basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

NOVEMBER 2011

MEMORANDUM

MARKS: 150

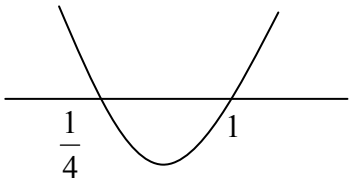
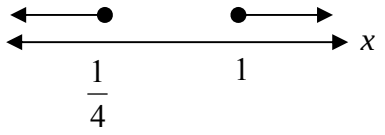
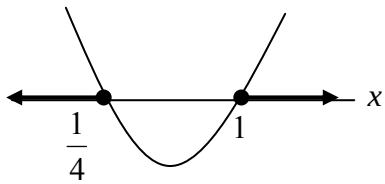
This memorandum consists of 28 pages.

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent Accuracy applies in all aspects of the marking memorandum.

QUESTION 1

1.1.1	$x(x+1)=6$ $x^2+x=6$ $x^2+x-6=0$ $(x+3)(x-2)=0$ $x=-3$ or 2 **OR** $x^2+x-6=0$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $=\frac{-1\pm\sqrt{1^2-4(1)(-6)}}{2(1)}$ $x=-3$ or 2	**Note:** Answers by inspection: award 3/3 marks **Note:** Answer only of $x=2$: award 1/3 marks **Note:** If candidate converts equation to linear: award 0/3 marks	✓ standard form ✓ factors ✓ answers (3) ✓ standard form ✓ substitution into correct formula ✓ answers (3)
1.1.2	$3x^2-4x=8$ $3x^2-4x-8=0$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $=\frac{-(-4)\pm\sqrt{(-4)^2-4(3)(-8)}}{2(3)}$ $=\frac{4\pm\sqrt{16+96}}{6}$ $=\frac{4\pm\sqrt{112}}{6}$ $=\frac{2\pm2\sqrt{7}}{3}$ $=2{,}43$ or $-1{,}10$	**Note:** Penalise 1 mark for inaccurate rounding off to ANY number of decimal places if candidate gives decimal answers. **Note:** If candidate uses incorrect formula: maximum 1/4 marks (for standard form) **Note:** If an error in subs and gets: $\frac{4\pm\sqrt{-80}}{6}$ and states "no solution": maximum 3/4 marks If doesn't conclude with "no solution": maximum 2/4 marks	✓ standard form ✓ substitution into correct formula ✓ $\sqrt{112}$ ✓ $\frac{4\pm\sqrt{112}}{6}$ or decimal answer (4)

	OR $3x^2 - 4x = 8$ $3x^2 - 4x - 8 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-8)}}{2(3)}$ $= 2{,}43 \text{ or } -1{,}10$ **Note:** Penalise 1 mark for inaccurate rounding off to ANY number of decimal places if candidate gives decimal answers	✓ standard form ✓ substitution into correct formula ✓ answer ✓ answer (4)
1.1.3	$4x^2 + 1 \geq 5x$ $4x^2 - 5x + 1 \geq 0$ $(4x - 1)(x - 1) \geq 0$ $+ \quad 0 \quad - \quad 0 \quad +$ $\frac{1}{4} \qquad 1$  $x \leq \frac{1}{4}$ or $x \geq 1$ **OR** $\left(-\infty; \frac{1}{4}\right] \cup [1; \infty)$ **OR**  **OR**  **Note:** If candidate gives either of these correct graphical solutions but writes down the incorrect intervals or uses AND: max 3/4 marks **NOTES:** If a candidate gives an answer of $1 \leq x \leq \frac{1}{4}$ then max 3/4 marks. If a candidate gives an answer of $\frac{1}{4} \leq x \leq 1$ then max 2/4 marks. If a candidate gives an answer of $x \leq \frac{1}{4}$ **and** $x \geq 1$ then max 3/4 marks. If the candidate leaves out the equality of the notation then penalty of 1 mark. If a candidate gives an answer of $x \leq \frac{1}{4}$; $x \geq 1$ then max 3/4 marks. If candidate gives $x \geq \frac{1}{4}$ and/or $x \geq 1$, BREAKDOWN: max 2/4 marks. If candidate gives : award 3/4 marks $\quad + \quad 0 \quad - \quad 0 \quad +$ over $\frac{1}{4} \qquad 1$	✓ factors ✓ both critical values of $\frac{1}{4}$ and 1 ✓ or **OR** $\cup$ ✓ answer (4)

1.2.1

$x^2 + 5xy + 6y^2 = 0$

$(x+3y)(x+2y) = 0$

$x + 3y = 0$ $\qquad$ $x + 2y = 0$

$x = -3y$ OR $x = -2y$

$\frac{x}{y} = -3$ $\qquad$ $\frac{x}{y} = -2$

✓ factors

✓✓ answers (3)

Note:
If a candidate gives $-\frac{x}{y} = 3$ or $-\frac{x}{y} = 2$ award 2/3 marks

OR

Let $k = \frac{x}{y}$

$x^2 + 5xy + 6y^2 = 0$

$\left(\frac{x}{y}\right)^2 + 5\left(\frac{x}{y}\right) + 6 = 0$

✓ factors

$k^2 + 5k + 6 = 0$

$(k+3)(k+2) = 0$

$k = -3$ or $k = -2$

$\frac{x}{y} = -3$ or $\frac{x}{y} = -2$

✓✓ answers (3)

OR

$x^2 + 5xy + 6y^2 = 0$

$x = \frac{-5y \pm \sqrt{(5y)^2 - 4(1)(6y^2)}}{2(1)}$

$x = \frac{-5y \pm \sqrt{y^2}}{2}$

✓ substitutes correctly into correct formula

$x = \frac{-5y \pm y}{2}$

$x = -3y$ $\qquad$ $x = -2y$

$\frac{x}{y} = -3$ or $\frac{x}{y} = -2$

✓✓ answers (3)

OR

$x^2 + 5xy + 6y^2 = 0$

$x^2 + 5xy + \left(\frac{5}{2}y\right)^2 = -6y^2 + \left(\frac{5}{2}y\right)^2$

$\left(x + \frac{5}{2}y\right)^2 = \frac{1}{4}y^2$

✓ completing the square

$x + \frac{5}{2}y = \pm\frac{1}{2}y$

$x = -\frac{5}{2}y \pm \frac{1}{2}y$

	$x=-3y \quad x=-2y$ $\frac{x}{y}=-3$ or $\frac{x}{y}=-2$ **OR** Let $k=\frac{x}{y}$ $x=ky$ $x^2+5xy+6y^2=0$ $(ky)^2+5y(ky)+6y^2=0$ $k^2y^2+5y^2k+6y^2=0$ $y^2(k^2+5k+6)=0$ $(k^2+5k+6)=0$ $(k+3)(k+2)=0$ $k=-3$ or $k=-2$ $\frac{x}{y}=-3$ or $\frac{x}{y}=-2$ Note: $(x;y) = (0;0)$ is also a solution, but in this case $\frac{x}{y}$ is undefined **OR** Let $y=1$, $x^2+5x+6=0$ $(x+2)(x+3)=0$ $x=-2$ or $x=-3$ $\frac{x}{y}=-2$ or $\frac{x}{y}=-3$	✓✓ answers (3) ✓ factors ✓✓ answers (3) ✓ factors ✓✓ answers (3)
1.2.2	$x+y=8 \quad x+y=8$ $-3y+y=8 \quad -2y+y=8$ $-2y=8$ OR $-y=8$ $y=-4 \quad y=-8$ $x=12 \quad x=16$ **OR** $\frac{8-y}{y}=-3$ OR $\frac{8-y}{y}=-2$ $8-y=-3y \quad 8-y=-2y$ $8=-2y \quad 8=-y$ $y=-4 \quad y=-8$ $x=12 \quad x=16$	✓ substitution $x=-3y$ ✓ subs $x=-2y$ ✓✓ y values ✓ both x values correct (5) ✓ $x=8-y$ ✓ substitution ✓✓ y values ✓ both correct x values (5)

	OR $x+y=8$ $y=8-x$ $\frac{x}{8-x}=-3$ OR $\frac{x}{8-x}=-2$ $x=-3(8-x)$ $x=-2(8-x)$ $x=-24+3x$ $x=-16+2x$ $-2x=-24$ $-x=-16$ $x=12$ $x=16$ $y=-4$ $y=-8$	✓ $y=8-x$ ✓ substitution ✓✓ x values correct ✓ both y values correct (5)
	OR $(x+2y)(x+3y)=0$ $x+y=8$ $x=8-y$ $(y+8)(2y+8)=0$ $y=-8$ or $y=-4$ $x=16$ $x=12$	✓ $x=8-y$ ✓ substitution ✓✓ y values correct ✓ both x values correct (5)
	OR $x=8-y$ $(8-y)^2+5(8-y)y+6y^2=0$ $64-16y+y^2+40y-5y^2+6y^2=0$ $2y^2+24y+64=0$ $y^2+12y+32=0$ $(y+8)(y+4)=0$ $y=-8$ or $y=-4$ $x=16$ $x=12$	✓ $x=8-y$ ✓ substitution ✓factors ✓ both y values correct ✓ both x values correct (5)
	OR	

OR

$x = 8 - y$

$(8-y)^2 + 5(8-y)y + 6y^2 = 0$

$64 - 16y + y^2 + 40y - 5y^2 + 6y^2 = 0$

$2y^2 + 24y + 64 = 0$

$y^2 + 12y + 32 = 0$

$y = \dfrac{-12 \pm \sqrt{12^2 - 4(1)(32)}}{2(1)}$

$= \dfrac{-12 \pm \sqrt{16}}{2}$

$y = -8$ or $y = -4$

$x = 16$ $\quad$ $x = 12$

Note:
If a candidate uses the formula and replaces *x* for *y* and then answers are swapped: maximum 4/5 marks

✓ $x = 8 - y$
✓ substitution
✓ substitutes into correct formula
✓ both *y* values correct
✓ both *x* values correct
(5)

OR

$y = 8 - x$

$x^2 + 5x(8-x) + 6(8-x)^2 = 0$

$x^2 + 40x - 5x^2 + 6\left(64 - 16x + x^2\right) = 0$

$2x^2 - 56x + 384 = 0$

$x^2 - 28x + 192 = 0$

$(x-16)(x-12) = 0$

$x = 12$ $\quad$ $x = 16$

$y = -4$ or $y = -8$

✓ $y = 8 - x$
✓ substitution
✓ factors
✓ both *x* values correct
✓ both *y* values correct
(5)

OR

$y = 8 - x$

$x^2 + 5x(8-x) + 6(8-x)^2 = 0$

$x^2 + 40x - 5x^2 + 6\left(64 - 16x + x^2\right) = 0$

$2x^2 - 56x + 384 = 0$

$x^2 - 28x + 192 = 0$

$x = \dfrac{-(-28) \pm \sqrt{(-28)^2 - 4(1)(192)}}{2(1)}$

$= \dfrac{28 \pm \sqrt{416}}{2}$

$x = 12$ $\quad$ $x = 16$

$y = -4$ or $y = -8$

✓ $y = 8 - x$
✓ substitution
✓ substitutes into correct formula
✓ both *x* values correct
✓ both correct *y* values
(5)
[19]

QUESTION 2

2.1.1	$x-4=32-x$ $2x=36$ $x=18$ **OR** $a=4$ $a+2d=32$ $2d=28$ $d=14$ $x=14+4$ $x=18$ **OR** $x=\frac{4+32}{2}=18$ **Note:** If answer only: award 2/2 marks **Note:** If candidate writes $x-4 \quad 32-x$ only (i.e. omits equality) : 0/2 marks	✓ $T_2-T_1=T_3-T_2$ ✓ answer (2) ✓ $a+2d=32$ and $a=4$ ✓ answer (2) ✓ substitutes correctly into arithmetic mean formula i.e. $\frac{4+32}{2}$ ✓ answers (2)
2.1.2	$\frac{x}{4}=\frac{32}{x}$ $x^2=128$ $x=\pm\sqrt{128}$ $x=\pm 8\sqrt{2}$ OR $x=\pm 11{,}31$ OR $x=\pm 2^{\frac{7}{2}}$ **OR** $a=4$ $r=\frac{x}{4}$ $ar^2=4\left(\frac{x}{4}\right)^2$ $32=4\left(\frac{x}{4}\right)^2$ $x^2=128$ $x=\pm\sqrt{128}$ $x=\pm 8\sqrt{2}$ or $x=\pm 11{,}31$ or $x=\pm 2^{\frac{7}{2}}$ **OR** $x=\pm\sqrt{4\times 32}$ $x=\pm\sqrt{128}$ or $x=\pm 8\sqrt{2}$ or $x=\pm 11{,}31$ or $x=\pm 2^{\frac{7}{2}}$ **Note:** If candidate writes $\frac{x}{4} \quad \frac{32}{x}$ only (i.e. omits equality) : 0/2 marks **Note:** If only $x=\sqrt{128}$ then penalty 1 mark	✓ $\frac{T_2}{T_1}=\frac{T_3}{T_2}$ ✓ $x^2=128$ ✓ both answers (surd or decimal or exponential form) (3) ✓ $32=4\left(\frac{x}{4}\right)^2$ ✓ $x^2=128$ ✓ both answers (surd or decimal or exponential form) (3) ✓✓ substitutes correctly into geometric mean formula i.e. $\pm\sqrt{4\times 32}$ ✓ both answers (surd or decimal or exponential form) (3)

2.2

$$P = \sum_{k=1}^{13} 3^{k-5}$$
$$= 3^{1-5} + 3^{2-5} + 3^{3-5} + ... + 3^{13-5}$$
$$= 3^{-4} + 3^{-3} + 3^{-2} + ... + 3^{8}$$
$$= \frac{3^{-4}\left(3^{13} - 1\right)}{3 - 1}$$
$$= 9841{,}49 \quad \text{or} \quad 9841\frac{40}{81} \quad \text{or} \quad \frac{797161}{81}$$

Note:
Correct answer only: 1/4 marks only

✓ $a = 3^{-4}$ or $\frac{1}{81}$
✓ $r = 3$
✓ subs into correct formula
✓ answer
(4)

OR

$$P = \sum_{k=1}^{13} 3^{k-5}$$
$$= 3^{1-5} + 3^{2-5} + 3^{3-5} + ... + 3^{13-5}$$
$$= 3^{-4} + 3^{-3} + 3^{-2} + ... + 3^{8}$$
$$= \frac{1}{81} + \frac{1}{27} + \frac{1}{9} + ... + 6561$$
$$= 9841{,}49 \quad \text{or} \quad 9841\frac{40}{81} \quad \text{or} \quad \frac{797161}{81}$$

Note: If the candidate rounds off and gets 9841,46 (i.e. correct to one decimal place): DO NOT penalise for the rounding off.

✓✓ expand the sum
✓ 13 terms in expansion
✓ answer
(4)

2.3

$$S_n = a + [a + d] + [a + 2d] + ... + [a + (n-2)d] + [a + (n-1)d]$$
$$S_n = [a + (n-1)d] + [a + (n-2)d] + ... + [a + d] + a$$
$$2S_n = [2a + (n-1)d] + [2a + (n-1)d] + ... + [2a + (n-1)d] + [2a + (n-1)d]$$
$$= n[2a + (n-1)d]$$
$$S_n = \frac{n}{2}[2a + (n-1)d]$$

✓writing out S_n
✓ "reversing" S_n
✓expressing $2S_n$
✓ grouping to get $2S_n = n[2a + (n-1)d]$
(4)

OR

$$S_n = a + [a + d] + [a + 2d] + ... + (T_n - d) + T_n$$
$$S_n = T_n + (T_n - d) + ... + [a + d] + a$$
$$2S_n = a + T_n + a + T_n + a + T_n + ... + a + T_n$$
$$= n[a + a + (n-1)d]$$
$$= [2a + (n-1)d]$$
$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Note:
If a candidate uses a circular argument (eg $S_{n+1} = S_n + T_n$):
max 1/4 marks
(for writing out S_n)

✓writing out S_n
✓ "reversing" S_n
✓expressing $2S_n$
✓ grouping to get $2S_n = n[a + a + (n-1)d]$
(4)

Note: If a candidate uses a specific linear sequence, then NO marks.

[13]

QUESTION 3

3.1	21; 24	**Note:** If candidate writes $T_8 = 21$ $T_7 = 24$: award 1/2 marks	✓ 21 ✓ 24 (2)
3.2	$T_{2k} = 3.2^{k-1}$ and so $T_{52} = 3.2^{26-1} = 100663296$ $T_{2k-1} = 3 + 6(k-1) = 6k - 3$ and so $T_{51} = 6(26) - 3 = 153$ $T_{52} - T_{51} = 100663296 - 153$ $= 100663143$ **OR** Consider sequence *P*: 3 ; 6 ; 12 … $P_n = 3.2^{n-1}$ $P_{26} = 3.2^{26-1} = 100663296$ Consider sequence *Q*: 3 ; 9 ; 15 … $Q_n = 6n - 3$ $Q_{26} = 6(26) - 3 = 153$ $T_{52} - T_{51} = P_{26} - Q_{26}$ $= 100663296 - 153$ $= 100663143$	**Note:** If candidate writes out all 52 terms **and** gets correct answer: award 5/5 marks **Note:** If candidate used $k = 52$: max 2/5 **Note:** if candidate interchanges order i.e. does $T_{51} - T_{52}$ **:** max 4/5 marks **Note:** writes out all 52 terms and subtracts $T_{51} - T_{52}$ **:** max 4/5 marks	✓ 3.2^{k-1} ✓ T_{52} ✓ $6k - 3$ ✓ T_{51} ✓ answer (5) ✓ $P_n = 3.2^{n-1}$ ✓ P_{26} ✓ $Q_n = 6n - 3$ ✓ Q_{26} ✓ answer (5)

3.3	For all $n \in \mathbf{N}$, $n = 2k$ or $n = 2k-1$ for some $k \in \mathbf{N}$ If $n = 2k$: $T_n = T_{2k} = 3.2^{k-1}$ If $n = 2k-1$: $T_n = T_{2k-1}$ $= 6k - 3$ $= 3(2k-1)$ In either case, T_n has a factor of 3, so is divisible by 3. **OR** $P_n = 3.2^{n-1}$ Which is a multiple of 3 $Q_n = 6n - 3$ $= 3(2n-1)$ Which is also a multiple of 3 Since $T_n = Q_{2k-1}$ or $T_n = P_{2k}$ for all $n \in \mathbf{N}$, T_n is always divisible by 3 **OR** The odd terms are odd multiples of 3 and the even terms are 3 times a power of 2. This means that all the terms are multiples of 3 and are therefore divisible by 3. **Note:** If a candidate only illustrates divisibility by 3 with a specific finite part of the sequence, not the general term: 0/2 marks	✓ factors 3.2^{k-1} ✓ factors $3(2k-1)$ (2) ✓ factors 3.2^{n-1} ✓ factors $3(2n-1)$ (2) ✓ odd multiples of 3 ✓ 3 times a power of 2 (2) **[9]**

QUESTION 4

4.1 The second, third, fourth and fifth terms are $1 ; -6 ; T_4$ and -14

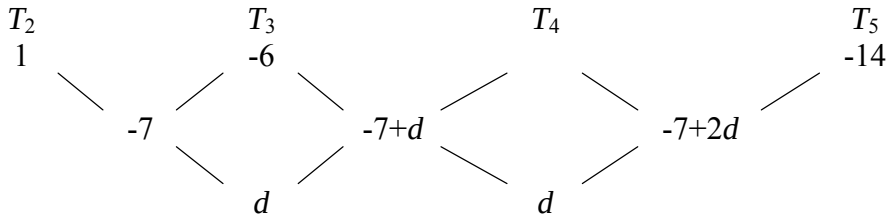
First differences are: $-7 ; T_4 + 6 ; -14 - T_4$
So $T_4 + 6 + 7 = -14 - 2T_4 - 6$
$T_4 = -11$
$d = -11 + 6 + 7 = 2$ or $-14 + 22 - 6 = 2$

Note: Answer only (i.e. $d = 2$) with no working: 3 marks

Note: Candidate gives $T_4 = -11$ and $d = 2$ only: award 5/5 marks

✓ -7
✓ $T_4 + 6$
✓ $-14 - T_4$
✓ setting up equation $T_5 - T_2 = (T_5 - T_4) + (T_4 - T_3) + (T_3 - T_2)$
✓ answer
(5)

OR



✓ -7
✓ $-7 + d$
✓ $-7 + 2d$

$T_5 - T_2 = (T_5 - T_4) + (T_4 - T_3) + (T_3 - T_2)$
$-15 = (-7 + 2d) + (-7 + d) + -7$
$-15 = -21 + 3d$
$6 = 3d$
$d = 2$

Note: Candidate uses trial and error **and** shows this: award 5/5 marks

✓ setting up equation $T_5 - T_2 = (T_5 - T_4) + (T_4 - T_3) + (T_3 - T_2)$
✓ answer
(5)

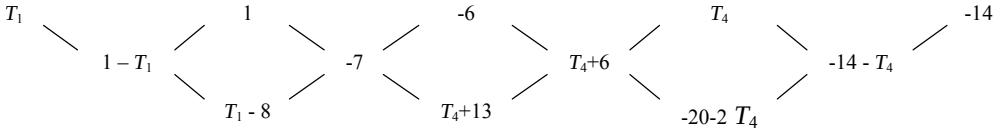
OR

$4a + 2b + c = 1$
$9a + 3b + c = -6$
$5a + b = -7$

$25a + 5b + c = -14$
$16a + 2b = -8$
$10a + 2b = -14$
$6a = 6$
$a = 1$
$d = 2a = 2$

✓ $4a + 2b + c = 1$
✓ $9a + 3b + c = -6$
✓ $25a + 5b + c = -14$
✓ solved simultaneously
✓ answer
(5)

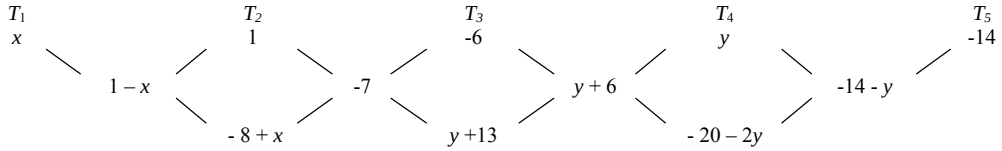
OR



$T_4 + 13 = -20 - 2T_4$
$3T_4 = -33$
$T_4 = -11$
$d = -11 + 13$
$d = 2$

✓ -7
✓ $T_4 + 6$
✓ $-14 - T_4$
✓ setting up equation
✓ answer
(5)

OR



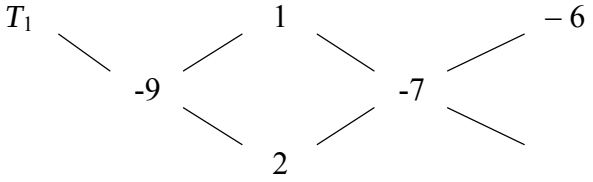
✓ -7
✓ $y+6$
✓ $-14-y$

$y+13=-20-2y$
$3y=-33$
$y=-11$

✓ setting up equation
✓ answer
(5)

Second difference = $y+13=-11+13=2$

4.2



✓ method
✓ $T_1=10$
(2)

Note: Answer only: award 2/2 marks

Note:
If incorrect d in 4.1, 2/2 CA marks for $T_1 = d + 8$ (since $1 - T_1 = -7 - d$)

$T_1=10$

OR

$a=1$
$5a+b=-7$
$5(1)+b=-7$
$b=-12$
$a+b+c=1$
$4(1)+2(-12)+c=1$
$c=21$
$T_n=n^2-12n+21$
$T_1=(1)^2-12(1)+21$
$=10$

✓ method
✓ $T_1=10$
(2)

OR

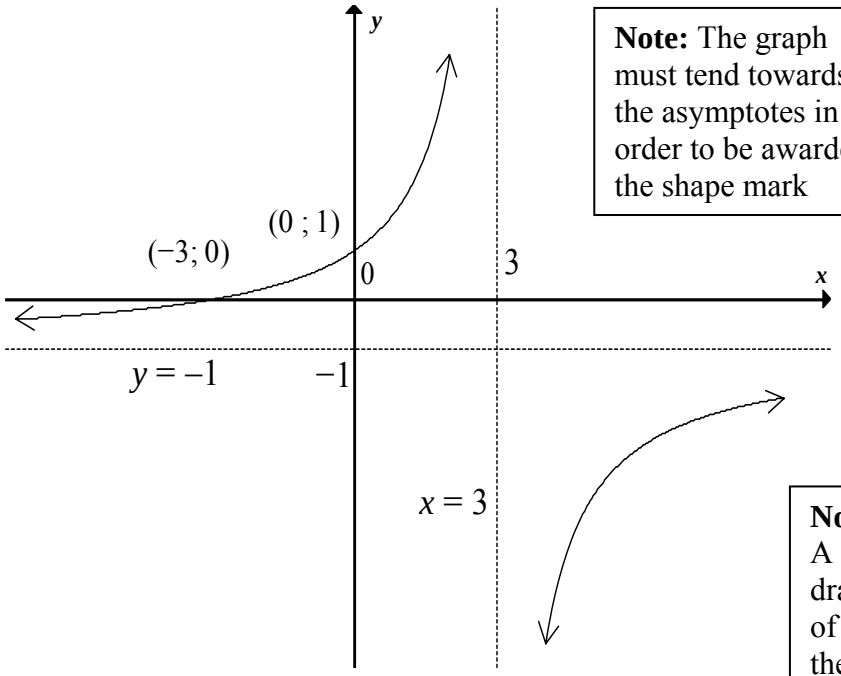
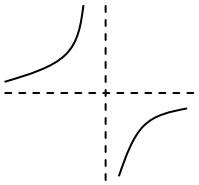
$T_4+13=-8+T_1$
$-11+13=-8+T_1$
$T_1=10$

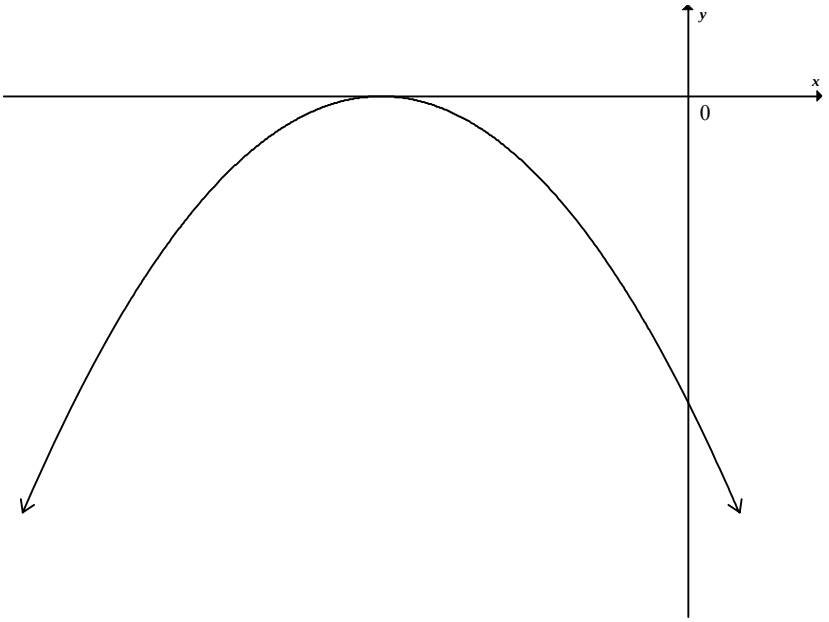
OR

$y+13=-8+x$
$-11+13=-8+x$
$x=10$

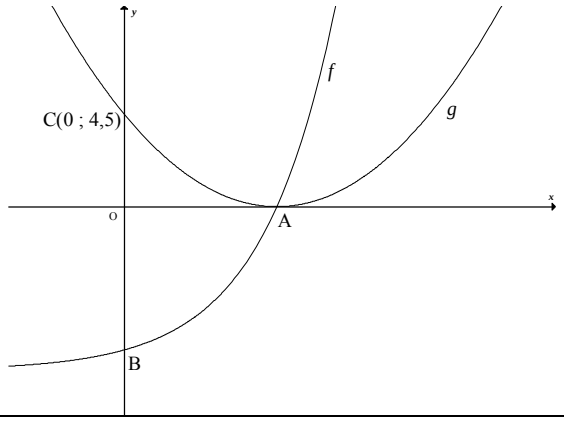
✓ method
✓ $T_1=10$
(2)
[7]

QUESTION 5

5.1.1	$y = f(0)$ $= \frac{-6}{0-3} - 1$ $= 1$ $(0\ ;\ 1)$ **OR** $x = 0$ and $y = 1$ **Note:** Mark 5.1.1 and 5.1.2 as a single question. If the intercepts are interchanged: max 3/5 marks	✓ $y = 1$ ✓ $x = 0$ (2)
5.1.2	$0 = \frac{-6}{x-3} - 1$ $1 = \frac{-6}{x-3}$ $x - 3 = -6$ $x = -3$ $(-3\ ;\ 0)$	✓ $y = 0$ ✓ $x - 3 = -6$ ✓ answer (3)
5.1.3	 **Note:** The graph must tend towards the asymptotes in order to be awarded the shape mark **Note:** A candidate who draws only one 'arm' of the hyperbola loses the 'shape' mark i.e. max 3/4 marks	✓ shape  ✓ both intercepts correct ✓ horizontal asymptote ✓ vertical asymptote (4)
5.1.4	$-3 < x < 3$ **OR** $(-3; 3)$ **OR** $-3 < x$ and $x < 3$ **Note**: if candidate writes $-3 < x$ only: 1/2 marks **Note**: if candidate writes $x < 3$ only: 1/2 marks	✓ −3 and 3 ✓ inequality OR interval notation (2)

5.1.5	$y = \frac{-6}{-2-3} - 1$ $= \frac{1}{5}$ $m = \frac{1-\frac{1}{5}}{0-(-2)}$ $= \frac{2}{5}$ **OR** $m = \frac{f(0)-f(-2)}{0-(-2)}$ $= \frac{1-\frac{1}{5}}{0+2}$ $= \frac{2}{5}$	✓ $\frac{1}{5}$ ✓ formula ✓ substitution ✓ answer (4) ✓ formula ✓ $f(-2) = \frac{1}{5}$ ✓ substitution ✓ answer (4)
5.2	$x = -\frac{b}{2a} < 0$ since $b < 0$ and $a < 0$  	✓*y*-intercept negative ✓ turning point on the *x* axis ✓ turning point on the left of the *y* axis ✓ **maximum** TP and quadratic shape (4) **[19]**

QUESTION 6



6.1	$0 = 2^x - 8$ $8 = 2^x$ $2^3 = 2^x$ $x = 3$ A(3 ; 0) $f(0) = 2^0 - 8$ $= 1 - 8$ $= -7$ B(0 ; –7)	✓ $y = 0$ ✓ answer for A ✓ $x = 0$ ✓ answer for B (4)
6.2	$y = -8$ **OR** $y + 8 = 0$ **Note**: no CA marks	✓ answer (1)
6.3	$h(x) = f(2x) + 8$ $= \left(2^{2x} - 8\right) + 8$ $= 4^x$ or 2^{2x} **Note**: answer only: award 2/2 marks	✓ $\left(2^{2x} - 8\right)$ ✓ answer of $h(x) = 4^x$ or 2^{2x} (2)
6.4	$x = 4^y$ **OR** $x = 2^{2y}$ $y = \log_4 x$ $2y = \log_2 x$ $y = \frac{1}{2}\log_2 x$ **OR** $y = \log_2 \sqrt{x}$ **OR** $y = \frac{\log x}{\log 4}$ **Note**: answer only award 2/2 marks **Note**: candidate works out f^{-1} and gets $y = \log_2(x + 8)$ award 1/2 marks	✓ switch x and y ✓ answer in the form $y =$… (2)
6.5	$p(x) = -\log_4 x$ **OR** $p(x) = \log_{\frac{1}{4}} x$ **OR** $p(x) = \log_4 \frac{1}{x}$ **OR** $p(x) = -\frac{1}{2}\log_2 x$ **OR** $y = -\log_2 \sqrt{x}$	✓answer (1)

6.6	$\sum_{k=0}^{3} g(k) - \sum_{k=4}^{5} g(k)$ $= g(0) + g(1) + g(2) + g(3) - g(4) - g(5)$ $x = 3$ is the axis of symmetry of g $\therefore$ by symmetry $g(2) = g(4)$ and $g(1) = g(5)$ Answer = $g(0) + g(3)$ $= 4{,}5 + 0$ $= 4{,}5$	✓ $= g(0) + g(1) + g(2) + g(3) - g(4) - g(5)$ ✓ $g(2) = g(4)$ and $g(1) = g(5)$ ✓ $g(0) + g(3)$ ✓ answer (4)
	OR	
	$\sum_{k=0}^{3} g(k) - \sum_{k=4}^{5} g(k)$ $\sum_{k=0}^{3} g(k) = g(0) + g(1) + g(2) + g(3)$ $\sum_{k=4}^{5} g(k) = g(4) + g(5)$ $x = 3$ is the axis of symmetry of g $\therefore$ by symmetry $g(4) = g(2)$ $g(5) = g(1)$ $\sum_{k=0}^{3} g(k) - \sum_{k=4}^{5} g(k)$ $= g(0) + g(3)$ $= 4{,}5 + 0$ $= 4{,}5$	✓ expansion ✓ $g(2) = g(4)$ and $g(1) = g(5)$ ✓ $g(0) + g(3)$ ✓ answer (4)
	OR	
	$g(x) = a(x-3)^2 + 0$ $4{,}5 = a(0-3)^2 + 0$ $4{,}5 = 9a$ $a = \frac{1}{2}$ $g(x) = \frac{1}{2}(x-3)^2$ $\sum_{k=0}^{3} g(k) - \sum_{k=4}^{5} g(k)$ $\sum_{k=0}^{3} g(k) = g(0) + g(1) + g(2) + g(3)$ $= 4{,}5 + 2 + 0{,}5 + 0$ $= 7$	✓ $g(x) = \frac{1}{2}(x-3)^2$ ✓ expansion

$\sum_{k=4}^{5} g(k) = g(4) + g(5)$

$= 0,5 + 2$

$= 2,5$

$\sum_{k=0}^{3} g(k) - \sum_{k=4}^{5} g(k)$

$= 7 - 2,5$ ✓ $7 - 2,5$

$= 4,5$ ✓ answer

(4)

OR

$g(x) = ax^2 + bx + c$

$g(k) = ak^2 + bk + c$

$g(0) = c$

$g(1) = a + b + c$

$g(2) = 4a + 2b + c$

$g(3) = 9a + 3b + c$

$\sum_{k=0}^{3} g(k) = 14a + 6b + 4c$

$g(4) = 16a + 4b + c$

$g(5) = 25a + 9b + c$

$\sum_{k=4}^{5} g(k) = 41a + 9b + 2c$

$\sum_{k=0}^{3} g(k) - \sum_{k=4}^{5} g(k) = -27a - 3b + 2c$ ✓✓ $-27a - 3b + 2c$

$g(x) = a(x-3)^2 + 0$

$4,5 = a(0-3)^2 + 0$

$4,5 = 9a$

$a = \frac{1}{2}$

$g(x) = \frac{1}{2}(x-3)^2$

$= \frac{1}{2}x^2 - 3x + \frac{9}{2}$ ✓ $g(x) = \frac{1}{2}(x-3)^2$

$\sum_{k=0}^{3} g(k) - \sum_{k=4}^{5} g(k) = -27a - 3b + 2c$

$= -27\left(\frac{1}{2}\right) - 3(-3) + 2\left(\frac{9}{2}\right)$

$= 4,5$ ✓ answer

(4)

[14]

QUESTION 7

7.1	$A = P(1-i)^n$ $\frac{P}{2} = P(1-0,07)^n$ $\frac{1}{2} = 0,93^n$ $\log\frac{1}{2} = n\log 0,93$ $n = \frac{\log\frac{1}{2}}{\log 0,93}$ $= 9,55$ years **OR** $A = P(1-i)^n$ $\frac{P}{2} = P(1-0,07)^n$ $\frac{1}{2} = 0,93^n$ $\log_{0,93}\frac{1}{2} = n$ $n = 9,55$ years **Note:** If candidate interchanges A and P i.e. uses $P = \frac{A}{2}$: max 2/4 marks **Note:** If candidate uses incorrect formula: max 1/4 marks for $A = \frac{P}{2}$	✓ $A = \frac{P}{2}$ ✓ subs into correct formula ✓ log ✓ answer (4)

7.2	**Radesh:** $A = P(1+in)$ $= 6\,000(1+0{,}085\times 5)$ OR $= 8\,550$ $A = 6\,000 + 8{,}5\%\text{ of }6000\times 5$ $= 6000 + 510\times 5$ $= 6000 + 2550$ $= 8\,550$ $\text{Bonus} = 0{,}05\times 6\,000$ $= 300$ $\text{Received} = 8\,550 + 300$ $= \text{R}8\,850$ **Thandi:** $A = P(1+i)^n$ $= 6\,000\left(1+\frac{0{,}08}{4}\right)^{20}$ $= \text{R}8\,915{,}68$ Thandi's investment is bigger.	✓ 8 550 ✓ **R**8 850 ✓ $n = 20$ ✓ $i = \frac{0{,}08}{4}$ ✓ answer ✓ choice made (6)
7.3	F_v = initial deposit with interest + annuity $= 1\,000\left(1+\frac{0{,}15}{12}\right)^{18} + 700\left(\frac{\left(1+\frac{0{,}15}{12}\right)^{18}-1}{\frac{0{,}15}{12}}\right)$ $= 1\,250{,}58 + 14\,032{,}33$ $= \text{R}15\,282{,}91$ **OR** F_v = initial deposit with interest + annuity $= 1\,000\left(1+\frac{0{,}15}{12}\right)^{18} + 700\left(\frac{1-\left(1+\frac{0{,}15}{12}\right)^{-18}}{\frac{0{,}15}{12}}\right)\left(1+\frac{0.15}{12}\right)^{18}$ $= 1\,250{,}58 + 11220{,}68\left(1+\frac{0.15}{12}\right)^{18}$ $= 1\,250{,}58 + 14\,032{,}33$ $= \text{R}15\,282{,}91$	✓ $i = \frac{0{,}15}{12}$ or $\frac{1}{80}$ or 0,0125 ✓ $n = 18$ ✓ $n = 18$ ✓ $1\,000\left(1+\frac{0{,}15}{12}\right)^{18}$ ✓ $700\left(\frac{\left(1+\frac{0{,}15}{12}\right)^{18}-1}{\frac{0{,}15}{12}}\right)$ ✓ answer (6) ✓ $i = \frac{0{,}15}{12}$ or $\frac{1}{80}$ or 0,0125 ✓ $n = 18$ ✓ $n = 18$ ✓ $1\,000\left(1+\frac{0{,}15}{12}\right)^{18}$ ✓ $700\left(\frac{1-\left(1+\frac{0{,}15}{12}\right)^{-18}}{\frac{0{,}15}{12}}\right)\left(1+\frac{0.15}{12}\right)^{18}$ ✓ answer (6)

OR

$$F_v = 300\left(1+\frac{0,15}{12}\right)^{18} + 700\left(\frac{\left(1+\frac{0,15}{12}\right)^{19}-1}{\frac{0,15}{12}}\right)$$

$$= 375,17 + 14\ 907,74$$

$$= \text{R}15\ 282,91$$

✓ $i = \frac{0,15}{12}$ or $\frac{1}{80}$ or 0,0125
✓ $n = 19$ (corresponding to 700)
✓ $n = 18$ (corresponding to 300)
✓ $300\left(1+\frac{0,15}{12}\right)^{18}$
✓ $700\left(\frac{\left(1+\frac{0,15}{12}\right)^{19}-1}{\frac{0,15}{12}}\right)$
✓ answer

(6)
[16]

QUESTION 8

8.1

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{-4(x+h)^2-\left(-4x^2\right)}{h}$$

$$= \lim_{h\to 0}\frac{-4\left(x^2+2xh+h^2\right)+4x^2}{h}$$

$$= \lim_{h\to 0}\frac{-4x^2-8xh-4h^2+4x^2}{h}$$

$$= \lim_{h\to 0}\frac{-8xh-4h^2}{h}$$

$$= \lim_{h\to 0}\frac{h(-8x-4h)}{h}$$

$$= \lim_{h\to 0}(-8x-4h)$$

$$= -8x$$

Note:
Incorrect notation:

no lim written:
penalty 2 marks

lim written before equals sign:
penalty 1 mark

Note:
A candidate who gives –8*x* only:
0/5 marks

Note:
A candidate who omits brackets in the line $\lim_{h\to 0}(-8x-4h)$:
NO penalty

✓ formula
✓ substitution
✓ expansion
✓ $-8x-4h$
✓ answer

(5)

OR

	$f(x) = -4x^2$ $f(x+h) = -4(x+h)^2$ $= -4x^2 - 8xh - 4h^2$ $f(x+h) - f(x) = -8xh - 4h^2$ $f'(x) = \lim_{h\to 0} \frac{-8xh - 4h^2}{h}$ $= \lim_{h\to 0} \frac{h(-8x-4h)}{h}$ $= \lim_{h\to 0} (-8x-4h)$ $= -8x$	✓ substitution ✓ expansion ✓ formula ✓ $-8x-4h$ ✓ answer (5)
8.2.1	$y = \frac{3}{2x} - \frac{x^2}{2}$ $= \frac{3}{2}x^{-1} - \frac{1}{2}x^2$ $\frac{dy}{dx} = -\frac{3}{2}x^{-2} - x$ $= -\frac{3}{2x^2} - x$ **Note:** Incorrect notation in 8.2.1 and/or 8.2.2: Penalise 1 mark	✓ $\frac{3}{2}x^{-1}$ ✓ $-\frac{3}{2}x^{-2}$ ✓ $-x$ (3)
8.2.2	$f(x) = (7x+1)^2$ $= 49x^2 + 14x + 1$ $f'(x) = 98x + 14$ $f'(1) = 98(1) + 14$ $= 112$ **OR** $f(x) = (7x+1)^2$ $f'(x) = 2(7x+1).(7)$ By the chain rule $f'(x) = 98x + 14$ $f'(1) = 98(1) + 14$ $= 112$	✓multiplication ✓ $98x$ ✓14 ✓ answer (4) ✓✓ chain rule ✓✓ answer (4) **[12]**

QUESTION 9

9.1

$f(x) = -2x^3 + ax^2 + bx + c$

$f'(x) = -6x^2 + 2ax + b$

$= -6(x-5)(x-2)$

$= -6(x^2 - 7x + 10)$

$= -6x^2 + 42x - 60$

✓ $f'(x) = -6x^2 + 2ax + b$

✓✓ $-6(x-5)(x-2)$

Note:
A candidate who substitutes the values of a, b and c and then checks (by substitution) that $T(2;-9)$ and $S(5;18)$ lie on the curve:
award max 2/7 marks

$2a = 42$

$a = 21$

$b = -60$

✓ $b = -60$

✓ $2a = 42$

$f(5) = -2(5)^3 + 21(5)^2 - 60(5) + c$

$18 = -25 + c$

$c = 43$

OR

$f(2) = -2(2)^3 + 21(2)^2 - 60(2) + c$

$-9 = -52 + c$

$c = 43$

✓ subs (5 ; 18) or (2 ; -9)

✓ $c = 43$

(7)

$a = 21$; $b = -60$; $c = 43$

Note:
A candidate who substitutes the values of a, b and c into the function i.e. gets $f(x) = -2x^3 - 21x^2 - 60x + 43$ and then shows by substitution that $T(2;-9)$ and $S(5;18)$ are on the curve **and** works out the derivative i.e. gets $f'(x) = -6x^2 - 42x - 60$ **and** shows (by substitution into the derivative) that the turning points are at $x = 2$ and $x = 5$ (assuming what s/he sets out to prove and proving what is given): **award max 4/7 marks** as follows:
✓ $x = 2$ from $f'(x) = 0$ OR subs $x = 2$ into the derivative and gets 0
✓ $x = 5$ from $f'(x) = 0$ OR subs $x = 5$ into the derivative and gets 0
✓ substitution of $x = 2$ in f and gets -9
✓ substitution of $x = 5$ in f and gets 18

OR

$f'(x) = -6x^2 + 2ax + b$

$f'(2) = -6(2)^2 + 2a(2) + b$

$0 = -24 + 4a + b$

$b = 24 - 4a$

$f'(5) = -6(5)^2 + 2a(5) + b$

$0 = -150 + 10a + b$

$0 = -150 + 10a + (24 - 4a)$

$0 = -126 + 6a$

$6a = 126$

$a = 21$

$b = -60$

✓ $f'(x) = -6x^2 + 2ax + b$

✓ $f'(2) = 0$

✓ $f'(5) = 0$

Note:
If derivative equal to zero is not written: penalize once only

✓ $6a = 126$

✓ $b = -60$

$f(5) = -2(5)^3 + 21(5)^2 - 60(5) + c$

$18 = -25 + c$

$c = 43$

OR

$f(2) = -2(2)^3 + 21(2)^2 - 60(2) + c$

$-9 = -52 + c$

$c = 43$

✓ subs (5 ; 18) or (2 ; -9)

✓ $c = 43$

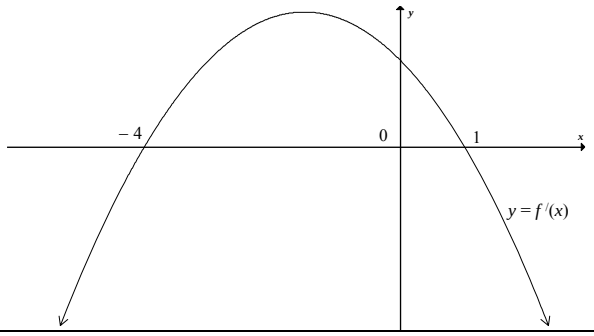
(7)

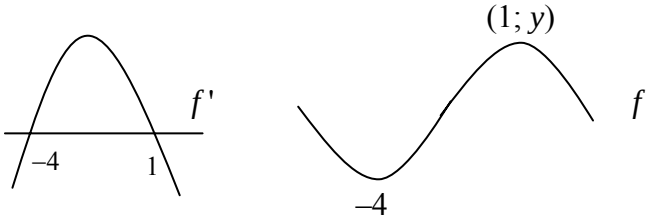
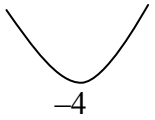
$a = 21$; $b = -60$; $c = 43$

	OR $f(2)=-9$ i.e. $-16+4a+2b+c=-9$ $4a+2b+c=7$ $f(5)=18$ i.e. $-250+25a+5b+c=18$ $25a+5b+c=268$ $21a+3b=261$ $f'(x)=-6x^2+2ax+b$ and $f'(2)=0$ OR $f'(5)=0$ $4a+b=24$ $10a+b=150$ $12a+3b=72$ $30a+3b=450$ $9a=189$ $9a=189$ $a=\frac{189}{9}$ OR $a=\frac{189}{9}$ $a=21$ $a=21$ $12(21)+3b=72$ $3b=-180$ $b=-60$ $4a+2b+c=7$ $25a+5b+c=268$ $4(21)+2(-60)+c=7$ OR $25(21)+5(-60)+c=268$ $c=43$ $c=43$	✓ $-16+4a+2b+c=-9$ and $-250+25a+5b+c=18$ ✓ $f'(x)=-6x^2+2ax+b$ ✓ $f'(2)=0$ or $f'(5)=0$ ✓ $9a=189$ ✓ $b=-60$ ✓ subs (5 ; 18) or (2 ; -9) ✓ $c=43$ (7)
9.2	$f'(x)=-6x^2+42x-60$ $m_{\tan}=-6(1)^2+42(1)-60$ $=-24$ $f(1)=-2(1)^3+21(1)^2-60(1)+43$ $=2$ Point of contact is (1 ; 2) $y-2=-24(x-1)$ $y=-24x+26$ **OR** $y=-24x+c$ $2=-24(1)+c$ $c=26$ $y=-24x+26$	✓ $f'(x)=-6x^2+42x-60$ ✓ subs $f'(1)$ ✓ $m_{\tan}=-24$ ✓ $f(1)=2$ ✓ $y-2=-24(x-1)$ OR $y=-24x+26$ (5)
9.3	$f'(x)=-6x^2+42x-60$ $f''(x)=-12x+42$ $0=-12x+42$ $x=\frac{7}{2}$ **OR**	✓ $f''(x)=-12x+42$ ✓ $x=\frac{7}{2}$ (2) ✓ $x=\frac{2+5}{2}$

	$x = \frac{2+5}{2}$ $x = \frac{7}{2}$ **OR** $x = \frac{-21}{3(-2)}$ $= \frac{7}{2}$	✓ $x = \frac{7}{2}$ (2) ✓ $x = \frac{-21}{3(-2)}$ ✓ $x = \frac{7}{2}$ (2) **[14]**

QUESTION 10



10.1	x-value of turning point: $x = \frac{-4+1}{2}$ $= -\frac{3}{2}$ $\therefore x > -\frac{3}{2}$ OR $\therefore x \in \left(-\frac{3}{2}; \infty\right)$	✓ $x > -\frac{3}{2}$ OR $\left(-\frac{3}{2}; \infty\right)$ (1)
10.2	f has a local minimum at $x = -4$ because:  **OR** $f'(x) < 0$ for $x < -4$, so f is decreasing for $x < -4$. $f'(x) > 0$ for $-4 < x < 1$, so f is increasing for $-4 < x < 1$. i.e.  $\therefore f$ has a local minimum at $x = -4$ **OR**	✓ $x = -4$ ✓✓ graph (3) ✓ $x = -4$ ✓ $f'(x) < 0$ for $x < -4$ ✓ $f'(x) > 0$ for $-4 < x < 1$ (3)

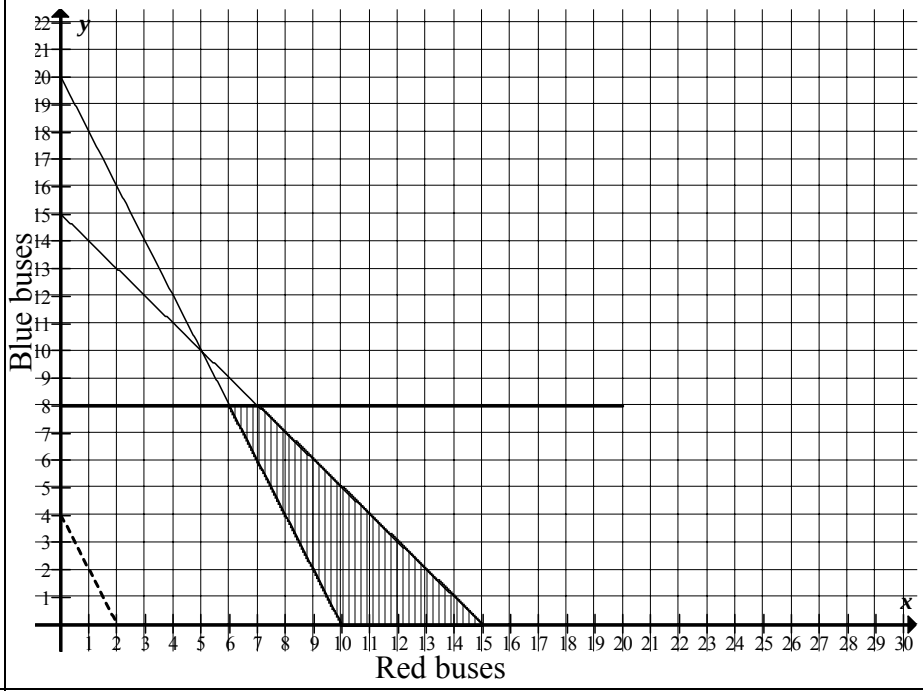
	OR Gradient of f changes from negative to positive at $x = -4$ **OR** $f'(-4) = 0$ $f''(-4) > 0$ so graph is concave up at $x = -4$, so f has a local minimum at $x = -4$.	✓ $x = -4$ ✓gradient negative for $x < -4$ ✓ gradient positive for $-4 < x < 1$ (3) ✓ $f'(-4) = 0$ ✓ $f''(-4) > 0$ ✓ $x = -4$ (3) **[4]**

QUESTION 11

11.1	$V(0) = 100 - 4(0)$ = 100 litres	✓ answer (1)
11.2	Rate in – rate out $= 5 - k$ l / min $V'(t) = -4$ l / min	✓ $5 - k$ ✓ -4 ✓ units stated once (3)
11.3	$5 - k = -4$ $k = 9$ l / min **OR** Volume at any time t = initial volume + incoming total – outgoing total $100 + 5t - kt = 100 - 4t$ $5t - kt = -4t$ $9t - kt = 0$ $t(9 - k) = 0$ At 1 minute from start, $t = 1$, $9 - k = 0$, so $k = 9$ **OR** Since $\frac{dV}{dt} = -4$, the volume of water in the tank is decreasing by 4 litres every minute. So k is greater than 5 by 4, that is, $k = 9$.	**Note:** Answer only: award 2/2 marks ✓ $5 - k = -4$ ✓ $k = 9$ (2) ✓ $100 + 5t - kt = 100 - 4t$ ✓ $k = 9$ (2) ✓✓ $k = 9$ (2) **[6]**

QUESTION 12

Note: If the wrong inequality $50x + 25y \leq 500$ is used, candidate wrongly says that there are more learners than available seats. Maximum of 10 marks.

12.1	$x, y \in \mathbf{N}$ $x + y \leq 15$ $50x + 25y \geq 500$ **OR** $y \leq -x + 15$ $y \geq -2x + 20$ $y \leq 8$ $y \leq 8$ **Note:** If candidate gives $50x + 25y = 500$: max 5/6 marks **Note:** for the inequality's marks to be awarded, the LHS and the RHS must be correct	✓✓ $x + y \leq 15$ ✓✓ $y \leq 8$ ✓✓ $50x + 25y \geq 500$ (6)
12.2		✓ $x + y \leq 15$ ✓ $50x + 25y \geq 500$ ✓ $y \leq 8$ ✓ feasible region (4)
12.3	$C = 600x + 300y$	✓ answer (1)
12.4.1	(6 ; 8) ; (7 ; 6) ; (8 ; 4) ; (9 ; 2) and (10 ; 0) **NOTE:** The gradient of the search line is $m = -\frac{2}{1}$	3 marks for all correct solutions 2 marks if only 3 or 4 correct solutions 1 mark if only 1 or 2 correct solutions (3)
12.4.2	$C = 6(600) + 8(300) = \text{R}6\,000$ or $C = 7(600) + 6(300) = \text{R}6\,000$ or $C = 8(600) + 4(300) = \text{R}6\,000$ or $C = 9(600) + 2(300) = \text{R}6\,000$ or $C = 10(600) + 0(300) = \text{R}6\,000$	✓ subs ✓ answer (2)
12.5	8 red ; 4 blue	✓ answer (1) **[17]**

TOTAL: 150

QUESTION 12.2