

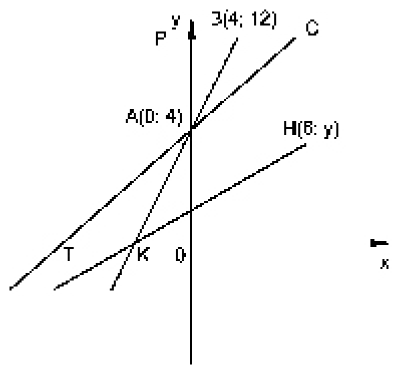
6. ABCD is a trapezium with co-ordinates A(−4 ; 3) B(x ; 6) C(4 ; y) and D(−2 ; −1) where $x < 4$.

If AD || BC and BC = 2AD

a) Find x and y.

b) Find the co-ordinates of E, the point of intersection of BA and CD.

7.



A(0 ; 4) B(4 ; 12) are the points seen above. Line TAC has a gradient of $\frac{1}{3}$

a) Find $C\hat{T}X$

b) Calculate $B\hat{A}C$

c) If H is the point (6 ; y) and $H\hat{K}X = 30°$, calculate the value of y.

(Leave answer in surd form)