

GRADE 12

**NATIONAL
SENIOR CERTIFICATE**

MATHEMATICS P1

PREPARATORY EXAMINATION 2008

MARKS: 150

TIME: 3 hours

This question paper consists of 9 pages, a formula sheet and 1 diagram sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 12 questions. Answer ALL the questions.
2. Clearly show ALL calculations, diagrams, graphs, et cetera you have used in determining the answers.
3. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
4. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
5. Graph paper is used in QUESTION 12 only. Write your name in the space provided and hand it in together with the ANSWER BOOK.
6. Diagrams are NOT necessarily drawn to scale.
7. Number the answers correctly according to the numbering system used in this question paper.
8. Write neatly and legibly.

QUESTION 1Solve for x :

1.1 $x^2 - x = 20$ (3)

1.2 $2x + \frac{8}{x+1} = 8$ (4)

1.3 Solve for x and y simultaneously:

$x - 2y = -1$ and $y^2 + xy + x^2 = 7$ (7)

1.4 $(x-1)(x+4) \geq 6$ (5)
[19]

QUESTION 2

2.1 Determine the time, taken in years, for a sum of money to double if the interest rate is 12,64% p.a., compounded half-yearly. (4)

2.2 Dudu wants to buy a house for R700 000,00. She has a deposit of R50 000,00 and takes out a loan for the balance at a rate of 18% p.a. compounded monthly.

2.2.1 How much money must Dudu borrow from the bank? (1)

2.2.2 Calculate the monthly payment if she wishes to settle the loan in 15 years. (4)

2.2.3 Dudu won a lottery and wishes to settle the loan after the 50th payment.
What is the outstanding balance? (4)
[13]**QUESTION 3**

Consider the following sequence of numbers: 1; 2; 1; 5; 1; 8; 1; 11; ...

3.1 What is the 10th term of the above sequence? (2)

3.2 Calculate the sum of the first 50 terms of the sequence. (4)
[6]

QUESTION 4

Consider the following sequence: 3; 6; 11; 18; 27; ...

- 4.1 Determine the 6th and 7th terms of the given sequence, if the sequence behaves consistently. (2)
- 4.2 Determine a formula for the general term, p , of the sequence. (4)
- 4.3 Use your formula to calculate p if the p^{th} term in the sequence is 627. (4)
- [10]**

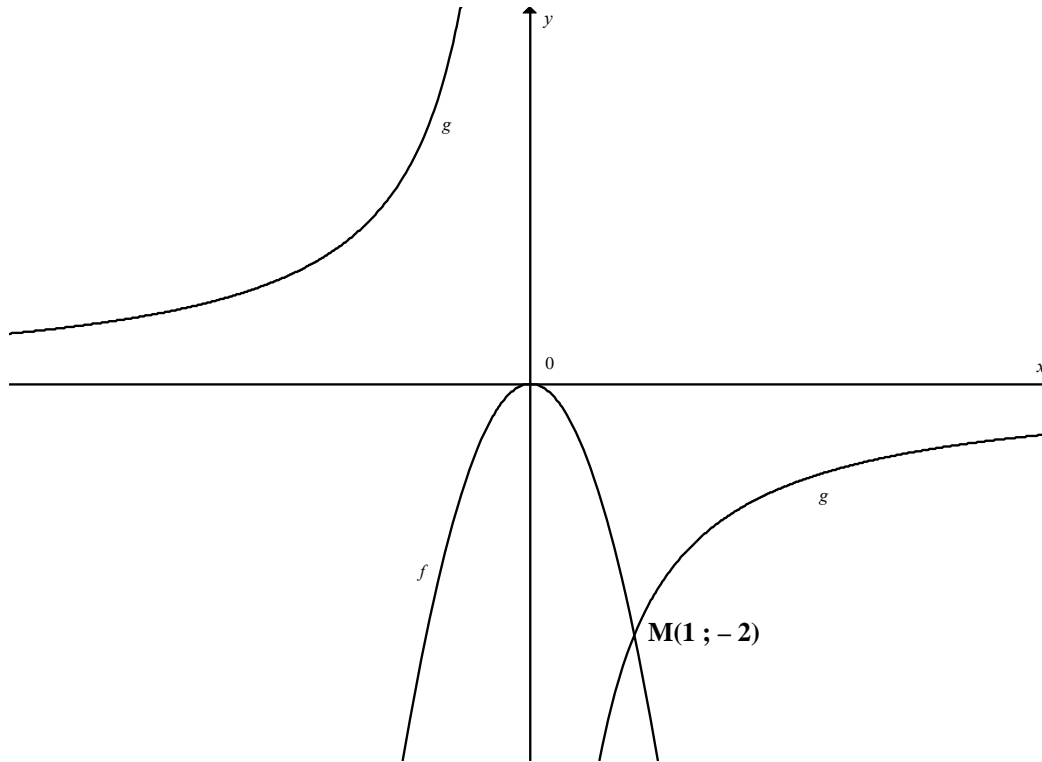
QUESTION 5

- 5.1 Kopano wants to buy soccer boots costing R800, but he only has R290,00. Kopano's uncle Stephen challenges him to do well in his homework for a reward. Uncle Stephen agrees to reward him with 50c on the first day he does well in his homework, R1 on the second day, R2 on the third day, and so on for 10 days.
- 5.1.1 Determine the total amount uncle Stephen gives Kopano for 10 days of homework well done. (5)
- 5.1.2 Is it worth Kopano's time to accept his uncle's challenge? Substantiate your answer. (2)
- 5.2 Consider the geometric sequence: $8(x-2)^2$; $4(x-2)^3$; $2(x-2)^4$; $x \neq 2$
- 5.2.1 Determine the value of x for which the sequence converges. (3)
- 5.2.2 Determine the sum to infinity of the series if $x = 2,5$. (3)
- [13]**

QUESTION 6

The diagram below shows the graphs of $f(x) = ax^2$ and $g(x) = \frac{-2}{x}$.

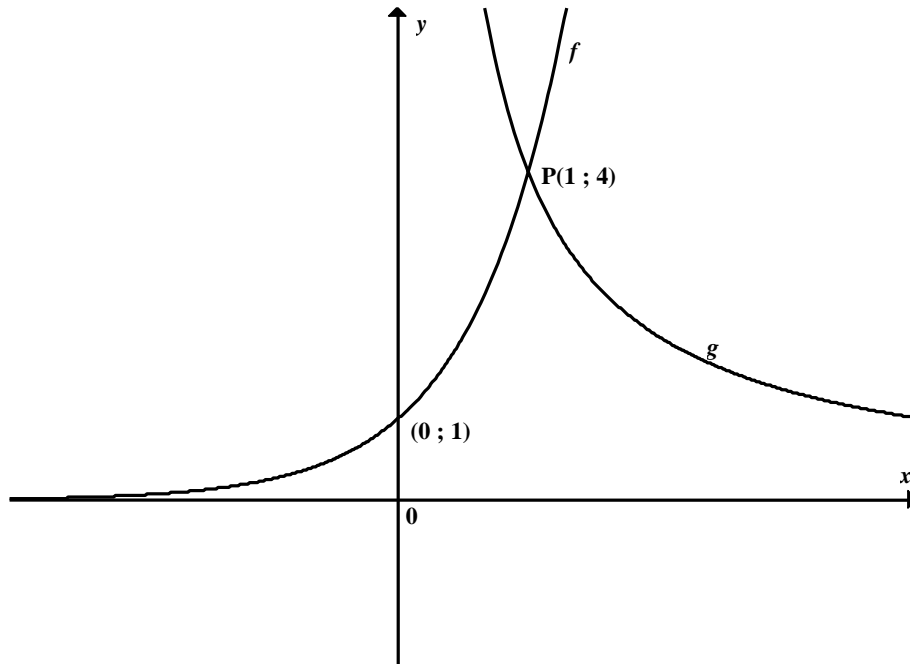
The point $M(1 ; -2)$ is the point of intersection of f and g .



- 6.1 Determine the value of a . (1)
- 6.2 If $g(x)$ is translated to give $h(x) = \frac{-2}{x-2} + 1$, write down the asymptotes of $h(x)$. (2)
- 6.3 Sketch the graph of $h(x) = \frac{-2}{x-2} + 1$. (3)
- [6]**

QUESTION 7

- 7.1 The diagram below shows the graphs of $f(x) = 4^x$ and $g(x) = \frac{a}{x}$.
The point P(1 ; 4) is the point of intersection of f and g .

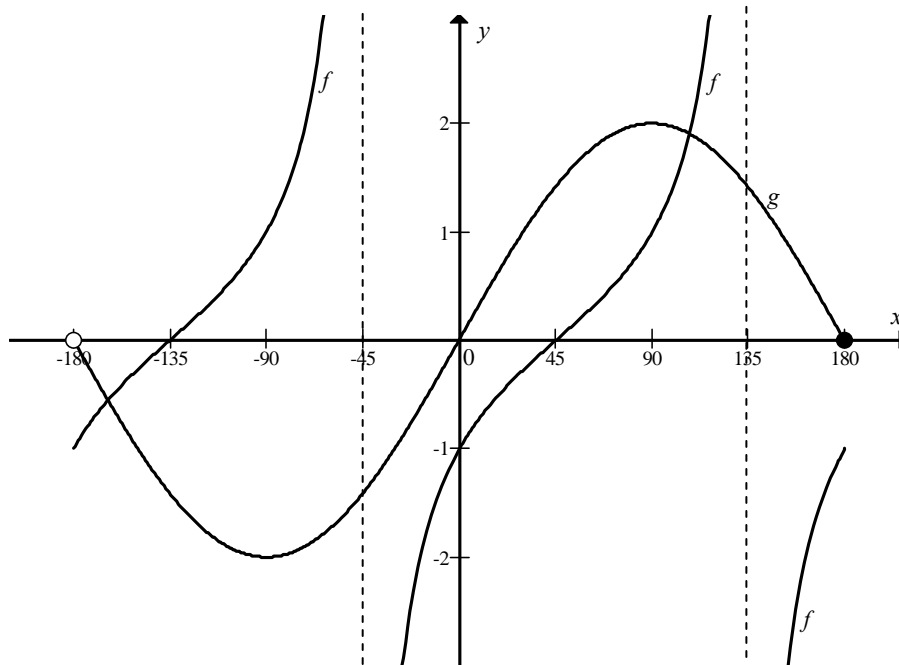


- 7.1.1 Write down the equation of f^{-1} in the form $y = \dots$. (2)
- 7.1.2 Is f^{-1} a function? Substantiate your answer. (2)
- 7.1.3 Determine the equation of $h(x)$, the resultant function when $f(x)$ is reflected about the y -axis. (2)
- 7.2 Determine the value of a in $g(x)$. (2)
- 7.3 Determine the equation of $m(x)$, the resultant function when $g(x)$ is shifted horizontally 2 units to the right and vertically 1 unit down. (2)
- 7.4 Calculate the intercepts of $m(x)$ with the axes. (3)
- [13]**

QUESTION 8

In the diagram below $f(x) = \tan(x - 45^\circ)$ and $g(x) = 2\sin x$.

The point O is the origin.



- 8.1 Write down the amplitude of $g(x)$. (1)
- 8.2 Write down the domain of $g(x)$. (2)
- 8.3 Write down the new equation of g if the translation of the graph is 60° horizontally to the right. (2)
- 8.4 Determine the value of x for which $g(x) - f(x) = 1$. (2)
- [7]**

QUESTION 9

9.1 Given: $f(x) = -x^2$

9.1.1 Determine $f'(x)$ from first principles. (5)

9.1.2 Determine the average gradient of $f(x)$ between $x = 1$ and $x = 3$. (3)

9.2 Determine $\frac{dy}{dx}$ if:

9.2.1 $y = (x^3 + 1)(x^2 - 2)$ (4)

9.2.2 $y = \frac{\sqrt{x} - 4}{\sqrt{x}}$ (4)
[16]

QUESTION 10

Given: $f(x) = x^3 + x^2 - 5x + 3$

10.1 Determine all intercepts of $f(x)$. (4)

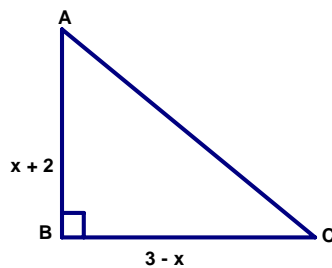
10.2 Determine the coordinates of the turning points of $f(x)$. (5)

10.3 Draw a sketch of $f(x)$. (4)

10.4 What is the equation of the tangent to $f(x)$ when $x = 2$? (3)

10.5 What are the coordinates of the turning point $f(x - 1)$? (2)

10.6 Determine the x -value of the point of inflection. (4)
[22]

QUESTION 11

11.1 Show that the area of the triangle in the figure above is given by:

$$A(x) = \frac{x^2}{2} + 4x - \frac{15}{2}$$
 (2)

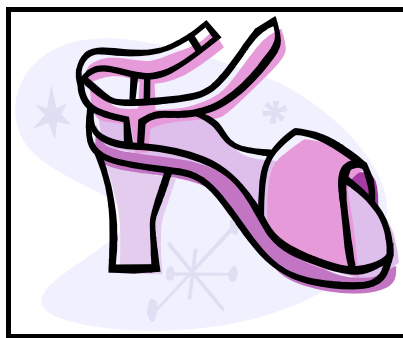
11.2 Calculate the value of x for which the area will be maximum. (3)

11.3 Hence, calculate the maximum area of the given triangle. (2)
[7]

QUESTION 12

Tsholanang is the manager of a small business that manufactures handmade sandals. Two types of pairs of sandals are manufactured: Elegance sandals and Classic sandals.

- The company can manufacture between 40 and 150 pairs of Elegance sandals per month.
- The company can manufacture between 10 and 120 pairs of Classic sandals per month.
- All together no more than 200 pairs of sandals can be manufactured per month.
- The profit on a pair of Elegance sandals is R60 and R100 on a pair of Classic sandals.



ELEGANCE SANDALS (x)



CLASSIC SANDALS (y)

Let the number of pairs of Elegance sandals be x and the number of pairs of Classic sandals y .

- 12.1 Write down the constraints of the above scenario. (6)
- 12.2 Sketch the constraint in QUESTION 12.1 on the grid on the DIAGRAM SHEET. Clearly indicate the feasible region. (6)
- 12.3 Using a search line and your graph, determine how many pairs of each type of sandal should be sold to generate maximum profit. (3)
- 12.4 Write down the objective function for maximising profit in the form $P = \dots$. (1)
- 12.5 Determine the maximum monthly profit. (2)

[18]

TOTAL: 150

FORMULA SHEET: MATHEMATICS
FORMULEBLAD: WISKUNDE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$\sum_{i=1}^n 1 = n$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^n (a + (i-1)d) = \frac{n}{2}(2a + (n-1)d) \quad \sum_{i=1}^n ar^{i-1} = \frac{a(r^n - 1)}{r - 1} \quad ; \quad r \neq 1$$

$$\sum_{i=1}^{\infty} ar^{i-1} = \frac{a}{1-r} \quad ; \quad -1 < r < 1$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

In $\triangle ABC$:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or/of } B) = P(A) + P(B) - P(A \text{ and/en } B)$$

NAME: _____

DIAGRAM SHEET

QUESTION 12

