



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

FEBRUARY/MARCH 2011

MEMORANDUM

MARKS: 150

This memorandum consists of 15 pages.

QUESTION 1

1.1	$\frac{55 + 55 + 50 + 47 + 42 + 3x}{8} = 48,375$ $\frac{249 + 3x}{8} = 48,375$ $3x = 138$ $x = 46$	$\checkmark \frac{249 + 3x}{8} = 48,375$ $\checkmark 3x = 138$ (2)
1.2		$\checkmark$ max and min $\checkmark$ median $\checkmark$ Q_1 and Q_3 $\checkmark$ whiskers (4) [6]

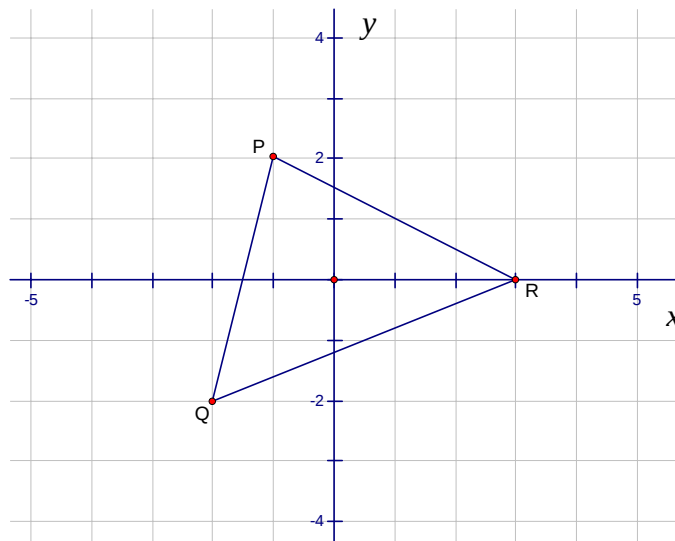
QUESTION 2

2.1	<table border="1"> <thead> <tr> <th>Mass (kg)</th><th>Frequency</th><th>Cumulative Frequency</th></tr> </thead> <tbody> <tr> <td>$60 \leq x < 70$</td><td>5</td><td>5</td></tr> <tr> <td>$70 \leq x < 80$</td><td>7</td><td>12</td></tr> <tr> <td>$80 \leq x < 90$</td><td>7</td><td>19</td></tr> <tr> <td>$90 \leq x < 100$</td><td>4</td><td>23</td></tr> <tr> <td>$100 \leq x < 110$</td><td>2</td><td>25</td></tr> </tbody> </table>	Mass (kg)	Frequency	Cumulative Frequency	$60 \leq x < 70$	5	5	$70 \leq x < 80$	7	12	$80 \leq x < 90$	7	19	$90 \leq x < 100$	4	23	$100 \leq x < 110$	2	25	$\checkmark\checkmark$ Frequencies $\checkmark\checkmark$ Cumulative Frequencies (4)
Mass (kg)	Frequency	Cumulative Frequency																		
$60 \leq x < 70$	5	5																		
$70 \leq x < 80$	7	12																		
$80 \leq x < 90$	7	19																		
$90 \leq x < 100$	4	23																		
$100 \leq x < 110$	2	25																		
2.2		$\checkmark\checkmark$ plotting points 1 mark: 3 – 5 points correctly 0 marks : 2 or less points correctly plotted $\checkmark$ graph (3)																		
2.3	Mean = 79,28	$\checkmark\checkmark$ answer (2)																		

2.4	Standard Deviation = 11,02 $79,28 - 11,02 = 68,26$ $79,28 + 11,02 = 90,3$ 17 players lie in this interval. $\frac{17}{25} = 68\%$	✓✓✓ sd = 11,02 ✓ 17 players ✓ 68% (5) [14]
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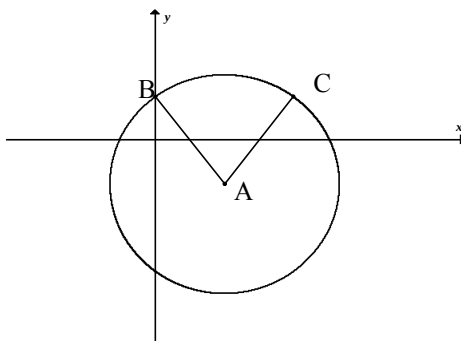
QUESTION 3

3.1 & 3.2	Scatter Plot showing Arm Span vs Height <table border="1"><caption>Data points from the scatter plot</caption><thead><tr><th>Arm Span (cm)</th><th>Height (cms)</th></tr></thead><tbody><tr><td>157</td><td>162</td></tr><tr><td>158</td><td>160</td></tr><tr><td>161</td><td>155</td></tr><tr><td>161</td><td>160</td></tr><tr><td>162</td><td>170</td></tr><tr><td>165</td><td>166</td></tr><tr><td>170</td><td>170</td></tr><tr><td>177</td><td>176</td></tr><tr><td>184</td><td>180</td></tr><tr><td>188</td><td>187</td></tr><tr><td>188</td><td>192</td></tr><tr><td>194</td><td>193</td></tr></tbody></table>	Arm Span (cm)	Height (cms)	157	162	158	160	161	155	161	160	162	170	165	166	170	170	177	176	184	180	188	187	188	192	194	193	Question 3.1 4 marks: All points plotted correctly. 3 marks: 9 – 11 points correct 2 marks: 6 – 8 points correct 1 marks: 3 – 5 points correct 0 marks if less than 3 points plotted correctly. (4) Question 3.2 ✓✓ linear best fit Line (2)
Arm Span (cm)	Height (cms)																											
157	162																											
158	160																											
161	155																											
161	160																											
162	170																											
165	166																											
170	170																											
177	176																											
184	180																											
188	187																											
188	192																											
194	193																											
3.3	Yes. The relationship between arm span and height is a positive, linear one so we can expect a person with below average arm span to have below average in height.	✓ Yes ✓ Reason (2) [8]																										

QUESTION 4

4.1	Let β be the angle of inclination of PQ. $\tan \beta = m_{PQ}$ $\tan \beta = \frac{2 - (-2)}{-1 - (-2)}$ $\tan \beta = 4$ $\beta = 75,96^\circ$	✓ $\tan \beta = m_{PQ}$ ✓ $\tan \beta = 4$ ✓ answer (3)
4.2	$M\left(\frac{-1+3}{2}; \frac{2+0}{2}\right)$ $M(1; 1)$	✓ x-value ✓ y-value (2)
4.3	$PQ = \sqrt{(-1+2)^2 + (2+2)^2}$ $= \sqrt{17}$ $PR = \sqrt{(-1-3)^2 + (2-0)^2}$ $= \sqrt{20}$ $QR = \sqrt{(0-(-2))^2 + (3-(-2))^2}$ $= \sqrt{29}$ $\text{Perimeter} = \sqrt{29} + \sqrt{20} + \sqrt{17}$ $= 13,98 \text{ units}$ $= 14 \text{ to the nearest whole number}$	✓ substitution into correct formula ✓ answer ✓ answer ✓ sum ✓ answer (5)
4.4	$y - 1 = 4(x - 1)$ $y = 4x - 3$	✓ $m = 4$ ✓ substitution of (1; 1) ✓ answer (3) [13]

Radius $AB = \sqrt{10}$ Distance from A to centre of circle is $= \sqrt{(6-3)^2 + (4+1)^2}$ $= \sqrt{9+25}$ $= \sqrt{34}$ $AB^2 = 34 - 10$ $AB^2 = 24$ $AB = \sqrt{24}$ $AB = 2\sqrt{6}$ $AB = 4,90$ OR $r^2 = 10$ $r = \sqrt{10}$ Radius $\perp$ tangent By Pythagoras $AB^2 = (6-3)^2 + (4+1)^2 - 10$ $= 24$ $AB = 4,90$	✓ subs into distance formula ✓ $\sqrt{34}$ ✓ $AB^2 = 34 - 10$ ✓ answer (5) ✓ $r = \sqrt{10}$ ✓✓ $AB^2 = (6-3)^2 + (4+1)^2 - 10$ ✓ $AB = 4,90$ (5) [14]
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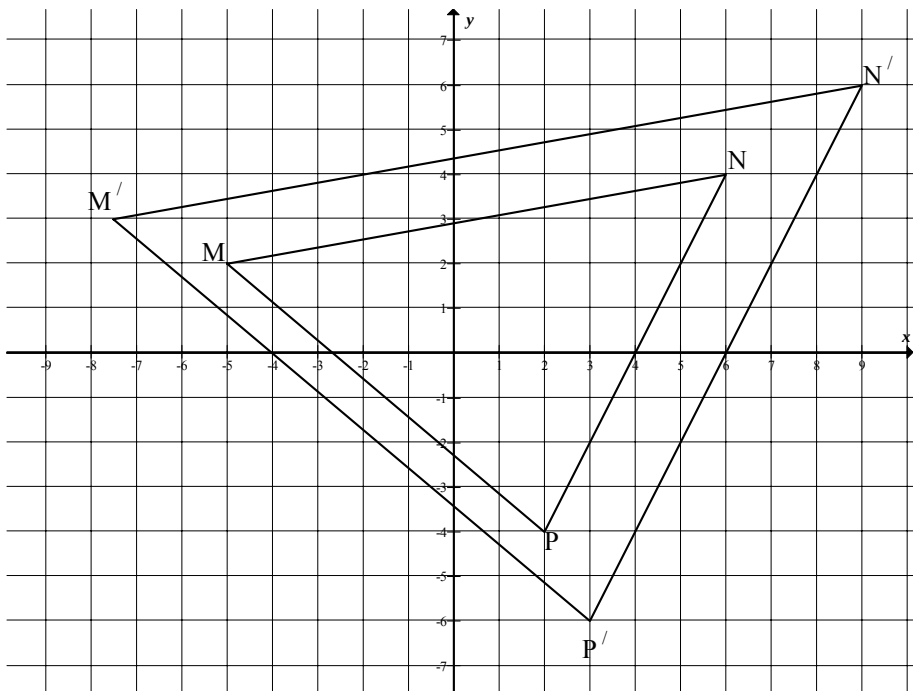
QUESTION 6

6.1	$9 + (y + 2)^2 = 25$ $(y + 2)^2 = 16$ $y + 2 = \pm 4$ $y = 2$ or $y = -6$ $B(0 ; 2)$ OR $x = 0$ $(0)^2 - 6(0) + y^2 + 4y = 12$ $y^2 + 4y - 12 = 0$ $(y + 6)(y - 2) = 0$ $y = -6$ or $y = 2$ $B(0 ; 2)$	✓ $x = 0$ ✓ factors ✓ answers ✓ answer for B (4) ✓ $x = 0$ ✓ factors ✓ answers ✓ answer for B (4)
6.2	$C(6 ; 2)$	✓✓ answer (2)
6.3	$\left(x - 3 \times \frac{3}{2}\right)^2 + \left(y + 2 \times \frac{3}{2}\right)^2 = \left(5 \times \frac{3}{2}\right)^2$ $\left(x - \frac{9}{2}\right)^2 + (y + 3)^2 = \left(\frac{15}{2}\right)^2$ $\left(x - \frac{9}{2}\right)^2 + (y + 3)^2 = 56,25$	✓ each part $\times \frac{3}{2}$ ✓ answer (2)
6.4.1	$AB = \sqrt{(12 - 3)^2 + (10 - (-2))^2}$ $= \sqrt{9^2 + 12^2}$ $= 15$	✓ substitution ✓ answer (2)
6.4.2	The radii are 5 and 10. $r_A + r_B = 5 + 10$ $= 15$ $= AB$ The circles will only intersect at one point.	✓ addition of radii ✓ answer (2) [12]

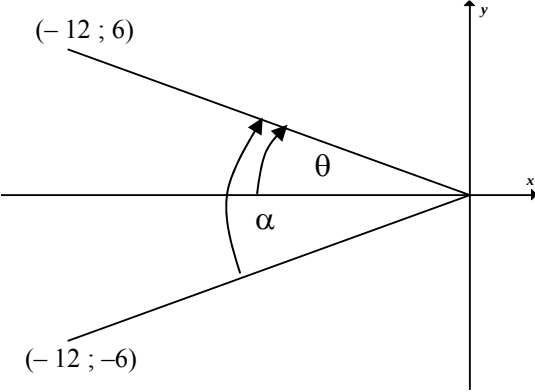
QUESTION 7

$-3 = x \cos 150^\circ - 2 \sin 150^\circ$ $-3 = -x \cdot \frac{\sqrt{3}}{2} - 2 \cdot \frac{1}{2}$ $\frac{\sqrt{3}}{2} x = 2$ $x = \frac{4}{\sqrt{3}}$ $y = x \cdot \sin 150^\circ + 2 \cdot \cos 150^\circ$ $y = \frac{4}{\sqrt{3}} \cdot \frac{1}{2} + 2 \left(-\frac{\sqrt{3}}{2} \right)$ $= \frac{2}{\sqrt{3}} - \sqrt{3}$ $= -\frac{\sqrt{3}}{3}$	✓ expansion ✓ substitution ✓ answer ✓ expansion ✓ answer
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[5]**QUESTION 8**

8.1		✓✓✓ coordinates of new points (3)
8.2.1	$\frac{MN}{M'N'} = \frac{2}{3}$	✓✓ (2)
8.2.2	$\frac{\text{area } \triangle MNP}{\text{area } \triangle M'N'P'} = \frac{4}{9}$	✓✓ (2)
8.2.3	$\frac{\text{area } \triangle MNP}{\text{area } \triangle M''N''P''} = \left(\frac{4}{9} \right)^{n+1}$	✓✓ (2) [9]

QUESTION 9

9.1	$A'(-12; -6)$	✓ answer (1)
9.2	$x' = x \cos \alpha - y \sin \alpha$ $-12 \cos \alpha - 6 \sin \alpha = -12$ $-2 \cos \alpha - \sin \alpha = -2 \dots (1)$ $y' = y \cos \alpha + x \sin \alpha$ $6 \cos \alpha - 12 \sin \alpha = -6$ $\cos \alpha = 2 \sin \alpha - 1 \dots (2)$ Substitute (2) into (1) $-2(2 \sin \alpha - 1) - \sin \alpha = -2$ $-4 \sin \alpha + 2 - \sin \alpha = -2$ $-5 \sin \alpha = -4$ $\sin \alpha = \frac{4}{5}$ $\alpha = 53,13^\circ$ OR  $\tan \theta = \frac{1}{2}$ $\theta = 26,565^\circ$ $\alpha = 2(26,565^\circ)$ $\alpha = 53,13^\circ$	✓ substitution ✓ substitution ✓ simplification ✓ substitution ✓ simplification ✓ answer (6)
		✓✓ $\tan \theta = \frac{1}{2}$ ✓ $\theta = 26,565^\circ$ ✓✓ $\alpha = 2(26,565^\circ)$ ✓ answer (6) [7]

QUESTION 10

10.1.1	$\cos 28^\circ = \sqrt{1 - \sin^2 28^\circ}$ $= \sqrt{1 - a^2}$	$\checkmark \sqrt{1 - \sin^2 28^\circ}$ $\checkmark$ answer (2)
10.1.2	$\cos 64^\circ$ $= \cos 2(32^\circ)$ $= 2 \cos^2 32^\circ - 1$ $= 2b^2 - 1$	$\checkmark \cos 2(32^\circ)$ $\checkmark 2 \cos^2 32^\circ - 1$ $\checkmark$ answer (3)
10.1.3	$\sin 4^\circ$ $= \sin(32^\circ - 28^\circ)$ $= \sin 32^\circ \cos 28^\circ - \cos 32^\circ \sin 28^\circ$ $= \sqrt{1 - b^2} \cdot \sqrt{1 - a^2} - ab$ OR $\sin 4^\circ$ $= \sin(60^\circ - 2 \times 28^\circ)$ $= \sin 60^\circ \cos(2 \times 28^\circ) - \cos 60^\circ \sin(2 \times 28^\circ)$ $= \frac{\sqrt{3}}{2} (1 - 2a^2) - \frac{1}{2} (2a) \sqrt{1 - a^2}$ $= \frac{\sqrt{3}}{2} - \sqrt{3}a^2 - a\sqrt{1 - a^2}$ OR $\sin 4^\circ$ $= \sin(2 \times 32^\circ - 60^\circ)$ $= \sin(2 \times 32^\circ) \cos 60^\circ - \cos(2 \times 32^\circ) \cdot \sin 60^\circ$ $= 2b\sqrt{1 - b^2} \cdot \frac{1}{2} - \frac{\sqrt{3}}{2} (2b^2 - 1)$ $= b\sqrt{1 - b^2} - \sqrt{3}b^2 + \frac{\sqrt{3}}{2}$ OR Using $\sin(A+B) + \sin(A-B) = 2 \cdot \sin A \cdot \cos B$ With $A = 28^\circ$ and $B = 32^\circ$ $\sin 60^\circ + \sin(-4^\circ) = 2ab$ $\sin 4^\circ = \frac{\sqrt{3}}{2} - 2ab$ OR	$\checkmark \sin(32^\circ - 28^\circ)$ $\checkmark$ expansion $\checkmark \checkmark$ answer (4)

	Using $\sin(A+B) + \sin(A-B) = 2.\sin A.\cos B$ With $A = 32^\circ$ and $B = 28^\circ$ $\sin 60^\circ + \sin(4^\circ) = 2\sqrt{1-b^2}.\sqrt{1-a^2}$ $\sin 4^\circ = 2\sqrt{1-b^2}.\sqrt{1-a^2} - \frac{\sqrt{3}}{2}$ OR Using $\sin 4^\circ = 2 \sin 2^\circ.\cos 2^\circ$ and $\sin 2^\circ = \sin(30^\circ - 28^\circ) = \frac{1}{2}(\sqrt{1-a^2} - \sqrt{3}a)$ and $\sin 2^\circ = \sin(32^\circ - 30^\circ) = \frac{1}{2}(\sqrt{3}\sqrt{1-b^2} - b)$ and $\cos 2^\circ = \cos(30^\circ - 28^\circ) = \frac{1}{2}(\sqrt{3}\sqrt{1-a^2} + a)$ and $\cos 2^\circ = \cos(32^\circ - 30^\circ) = \frac{1}{2}(\sqrt{3}b + \sqrt{1-b^2})$ then $\sin 4^\circ = \frac{1}{2} \left\{ \sqrt{3}b\sqrt{1-a^2} - 3ab + \sqrt{1-a^2}.\sqrt{1-b^2} - \sqrt{3}a\sqrt{1-b^2} \right\}$ OR $\sin 4^\circ = \frac{1}{2} \left\{ 3\sqrt{1-b^2}\sqrt{1-a^2} + \sqrt{3}a\sqrt{1-b^2} - \sqrt{3}b\sqrt{1-a^2} - ab \right\}$	
10.2	$b\sqrt{1-a^2} - a\sqrt{1-b^2}$ $= \cos 32^\circ.\sqrt{1-\sin^2 28^\circ} - \sin 28^\circ\sqrt{1-\cos^2 32^\circ}$ $= \cos 32^\circ.\cos 28^\circ - \sin 28^\circ.\sin 32^\circ$ $= \cos(32^\circ + 28^\circ)$ $= \cos 60^\circ$ $= \frac{1}{2}$	✓ substitution ✓ $\cos 28^\circ$ ✓ $\sin 32^\circ$ ✓ compound angle formula (4)
10.3.1	$\frac{\sin 130^\circ.\tan 60^\circ}{\cos 540^\circ.\tan 230^\circ.\sin 400^\circ}$ $= \frac{\sin 50^\circ \times \tan 60^\circ}{\cos 180^\circ \times \tan 50^\circ \times \sin 40^\circ}$ $= \frac{\sin 50^\circ \times \sqrt{3}}{-1 \times \frac{\sin 50^\circ}{\cos 50^\circ} \times \cos 50^\circ}$ $= -\frac{\sqrt{3} \cos 50^\circ}{\cos 50^\circ}$ $= -\sqrt{3}$	✓ $\sin 50^\circ$ ✓ $\tan 50^\circ$ ✓ $\sin 40^\circ$ ✓ $\cos 50^\circ$ ✓ $\frac{\sin 50^\circ}{\cos 50^\circ}$ ✓ -1 ✓ answer (7)

10.3.2	$(1 - \sqrt{2} \sin 75^\circ)(1 + \sqrt{2} \sin 75^\circ)$ $= 1 - 2 \sin^2 75^\circ$ $= \cos 150^\circ$ $= -\frac{\sqrt{3}}{2}$ OR $\sin 75^\circ$ $= \sin(45^\circ + 30^\circ)$ $= \sin 45^\circ \cdot \cos 30^\circ + \cos 45^\circ \cdot \sin 30^\circ$ $= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$ $\sqrt{2} \sin 75^\circ = \frac{\sqrt{3}}{2} + \frac{1}{2} = a$ $(1 - \sqrt{2} \sin 75^\circ)(1 + \sqrt{2} \sin 75^\circ)$ $= (1 - a)(1 + a)$ $= 1 - a^2$ $= 1 - \left(\frac{3}{4} + \frac{1}{4} + 2 \cdot \frac{\sqrt{3}}{2} \cdot \frac{1}{2} \right)$ $= -\frac{\sqrt{3}}{2}$	✓ simplification ✓ $1 - 2 \sin^2 75^\circ$ ✓ $\cos 150^\circ$ ✓ answer (4)
10.4	$\sin^2 x + \cos 2x - \cos x = 0$ $\sin^2 x + (\cos^2 x - \sin^2 x) - \cos x = 0$ $\cos^2 x - \cos x = 0$ $\cos x(\cos x - 1) = 0$ $\cos x = 0 \text{ or } \cos x = 1$ $x = \pm 90^\circ + k.360^\circ \text{ or } x = 0^\circ + k.360^\circ \quad k \in \mathbb{Z}$ $= k.360^\circ$ (i.e. $x = 90^\circ + k.180^\circ$ or $x = k.360^\circ \pm 90^\circ, k \in \mathbb{Z}$)	✓ $(\cos^2 x - \sin^2 x)$ ✓ $\cos^2 x - \cos x = 0$ ✓ factors ✓ $\cos x = 0$ or $\cos x = 1$ ✓ $90^\circ + k.360^\circ$ ✓ $k.360^\circ$ ✓ $x = -90^\circ + k.360^\circ$ (7)
10.5.1	$x = 0^\circ; 90^\circ; 180^\circ$	✓✓✓ each value (3)

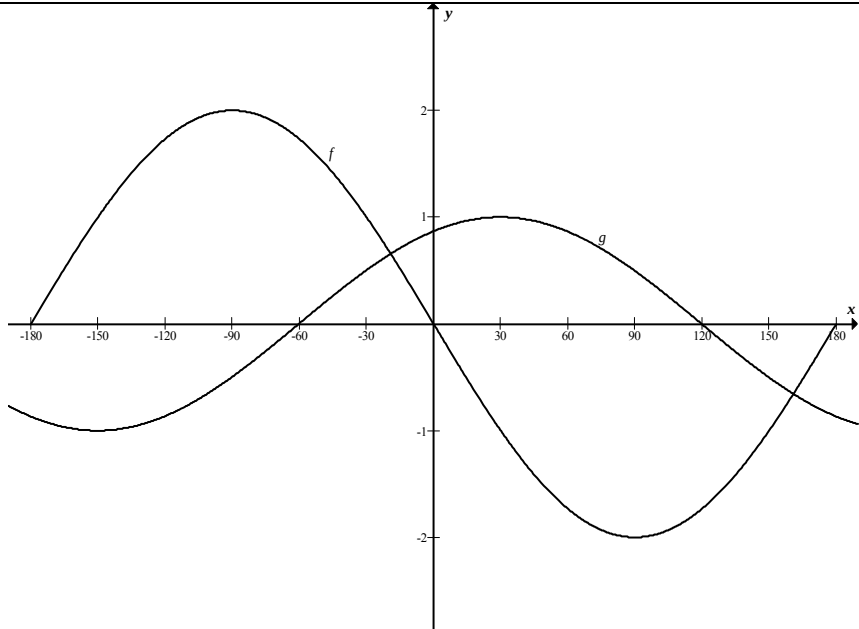
10.5.2	$\frac{\cos 2x \cdot \tan x}{\sin^2 x} = \frac{(\cos^2 x - \sin^2 x) \cdot \frac{\sin x}{\cos x}}{\sin^2 x}$ $= \frac{\cos^2 x - \sin^2 x}{\cos x \cdot \sin x}$ $= \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}$ $= \frac{\cos x}{\sin x} - \tan x$	✓ $(\cos^2 x - \sin^2 x)$ ✓ $\frac{\sin x}{\cos x}$ ✓ answer ✓ $\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}$ ✓ answer (5) [39]
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QUESTION 11

11.1	$EC^2 = DE^2 + DC^2 - 2DE \cdot DC \cos \hat{C}$ $= (7,5)^2 + (9,4)^2 - 2 \cdot (7,5)(9,4) \cos 32^\circ$ $= 25,03521844...$ $EC = 5,0 \text{ metres}$	✓substitution into cosine rule ✓25,03521844... ✓answer (3)
11.2	$\frac{\sin \hat{DCE}}{7,5} = \frac{\sin 32^\circ}{5,0}$ $\sin \hat{DCE} = \frac{7,5 \cdot \sin 32^\circ}{5,0}$ $= 0,7948788963$ $\hat{DCE} = 52,6^\circ$	✓ sin rule ✓0,7948788963 ✓answer (3)
11.3	Area of $\triangle DEC$ $= \frac{1}{2} DE \cdot DC \sin \hat{D}$ $= \frac{1}{2} (7,5)(9,4) \sin 32^\circ$ $= 18,7m^2$ OR Area of $\triangle DEC$ $= \frac{1}{2} CE \cdot DC \sin 52,6^\circ$ $= \frac{1}{2} (5,0)(9,4) \sin 52,6^\circ$ $= 18,7m^2$	✓ substitution ✓answer (2)
11.4	$\sin 32^\circ = \frac{EG}{7,5}$ $EG = 7,5 \cdot \sin 32^\circ$ $= 4,0$ $EF = (4 + 3,5)$ $= 7,5 \text{ metres}$ OR $EG = EC \cdot \sin 52,6^\circ$ $= (5,0) \cdot \sin 52,6^\circ$ $= 4,0$ $EF = 4,0 + 3,5$ $= 7,5$ OR	✓ratio ✓substitution ✓answer (3) [11]

	$\frac{1}{2} \cdot DC \cdot EG = \text{area } \triangle DEC$ $\frac{1}{2} (9,4) EG = 18,7$ $\therefore EG = \frac{18,7 \times 2}{9,4}$ $= 4,0$ $EF = 4,0 + 3,5$ $= 7,5$	
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QUESTION 12

12.1	Period = 360°	✓ answer (1)
12.2	Amplitude = $\frac{1}{2}$	✓✓ answer (2)
12.3		✓ shape ✓ x intercepts ✓ amplitude (3)
12.4	2 solutions	✓ answer (1)
12.5	$-60^\circ \leq x \leq 120^\circ$ or $x \in [-60^\circ; 120^\circ]$	✓ $-60^\circ; 120^\circ$ ✓ notation (2)
12.6	$-90^\circ < x < 30^\circ$ or $x \in (-90^\circ; 30^\circ)$	✓✓ $-90^\circ; 30^\circ$ ✓ notation (3) [12]

TOTAL: 150