LEARNING AREA **MATHEMATICS**

GRADE 9

TEACHER'S GUIDE

Module 1 - 4

MODULE 1

HINTS

- Converting certain decimal fractions to ordinary fractions is not in the syllabus. However, it has great value for the understanding of rational numbers, and learners will benefit from the exercise, particularly those that intend continuing with mathematics.

ASSESSMENT

- Please note that the teacher can make changes to the assessment instruments (scales, rubrics, etc.) below, according to the circumstances.
- The class work may be done as group work while the teacher provides explanations and guidance. This provides a useful opportunity to evaluate and mark their group skills.
- Learners can evaluate each other, especially when the assessment mechanism is simple. This will give them better judgement of their own work.
    - ♣ Assessment grids are included in the module.

LEARNING UNIT 1.1
Numbers, operations and relationships

TEST 1

TEST 1 Marked with this analytical rubric:

OUTCOMES	1	2	3	4
Distinguishes between numbers in unsimplified and simplified forms	Confuses them completely	Can tell simple differences apart	Distinguishes kinds of numbers in most cases	Recognises all disguised numbers
Classifies numbers in simplest form	Cannot classify simple numbers at all	Correctly classifies some simple numbers	Correctly classifies most simple numbers	Correctly classifies all the simple numbers
Simplifies correctly	No correct simplifications	Some correct simplifications	Most simplifications correct	All simplifications correct
Classifies consistently (crosses in correct places)	No or few correct crosses	Some crosses correct	Most classifications correct	All classifications correct
Classifies completely (crosses in all correct places)	Most extensions left out	Few extensions complete	Few extensions left out	All extensions completed

TEST NUMBER SYSTEMS DATE:______________ NAME:_________________

In which set (s) does each number belong? Simplify the numbers, if necessary, and complete the table by making crosses under all appropriate column headings.

	SIMPLIFIED	N	N_0	Z	Q	Q`	R	R`
$-\sqrt{9}$								
5								
$\frac{355}{113}$								
$-\sqrt{11}$								
$\sqrt{-16}$								
$-18+22$								
0								
$\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{3}}$								
$\frac{3}{4}$								
$1+\sqrt{10}$								
-25								
$-12 \div 2$								
$\frac{\sqrt{5}}{5}$								
$4-\sqrt{16}$								

TEST – Memorandum

	SIMPLIFIED	N	N_0	Z	Q	Q`	R	R`
$-\sqrt{9}$	–3			x	x		x	
5		x	x	x	x		x	
$\frac{355}{113}$					x		x	
$-\sqrt{11}$						x	x	
$\sqrt{-16}$								x
$-18+22$	4	x	x	x	x		x	
0			x	x	x		x	
$\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{3}}$	$\frac{2}{\sqrt{3}}$					x	x	
$\frac{3}{4}$					x		x	
$1+\sqrt{10}$						x	x	
-25				x	x		x	
$-12 \div 2$	–6			x	x		x	
$\frac{\sqrt{5}}{5}$						x	x	
$4-\sqrt{16}$	0		x	x	x		x	

Memoranda

... CLASS WORK

1.1 Yes, any informal "proof" is acceptable.
1.2 As 1.1
1.3 Zero and negative numbers make an appearance. The explanation is not important – only the thinking that the learner does.
2.1 $N_0 = \{0 ; 1 ; 2 ; . . . \}$ and $Z = \{ . . . –3 ; –2 ; –1 ; 0 ; 1 ; 2 ; 3 ; . . .\}$
3. Here are the fractions. Explain carefully that integers can also be written as fractions – in fact it is quite often a useful technique.
4.1 Not everyone will be able to cope with this. *R*` gives the answers that are obtained when square roots of negative numbers (inter alia) is taken.

.. TASK

2. Point out to learners that zero is missing from the table.

... HOMEWORK ASSIGNMENT

1. Zero is needed because:
The principle behind place values is totally dependent on having a symbol for zero.
It separates positive and negative numbers.
It symbolises "nothing".
Algebraically it is defined as: $a + (-a)$
2. Complex numbers – don't expect too much.
If one uses the symbol i for $\sqrt{-1}$, then we can represent non–real numbers as follows: $\sqrt{-12} = 2\sqrt{-3} = 2\sqrt{3\times-1} = 2\sqrt{3}\sqrt{-1} = 2\sqrt{3}\,i$
3 + 5i and 2,5 – 16i are examples of non–real numbers, and each consists of two parts: a real part and a non-real part. The most important consequences of this are that one must be careful when doing arithmetic calculations, and that these numbers cannot be arranged in ascending order!
3. Any reasonable answer can be accepted. This might be a good opportunity to have learners evaluating each other's number systems.

.. ENRICHMENT ASSIGNMENT

If there is time, one can go through this work, particularly with a strong group.
4.1 Non-repeating; although 3,030030003000030… has a pattern, it does not repeat.

4.2 Emphasise that the first one is NOT equal to π. The two others must be simplified properly.

4.3.1 $\frac{1553}{1000}$ 4.3.2 $\frac{51}{90}$ 4.3.3 $\frac{30311}{999}$ 4.3.4 $\frac{2403}{990}$

... CLASS ASSIGNMENT

The aim of this exercise is to familiarise learners with unsimplified values, so that they can learn to estimate. It is very important that they mentally simplify correctly so that they can start guessing the magnitudes. Then the values have to be arranged in at least the correct order. If the spaces in between are in reasonable proportion, that is a bonus. This shows the order:

1.1 0,00 ; 1 ; 2 ; 3,0 ; 4 ; 5,0000 ; 5+2 ; 6 ; 9–1

1.2 –4 ; –3 ; –1 ; 3–3 ; 2 ; 5

1.3 $\frac{1}{5}$; $\frac{1}{4}$; 0.666 ; $\frac{2}{3}$; $\frac{2}{2}$; 1,000 ; 0,2+1 ; 1.75

1.4 $\frac{3}{2}-12$; $-8+\frac{7}{5}$; –5,5 ; $-\frac{5}{2}$; –2,5 ; 5,55 ; $\frac{14}{2}$

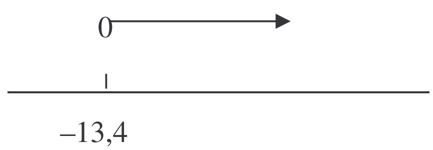
1.5 $-\sqrt{9}$; $-\sqrt{4}$; $\sqrt{0}$; $\sqrt{1}$; $\sqrt{\frac{9}{4}}$; $\frac{\sqrt{16}}{2}$; $\sqrt{9}$; $\sqrt{36}-1$

1.6 $\sqrt{4}$; $\sqrt{6}$; $\sqrt{9}$; $\sqrt{16}$; $\sqrt{25}$; $\sqrt{20}+1$; $\sqrt{32}$; $\sqrt{36}$

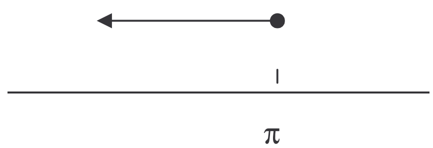
..........ENRICHMENT ASSIGNMENT

1.1 5,6 < 5,7 3+9 = 4×3 –1 > –2 3 > –3 $\sqrt[3]{27} < \sqrt{15}$

2.1 $y > -13,4$



$y \leq \pi$



..........GROUP ASSIGNMENT

These are the simplified values in the original order:

1.1 –8 ; 12 ; –6 ; 2 ; 10 ; 3 ; 5 ; 3,44… ; 3

1.2 2 ; 0,3… ; 1,3… ; 0,5 ; 0,5 ; 0,05 ; 0,005

1.3 3 ; 3,5 ; 3,14 ; 3,142857… ; 3,1415929… ; 3,1415926… (the last one is π)

These are the same values in the correct order:

1.1 –8 ; –6 ; 2 ; 3 ; 3,44… ; 5 ; 10 ; 12

1.2 0,005 ; 0,05 ; 0,3… ; 0,5 ; 1,3… ; 2

1.3 3 ; 3,14 ; π ; 3,1415929… ; 3,142857…

..........CLASS WORK

This exercise has been designed to give learners a feeling for the consequences of rounding (approximated answers). They often put complete unthinking faith in their calculators' answers.

1.1 Note the notation as well as the number of decimal places.

1.2 Again, notation as well as number of decimal places.

1.3 Emphasise once again that an approximation to π is not *equal* to π.

Discuss the meaning of the term "approximately equal to".

3. Answers: 1,03 ; 2,83 ; 15,71 ; 12 ; 1,0 (the zero must be there).

...CLASS WORK

Learners often have difficulties with conversions – you might have to supply lots of help and guidance.

1.3 The months don't have the same number of days; simply multiplying will not give the best answer. Find out which months are meant and don't forget leap years!

1.4 Why division? Help them develop strategies.

3. Similar problems to 1.3. The answer can be approximated. Explain why this acceptable. This problem will motivate them to appreciate the advantages of scientific notation: ≈ 3 157 056 000 seconds.

5.1 $9{,}1 \times 10^{28}$ 5.2 24 km 5.3 100 liter

LEARNING UNIT 1.2
Exponents

TEST 2

1. Scientific Notation

1.1 Write the following values as ordinary numbers:

1.1.1 $2{,}405 \times 10^{17}$

1.1.2 $6{,}55 \times 10^{-9}$

1.2 Write the following numbers in scientific notation:

1.2.1 5 330 110 000 000 000 000

1.2.2 0,000 000 000 000 000 013 104

1.3 Do the following calculations and give your answer in scientific notation:

1.3.1 $(6{,}148 \times 10^{11}) \times (9\ 230\ 220\ 000\ 000\ 000)$

1.3.2 $(1{,}767 \times 10^{-6}) \div (6{,}553 \times 10^{-4})$

2. Exponents
Simplify and leave answers without negative exponents. (Do not use a calculator.)

2.1 $3a^2xy\left(3ab^2x^2y\right)^3$ 2.2 $\dfrac{\left(a^0b^0c\right)^3}{6c^2\left(ab^3c^5\right)^2} \times \dfrac{2\left(3a^2c^3\right)^0}{4abc^2} \times 18b^4\left(2a^3c^4\right)^2$

3. Substitution

3.1 Simplify: $2x^2y^3 + (xy)^2 - 4x$

3.2 Calculate the value of $2x^2y^3 + (xy)^2 - 4x$ as $x = 4$ and $y = -2$

4. Formulae

the formula for the area of a circle is: area = πr^2 (r is the radius).

4.1 Calculate the areas of the following circles:

4.1.1 A circle with radius = 12 cm ; round answer to 1 decimal place.

4.1.2 A circle with a diameter 8 m ; approximate to the nearest metre.

TEST 2 – Memorandum

1.1.1 240 500 000 000 000 000

1.1.2 0,000 000 006 55

1.2.1 $5{,}330\ 110 \times 10^{18}$

1.2.2 $1{,}3104 \times 10^{-17}$

1.3.1 $6{,}148 \times 10^{11} \times 9{,}23022 \times 10^{15}$

$= 6{,}148 \times 9{,}23022 \times 10^{11} \times 10^{15}$

$\approx 56{,}74 \times 10^{26}$

$= 5{,}674 \times 10^{27}$

1.3.2 $\dfrac{1{,}767\times10^{-6}}{6{,}553\times10^{-4}} = \dfrac{1{,}767}{6{,}553}\times10^{-6-(-4)} \approx 0{,}26 \times 10^{-2} = 2{,}6 \times 10^{-1}$

2.1 $3^4a^5x^7y^4 = 81a^5x^7y^4$

2.2 $\dfrac{c^3\times2\times18a^6b^4c^8}{6a^2b^6c^{12}\times4abc^2} = \dfrac{36a^6b^4c^{11}}{24a^3b^7c^{14}} = \dfrac{3a^3}{2b^3c^3}$

3.1 $2x^2y^3 + x^2y^2 - 4x$

3.2 $2(4)^2(-2)^3 + (4)^2(-2)^2 - 4(4) = 2(16)(-8) + (16)(4) - 16 = -256 + 64 - 16 = -208$

4.1.1 opp = $\pi \times 12^2 = 452{,}38934\ldots \approx 452{,}4\ \text{cm}^2$

4.1.2 opp = $\pi \times 4^2 = 50{,}26548\ldots \approx 50\ \text{m}^2$

Mark test according to this scale:

OUTCOME	1	2	3	4
Converts between ordinary and scientific notations	Unable to do any successful conversions	Converts in only one direction, or makes many errors	Converts in both directions, but with errors	All conversions correct
Calculates using scientific notation	Unable to do successful calculations	Manages only simple work	Only minor errors	All answers correct
Uses laws of exponents	Little or no correct work	Many errors	Most answers correct	Understands and uses laws correctly
Substitutes constants for variables	No correct substitutions	Only simplest parts correct	Makes few major errors	All substitutions correct
Uses given formula	Cannot use formula correctly	Wrong answers	Formula correct, answers without units	Formula and answers correct

Memoranda

... CLASS WORK

The learners are likely to know the work in the first part already. Those who have not mastered the simplest laws of exponents now have an opportunity to catch up. For the rest it serves as revision in preparation for the new work in the second part.

1.1 $4 \times 4 \times 4$ $(p+2) \times (p+2) \times (p+2) \times (p+2) \times (p+2)$ etc.

1.2 7^4 y^5 etc.

1.3 $(-7)^6 = 7^6$, $(-7)^6 \neq -7^6$ etc.

2.1 7^{14} $(-2)^{17} = -2^{17}$ etc.

3.1 a^{6-y} 3^2 $(a+b)^{p-12}$ a^0

4.1 a^{5a} etc.

TUTORIAL

The tutorial should be done in silence in class in a fixed time. Recommendation: Mark it immediately – maybe the learners can mark one another's work.

Answers: 1. a^3 2. xy 3. $a^6b^8c^8$ 4. a^8b^6 5. $4x^8y^9$ 6. 1

... CLASS WORK

New for most learners in grade 9.

.. HOMEWORK ASSIGNMENT

Answers: 1. $18x^6y^7$ 2. $24x^{11}y^3$ 3. 0 4. $32a^6b^{14}c^{14}d^{11}$

5. $\frac{16x^{10}}{y^{12}}$ 6. 1 7. $\frac{108y^5}{x}$

.. CLASS WORK

Substitution gives lots of trouble because it looks so easy. Learners who leave out steps (or don't write them down) often make careless mistakes. Force them to use brackets.

2. They should conclude that simplification should come first – after all, this is why we simplify!

3.1 26 cm 3.2 229,5 cm^2 3.3 $\approx 1{,}45 \times 10^{15}$

LEARNING UNIT 1.3
Pythagoras

TEST

Where appropriate, give answers accurate to one decimal place.

1. Write the complete Theorem of Pythagoras down in words.

2. Calculate the hypotenuse of ΔABC where $\angle A$ is a right angle and b = 15 mm and c = 20 mm.

3. ΔPQR has a right angle at R. $PR = QR$. Calculate the lengths of sides PR and QR if QP = 15 cm.

4. Is ΔDEF right–angled if DF = 16cm, DE = 14 cm and EF = 12 cm?

5. What kind of triangle is ΔXYZ if YZ = 24 cm, XY = 10cm and XZ = 26 cm? Give complete reasons.

TEST 3 – Memorandum

1. In a right–angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

2. Hypotenuse = a. $a^2 = 15^2 + 20^2 = 225 + 400 = 625$ $a = 25$ Hypotenuse is 25 mm

3. $PR^2 + QR^2 = QP^2$ $2(PR)^2 = 15^2$ $2(PR)^2 = 225$ $PR^2 = 112{,}5$ $PR \approx 10{,}6$ cm

4. $LK = 16^2 = 256$
   $RK = 14^2 + 12^2 = 196 + 144 = 340$
   $LK \neq RK$, so ΔDEF is not right–angled.

5. $LK = 26^2 = 676$
$RK = 24^2 + 10^2 = 576 + 100 = 676$
$LK = RK$, so ΔXYZ is right–angled with Y the right angle.

6. Write the following roots in the simplest form:

6.1 $\sqrt{12}$ 6.2 $\sqrt{50a^3b^5}$ 6.3 $\sqrt[4]{64(a-1)^4 b}$

RUBRIC:

OUTCOME	1	2	3	4
States theorem in words	Cannot put theorem in words	Statement vague and incomplete	Statement has minor errors	Statement complete and correct
Applies theorem to calculate side lengths	Cannot use theorem	Makes major errors (e.g. no squaring)	Makes minor errors	Correct application
Applies theorem to determine whether Δ is right–angled	Cannot use theorem	Tried, but unsuccessful	Correct, but not *RHS* vs. *LHS*	Applies theorem correctly
Uses type of Δ	Uses type wrongly or not at all	Manages only question 2	Few errors	Understands how to use type of Δ
Gives answer in correct format	Few correct roundings or units	Many errors in roundings or units	Some errors in roundings or units	Roundings and units correct
Uses powers in simplifying (problem 6)	Uses powers wrongly or not at all	Few correct	Some errors	All powers used correctly
Simplest form (pr. 6)	None correct	Only 1 correct	Only 2 correct	All correct

Memoranda

INVESTIGATION

- If there is confusion about the *a*, *b*, *c* symbols, do draw a triangle as guidance while learners complete the table Learners with poor measuring skills might need individual support, if they cannot get reasonable answers.
- Photocopy the squares so that they can be cut out and fitted.

2.1 This is the well-known "proof" of the Theorem of Pythagoras. This work is addressed again when working with similarity.

.. CLASS WORK

Encourage learners to get into the habit of making realistic sketches.

2.1.1 $EF = d$ $d^2 = 12^2 + 5^2 = 144 + 25 = 169 = 13^2$ $d = 13$

2.1.2 $XY = 4$

3.1.1 hypotenuse$^2 = 81 + 81 = 162$ hypotenuse ≈ 12,73 cm

3.1.2 $PR^2 + RQ^2 = 2\ (PR)^2$ – isosceles $2(PR)^2 = 13{,}5^2$ $PR \approx 9{,}55$ cm

4. Because GH is the longest side, it has to be the hypotenuse – so $\angle K$ is a right angle.

4.1.1 $LK = c^2 = 50^2 = 2500$ mm^2

$RK = a^2 + b^2 = 30^2 + 40^2 = 2500$ mm^2

$LK = RK$, triangle is right–angled; $\angle C$ is the right angle.

4.1.2 $LK = 225$ cm^2 $RK = 64 + 169 = 233$ cm^2

$LK \neq RK$ so triangle is not right–angled.

4.1.3 $LK = 242{,}11$ cm^2 $RK = 121 + 121 = 242$ cm^2

$LK \neq RK$ but almost! $\angle P$ is very close to 90°.

.. HOMEWORK ASSIGNMENT

1.1 $a = 12$ mm 1.2 $o = 10$ cm

2.1 No 2.2 Very close –$Z \approx 90°$

... CLASS WORK

Memorandum:

1. $\sqrt{64} = 8$ does not fit the table.

c	$9 = 3^2$	$25 = 5^2$	$7^2 = 49$	$3^4 = 81$	$b \times b = b^2$	$64 = 2^6$	$a \times a \times a = a^3$
$\sqrt[3]{8} = 2$	$\sqrt{9} = 3$	$\sqrt{25} = 5$	$\sqrt{49} = 7$	$\sqrt[4]{81} = 3$	$b = \sqrt{b^2}$	$\sqrt[6]{64} = 2$	$\sqrt[5]{a^5} = a$

2.1 $5a^2bc\sqrt{ab}$ 2.2 $3x^3y^4\ \sqrt[3]{3}$ 2.3 $4(a+b)$

... ENRICHMENT ASSIGNMENT

- Group learners to check one another's work so that the whole class can decide on the answer.

LEARNING UNIT 1.4
Measurement

TEST 4

- There is no test for this learning unit.

Memoranda

..CLASS WORK

1.1 One
1.2.1 cm of m
1.2.2 light years
1.2.3 months
1.2.4 litres
1.2.5 milligrams
1.2.6 degrees Fahrenheit
1.2.7 km^2 or hectares
1.2.8 kilometres per hour
1.2.9 m^3
1.2.10 Rand or millions or billions of rand

PROJECT

Encourage originality.

... HOMEWORK ASSIGNMENT

2.1 ruler or measuring tape
2.2 scale
2.3 millilitres
2.4 litres
2.5 hygrometer
2.6 speedometer

..ASSIGNMENT

- Accept any reasonably practical answers. This exercise can be addressed again when the learners have mastered graphs, formulae and tables. They should then be able to improve their answers.

..CLASS WORK

- The intention of this exercise is to illustrate the consequences of inaccurate calculations. If time allows, the learners can be given photocopies of squares to measure. They can complete another table and compare answers.

LEARNING UNIT 1.5
Financial calculations

TEST 5

1. A toy shop:
   Calculate the net income from the following information, and say whether it is a profit or a loss.

Expenses:	Hiring shop:	R1 450
	Water and electricity:	R380
	Telephone:	R675
	Staff salaries:	R7 530
	Purchases of toys from wholesaler:	R67 550
	Packaging material:	R1 040
	Income from sales:	R92 406

2. Joey borrows R780 for six months from his dad to fix his bicycle. His dad requires 8% interest per year. What is the sum of money Joey pays back after six months?

3. You receive a bequest of R12 000 from an aunt. But you have to wait five years until you are 19 before receiving it. In the meantime it is invested at 13,5% per annum compounded. What sum do you receive after five years?

4. The rand-euro exchange rate is 8,75. How many rand do you have to exchange if you need 11 500 euros for a holiday in Europe?

TEST 5 – Memorandum

1. Net Income = Total Income – Total Expenses = R92 406 – R78 625 = R13781

   and this is a profit.

2. For one year the interest comes to R62,40. For six months he owes his dad R811,20 in total.

3. After one year there is R13 620 in the bank.

   After two years: R15458,70

   After three years: R17 545, 62…

   After four years: R19 914,28…

   After five years: R22 602,71 rounded to the nearest cent

4. Rand = 8,75 × 11 500 = R100 625

RUBRIC:

SKILL	NOT MASTERE	PARTLY MASTERED	ADEQUATEL Y	COMPLETEL Y
	1	2	3	4
Calculates profit / loss				
Calculates loans				
Calculates investment				
Uses exchange rate				

Memoranda

- The teacher should adapt and extend this learning unit according to the background and experience of his learners, if they are not familiar with this environment.

..CLASS WORK

1.1.1 Net income = (36 000 + 1 250 + 9 500) – 49 000 = –2 250 (R2 250 loss)

1.1.2 (85 000 + 95 000 + 63 550) – (120 560 + 15 030 + 55 250) = 52 710 rand profit

1.1.3 Loss = R1 100

2.1 There is no right or wrong answer – it is the process that matters.

3.1 R3 972,50

4. The last example (Kevin) is actually more than compound interest. It illustrates the mechanism of an annuity – a popular saving mechanism. It can be taught as enrichment.

   When learners learn about common factors, they will appreciate the pattern found in compound interest.

4.1 1 375 + 3 100 + 910,42 = R5 385,42

5.1 About R105 100

5.2 About r R85 000

6.1 £2,36

.. HOMEWORK ASSIGNMENT

1. Turnover is the total amount made from sales, before any deductions (gross amount).
2. Judge the process and not the answer.
3. 15 months (the amount in the 15th month is less).
4. Annually: R315 000; monthly: R26 250; weekly: R6 058; daily: R865
5. A little less than two years (in two years she saves nearly R70 000).
6. R16 617

MODULE 2

CONTENT OF MODULE

- Module 2 contains mostly very traditional algebra and geometry.
- The unit on algebra is essential preparation for solution of simple equations, as well as the basis for algebra in the next phase. Please note that the factorising of some basic trinomials is included in this module, although it is not specified in the syllabus.
- Learners should be able to recognise and name the basic two-and three-dimensional shapes as a component of their basic mathematical literacy. This is addressed in the second unit.
- The third unit will supply some very important skills for building further geometrical explorations.

ASSESSMENT

Supplement the assessment in this module according to the needs of the teacher and the learners

LEARNING UNIT 2.1

Algebra

DISCUSSION

Terminology

- Learners often use procedures defined for *expressions* when working with *equations* (for instance, they may ‘drop’ denominators in expressions), and vice versa. Be alert for this and encourage them to check the context before blindly going ahead.
- As multiplying and factorising are the reverse of one another, the learners should be made aware of this connection. In most cases it will make the work easier to master.
- If learners cannot tell terms and factors apart, they will not be able to manipulate expressions effectively. If necessary, they can be given more examples to help them recognise the difference.
- Learners in general find fractions problematic. If necessary, start with non-algebraic fractions to make sure that they understand the basics.

TEST 1

1. Simplify the following expressions by collecting like terms:

1.1 $3a^2 + 3a^2 - 6a + 3a - 4 + 1$

1.2 $2y^2 - 1y + 2y^2 - 6 + 2y - 9$

1.3 $8x^2 - (5x + 12x^2 - 1) + x - 4$

1.4 $(3a - a^2) - [(2a^2 - 11) - (5a - 3)]$

2. Give the answers to the following problems in the simplest form:

2.1 Add $3x^2 + 5x - 1$ to $x^2 - 3x$

2.2 Find the sum of $2a + 3b - 5$ and $3 + 2b - 7a$

2.3 Subtract $6a + 7$ from $5a^2 + 2a + 2$

2.4 How much is $3a - 8b + 3$ less than $a + b + 2$?

3. Simplify by multiplication, leaving your answer in the simplest form:

3.1 $(3x^2) \times (2x^3)$

3.2 $(ab\mathrm{c})\ (a^2\mathrm{c})\ (2b^2)$

3.3 $ab\mathrm{c}(a^2\mathrm{c} + 2b^2)$

3.4 $-3a(2a^2 - 5a)$

3.5 $(a - 2b)\ (a + 2b)$

3.6 $(3 - x^2)\ (2x^2 + 5)$

3.7 $(x - 5y)^2$

3.8 $(2 - b)\ (3a + c)$

TEST 1 – Memorandum

1.1	$6a^2 - 3a - 3$	1.2	$4y^2 + y - 15$	1.3	$-4x^2 - 4x - 3$
1.4	$-3a^2 + 8a + 8$	2.1	$4x^2 + 2x - 1$	2.2	$-5a + 5b - 2$
2.3	$5a^2 - 4a - 5$	2.4	$-2a + 9b - 1$	3.1	$6x^5$
3.2	$2a^3b^3c^2$	3.3	$a^3bc^2 + 2ab^3c$	3.4	$-6a^3 + 15a^2$
3.5	$a^2 - 4b^2$	3.6	$-2x^4 + x^2 + 15$	3.7	$x^2 - 10xy + 25y^2$
3.8	$6a + 2c - 3ab - bc$				

Assess test according to this scale:

OUTCOME	1	2	3	4
Identifies like terms	Does not identify like terms at all	Recognises some like terms	Most like terms identified	Identifies and handles like terms perfectly
Handles negative terms	Ignores negative terms completely	Manages only simple work	Few errors only	All answers correct
Understands problem statement	Most wrong	Manages only simple cases	Most understood	Understands all cases
Gives exponents correctly	Hardly any correct work	Can do only easy ones	Not many mistakes	No errors
Simplifies answer	No attempt to simplify	Simplification attempted	Simplifies, but with errors	All answers in correct simplest form

TEST 2

1. Find the Highest Common Factor of these three expressions: $6a^2c^2$ and $2ac^2$ and $10ab^2c^3$.

2. Completely factorise these expressions by finding common factors:

2.1 $12a^3 + 3a^4$

2.2 $-5xy - 15x^2y^2 - 20y$

2.3 $6a^2c^2 - 2ac^2 + 10ab^2c^3$

3. Factorise these differences of squares completely:

3.1 $a^2 - 4$

3.2 $\frac{1}{9}a^2 - 9b^2$

3.3 $x^4 - 16y^4$

3.4 $1 - a^4b^4$

4. Factorise these expressions as far as possible:

4.1 $3x^2 - 27$

4.2 $2a - 8ab^2$

4.3 $a^2 - 5a - 6$

4.4 $a^2 + 7a + 6$

5. Simplify the following fractions by making use of factorising:

5.1 $\frac{3a^2 - 3}{6a + 6}$

5.2 $\dfrac{6x^2y-6y}{2x-2}$

5.3 $\dfrac{a^2-9}{2}\times\dfrac{1}{4a^2-12a}$

5.4 $\dfrac{3x+6}{5}\div\dfrac{x^2-4}{15}$

5.5 $\dfrac{abx}{2cx}+\dfrac{2ac}{3x}+\dfrac{3cx}{2a}$

5.6 $\dfrac{2a}{x^2}-\dfrac{3a}{x}+\dfrac{a}{2x}$

5.7 $\dfrac{4a-4b}{2a^2-2b^2}-\dfrac{3}{2a-2b}$

5.8 $\dfrac{2}{3}(a+2)+\dfrac{1}{3}(a-1)-\dfrac{1}{4}(a-5)$

TEST 2 – Memorandum

1. $2ac^2$

2.1 $3a^3(4+a^2)$

2.2 $-5y(x+3x^2y+4)$

2.3 $2ac^2(3a-1+5b^2c)$

3.1 $(a+2)(a-2)$

3.2 $\left(\dfrac{1}{3}a+3b\right)\left(\dfrac{1}{3}a-3b\right)$

3.3 $(x^2+4y^2)(x+2y)(x-2y)$

3.4 $(1+a^2b^2)(1+ab)(1-ab)$

4.1 $3(x+3)(x-3)$

4.2 $2a(1+4b)(1-4b)$

4.3 $(a+1)(a-6)$

4.4 $(a+1)(a+6)$

5.1 $\dfrac{a-1}{2}$

5.2 $3y(x+1)$

5.3 $\dfrac{a+3}{8a}$

5.4 $\dfrac{9}{x-2}$

5.5 $\dfrac{3a^2bx+4a^2c^2+9c^2x^2}{6acx}$

5.6 $\dfrac{4a-5ax}{2x^2}$

5.7 $\dfrac{a-7b}{2(a+b)(a-b)}$

5.8 $\dfrac{3a+9}{4}$

Assess test according to this scale:

OUTCOME	**1**	**2**	**3**	**4**
Identifies common factors	No common factors found	Easiest common factors only	Most handled correctly	All common factors correctly found
Gives second factors	No correct work	Many errors	Only minor errors	All answers correct
Handles negatives	Ignores negatives	Many errors	All simple cases correct	No errors
Completes factorising	Never past first step	Easiest only	All simple cases completed	All correct
Recognises squares	Does not know squares	Recognises basic types	Most correct	All squares properly handled
Handles division of fractions	Wrong procedure used	Attempts appropriate method	Uses correct method, with errors	Always divides fractions properly
Finds lowest common divisor	Haphazard choice of divisor	Find a divisor	Finds lowest divisor	Correct divisor and numerators

LEARNING UNIT 2.2

Space and Shape

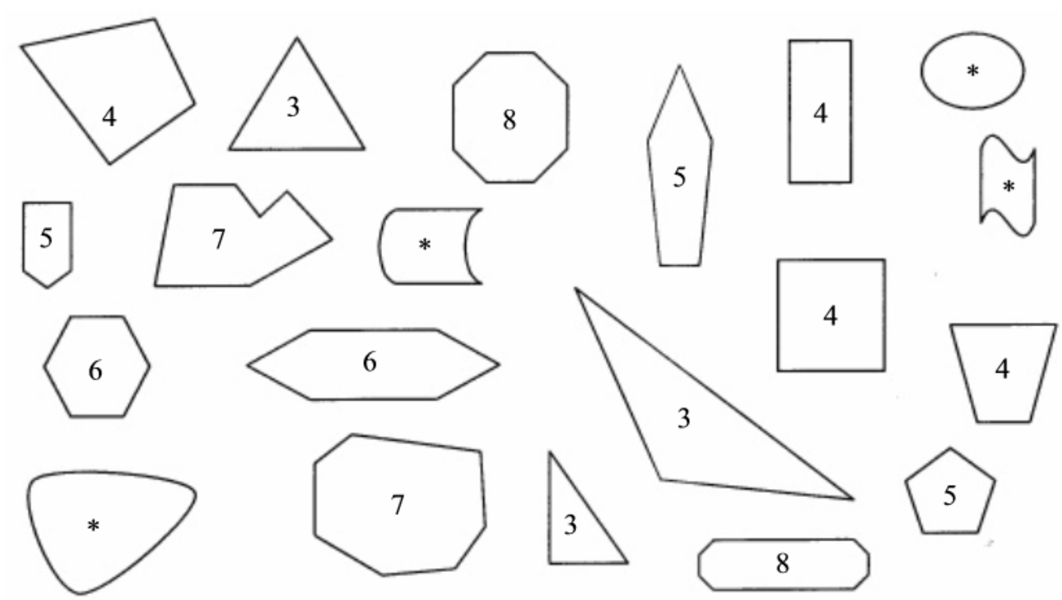
DISCUSSION

- This guide includes two pages of figures for constructing simple right prisms. Photocopy enough for the learners to make at least two of the figures. It would be best if the copies could be made on very light card (or heavy paper). If they are asked to colour some of the parts (e.g. the base and top) it might make it easier to explain some of the more difficult formulae.

- The two formulae for right prisms are, in general:
- Total Surface Area = double the base area + height of prism × perimeter of base
- Volume = base area × height of prism
- Ensure that learners are clear on the units (squared or cubed) appropriate to each formula.
- Another difficulty that learners might encounter is that the word *height* is used in calculating the area of triangles as well as being one of the dimensions of right prisms. A useful trick is to use **h** for the triangle case and **H** for the prism case.
- Breaking down the steps required for the calculations is a useful method for learners who get confused by the components in the formula. Of course, very competent learners will substitute values straight into the formula. This is an effective system, and should be encouraged where appropriate.

SOLUTIONS – EXERCISE:

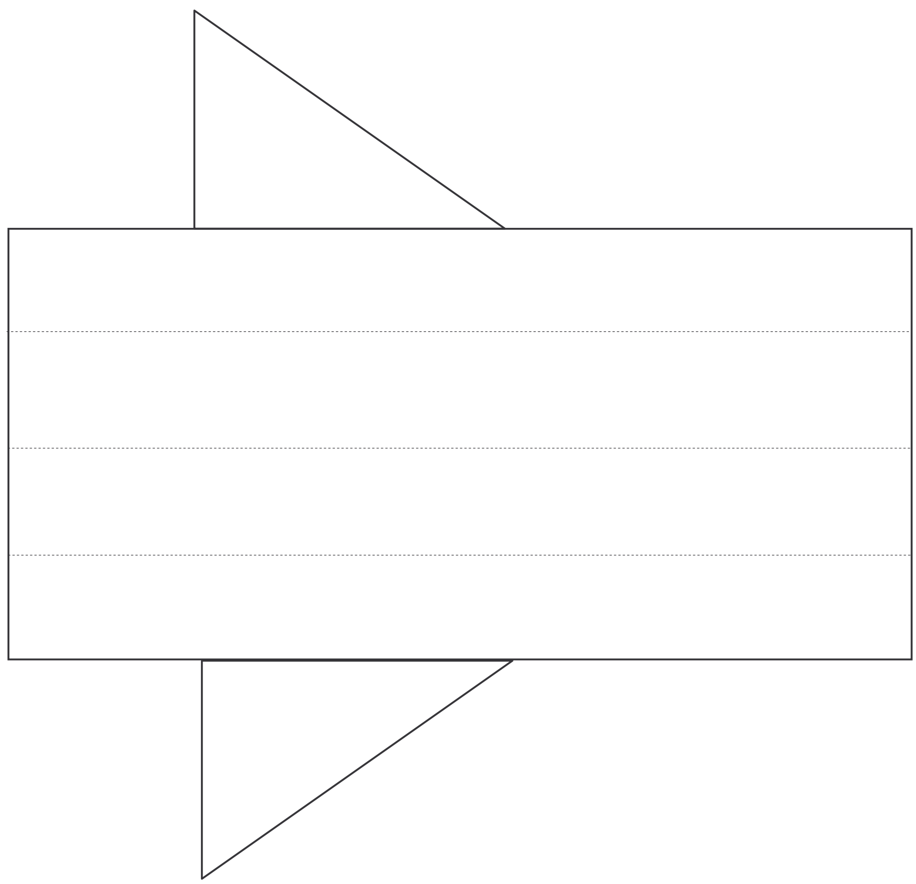
Rectangular prism: *TBO* = 412 cm^2 Vol = 480 cm^3
Triangular prism: *TBO* = 307,71 cm^2 Vol = 360 cm^3
Cylinder: *TBO* = 402,12 cm^2 Vol = 603,19 cm^3
Granny's Jam: Pot: Vol = 8 595,40 cm^2
11 Square–based jars: Vol = 8 096 cm^2
11 Rectangle–based jars: Vol =8 633,63 cm^2
So, granny must use the recgtanular–based jars if she wants to fit all the jam in!

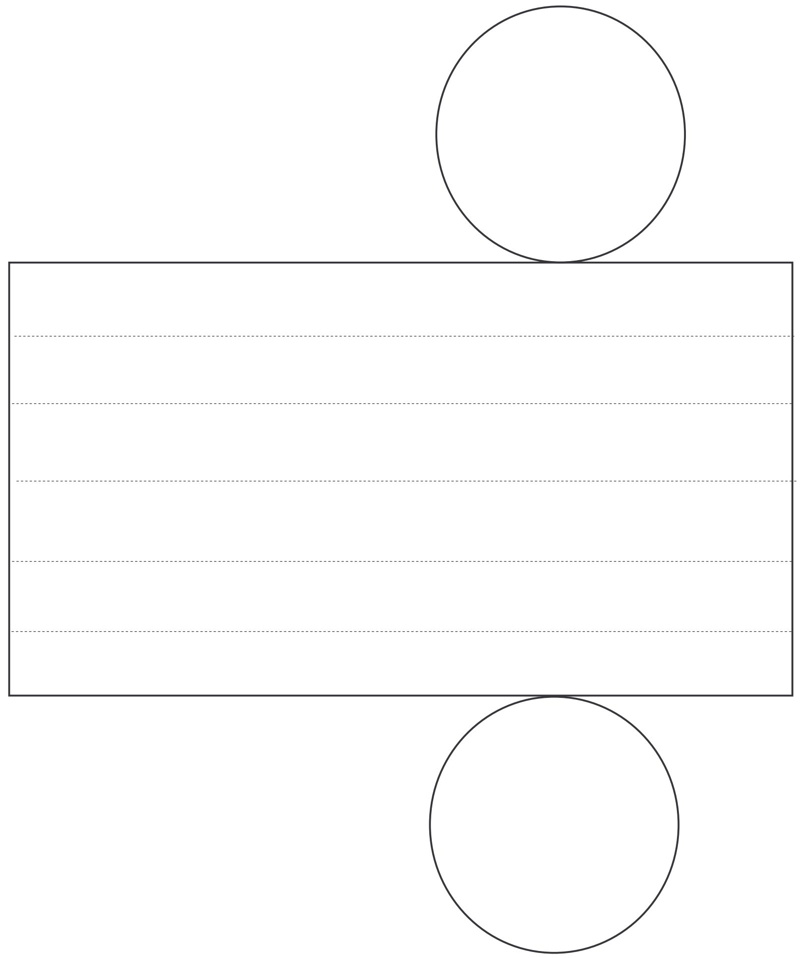


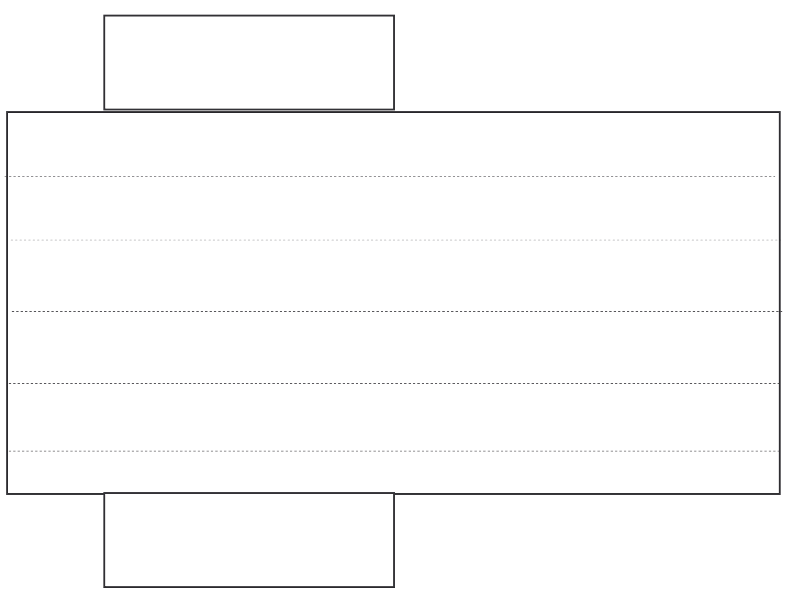
3 = triangle; 4 = tetragon; 5 = pentagon; 6 = hexagon; 7 = heptagon; 8 = octagon; * = not polygon

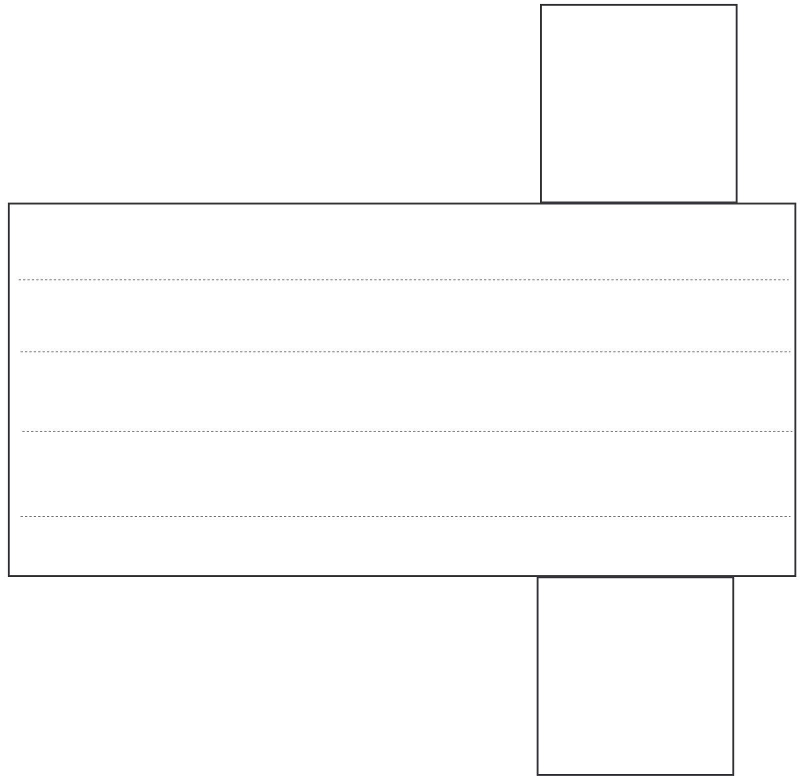
No of sides	*a* = internal angle size	*b* = 360° – *a*	*c* = *b* – 180°	Total of *a*	Total of *c*
Three	60°	300°	120°	3×*a* = 180°	3×*c* = 360°
Four	90°	270°	90°	4×*a* = 360°	4×*c* = 360°
Five	108°	252°	72°	5×*a* = 540°	5×*c* = 360°
Six	120°	240°	60°	6×*a* = 720°	6×*c* = 360°
Seven	308,57°	51,43°	–128,57°	7×*a* = 2160°	7×*c* = –360°
Twelve	330°	30°	–150°	12×*a* = 3960°	12×*c* = –360°

The test for this unit follows the figure sheets.



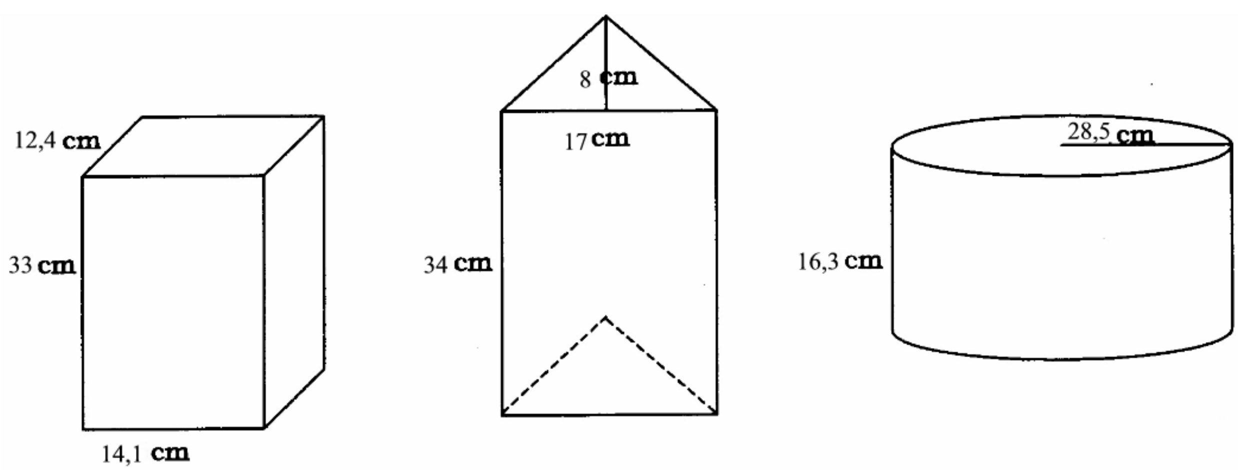






TEST 1

1. Explain how you would recognise a right prism.
2. Explain how you could find the base of a right prism.
3. Calculate the total surface area and the volume of each of the following three prisms. Give your answers accurate to two decimal places.



TEST 1 – Memorandum

1. Essential points in the explanation: three-dimensional; top and base congruent plane shapes; side(s) at right angles to base.
2. Any reasonable explanation, e.g. if the chosen side is placed at the bottom, the description of a right prism fits what you see.
3. Rectangular right prism: *TBO* = 1 939,68 cm^2 Volume = 5 769,72 cm^3

   Triangular right prism: *TBO* = 1 507,74 mm^2 Volume = 2 312 mm^3

   Cylinder: *TBO* = 8 022,37 m^2 Volume = 41 593,67 m^3

Assess test according to this scale:

OUTCOME	1	2	3	4
Understands what a right prism is.	Cannot identify right prisms	Recognises only basic prisms in easy orientation	Needs to have orientation to help	Can identify right prism perfectly
Can identify base of prism	Takes bottom side regardless	Only three basic shapes when orientation helpful	Only if orientation not confusing	Always correct
Calculates *TSA* correctly	Wrong from beginning	Starts properly, but cannot complete	Minor errors only	Always correct
Calculates Volume correctly	Does not know formula	Can do only easy ones	Minor errors only	No errors
Works accurately	Many steps wrong	Inaccuracies in intermediate steps	Decimal places wrong	All answers correct

LEARNING UNIT 2.3

Geometry

DISCUSSION

Point out to learners the relationships between angle sizes and lengths of opposite sides, i.e. that the largest angle lies opposite the longest side, etc. In this regard the following theorems are interesting:

In ΔABC:

As $b^2 = a^2 + c^2$, then $\hat{B} = 90°$

As $b^2 > a^2 + c^2$, then $\hat{B} > 90°$

As $b^2 < a^2 + c^2$, then $\hat{B} < 90°$

- These relationships are easily confirmed by construction. If time allows, this could be a good basic familiarisation exercise.
- Many learners enter grade 9 without knowing that the relationship between the height and the base of a triangle is crucial in the formula for the area of a triangle. This is a good point to emphasize that the height is measured from the vertex opposite the base, perpendicular to the base. When they construct scalene triangles with three heights and then use these heights and the corresponding three bases to work out the area three times, this usually makes the point very clear.

Congruence

- In the first exercise in activity 3.2 the learners should not work in very large groups – pairs would be best. The aim is to obtain as many versions of the triangles as possible, so as to confirm when they are congruent and when not. It would be best to give this exercise as a homework assignment if, in the teacher's judgement, the learners can do it without help.
- When discussing the four cases of congruence, point out that two right–angled triangles are congruent if the two non-hypotenuse sides are respectively equal, not by the *RHS*-rule, but by the *SAS*-rule.
- Learners have to become comfortable with the idea that figures need not be drawn to scale – and that the information given on the figure must be used. They must not measure attributes unless specifically asked to do so.

Answers to matching exercise:

$A \equiv O$ (SSS or *RHS* from calculated hypotenuse, Pythagoras)

$B \equiv G$ (SAS) They are not congruent to *I*, as the given angle is not included

$C \equiv F \equiv N$ (*SSS* or *RHS* from calculated hypotenuse)

D, *L* and *K* are not congruent as only angles are given

$E \equiv H$ (*AAS*) They are not congruent to *M*, as the given side is not in a corresponding position

- In doing proofs, rigour is not required from grade 9 learners. On the other hand, one of the strengths of learning geometry is that it teaches the learners to work in a logical and rigorous order. Make a point of encouraging this style from the type of learner who might benefit from it, particularly in further mathematics.

Congruency proofs:

1. $\angle C = 180° - 88° - 43° = 49° = \angle F$ (angles of Δ sum to 180°)

   $\angle B = 43° = \angle E$ (given)

   $AB = 15 = DE$ opposite 49°–angles (given)

   $\Delta ABC \equiv \Delta DEF$ ($\angle\angle S$)

2. $BC = 12 = FE$ (Pythagoras)

   $AC = 15 = DE$ (given)

   Right–angled triangles

   $\Delta ABC \equiv \Delta D\underline{FE}$ (RHS)

3. $BC = BC$ (common or shared side)

   $\angle B = 55° = \angle C$ (given)

   $\angle A = \angle D$ (given; shown by little arc)

   $\Delta ABC \equiv \Delta D\underline{CB}$ (RHS)

Similarity

- If a photocopier is available, teachers can design more exercises that will illustrate the principles of similarity by direct measurement.

EXERCISE:

$\angle Q = 55°$ en $\angle Z = 65°$

$\Delta PQR \,|||\, \Delta XYZ$ (equiangular)

$213 = 3(71)$

$PR = 201 \div 3 = 67$ en $XY = 74 \times 3 = 222$

$DE = AB \times 4$, $DF = AC \times 4$ and $EF = BC \times 4$

$\Delta ABC \,|||\, \Delta DEF$ (sides are in proportion)

Corresponding angles must be equal

$\angle A = \angle D = 62°$; $\angle E = \angle B = 49°$ en $\angle C = \angle F = 69°$ (angles of Δ sum to 180°)

Exercise from activity 3.5:

1.1 Yes, Pythagoras gives *BG* = 12 cm and *PT* = 3cm; sides in proportion

1.2 Yes, $\angle E = 70° = \angle U$ and $\angle P = 60° = \angle R$ (angles of triangle); equiangular

1.3 No, *LP* = 5 cm and *AT* = 20 cm (Pythagoras); but sides are not in proportion

2. Proportional constant = 36 ÷ 12 = 3

3. Short flagpole is 6,67 m tall

4. Pile of books on photocopy is 18 ÷ 2 × 3 = 27 cm high

   54 ÷ 27 = 2 is the factor by which the design is made smaller.

TEST

There is no test for this unit.

MODULE 3

CONTENT OF MODULE

- The work on the straight line (learning unit 3.2) is not directly specified in the syllabus. However, learners who intend continuing with mathematics as a subject in the senior phase will need the background. This is the main reason why it has been included in this module.
- There is great scope for investigations and creative analysis in this module – particularly in learning units 3.1, 3.4 and 3.5. Teachers should take this opportunity to bring excitement and appreciation for mathematics into the learning experience, as well as to make a connection with other contexts in everyday life.

ASSESSMENT

- The assessment instruments used for general class assignments are included with the learner's module. The educator may wish to use these, or to make others to suit her particular purpose.
- Standardised tests for summative assessment are included in the teacher's guide. Memoranda and assessments grids for these tests are included here as well.

LEARNING UNIT 3.1
Number patterns

DISCUSSION

Answers:

1 480 black; 960 white; 300 red; 240 yellow; 240 blue and 180 green
2 480 black; 960 white; 675 red; 540 yellow; 540 blue and 405 green

5.

Side length of triangle in centimetres	2	3	4	5	6	7	8
Number of beads per triangle	3	6	10	15	21	28	36
Perimeter of triangle	6	9	12	15	18	21	24

6.

Size of necklace	1	2	3	4	5	x
Number of triangular motifs	1	3	6	10	15	$x+(x-1)+(x-2)+ \ldots +1$
Number of triangular spaces	0	1	3	6	10	$(x-1)+(x-2)+ \ldots +1$
Number of beads on each side of triangular motif	4	8	12	16	20	$4x$
Total number of beads in necklace	10	30	60	100	150	$10\{x+(x-1)+(x-2)+ \ldots +1\}$
Number of black beads	1	3	6	10	15	$(x-1)+(x-2)+ \ldots +1$
Total perimeter of pendant with 1cm-diameter beads	9	21	33	45	57	$3(4x-1)$

7.

Size of necklace	1	2	3	4	5	x
Number of triangular motifs	1	3	6	10	15	$x+(x-1)+(x-2)+ \ldots +1$
Number of triangular spaces	0	1	3	6	10	$(x-1)+(x-2)+ \ldots +1$
Number of beads on each side of triangular motif	5	10	15	20	25	$5x$
Total number of beads in necklace	15	45	90	150	225	$15\{x+(x-1)+(x-2)+ \ldots +1\}$
Number of black beads	3	9	18	30	45	$3\{x+(x-1)+(x-2)+ \ldots +1\}$
Total perimeter of pendant with 1cm-diameter beads	12	27	42	57	72	$3(5x-1)$

...ACTIVITY 1.2

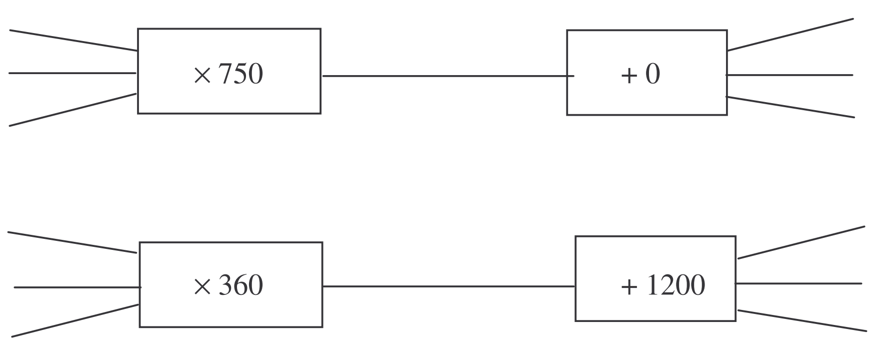
1.

Number of days:	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
Away-van:	R750	1500	2250	3000	3750	4500	5250	6000	6750	7500	8250
Best Caravans:	R1560	R1920	2280	2640	3000	3360	3720	4080	4440	4800	5160
Car-a-holiday:	R1490	R2030	2570	3110	3650	4190	4730	5270	5810	6350	6890

If they want to go for only three days then Away-van is the cheapest. Best Caravans is the cheapest for holidays of from 4 to 11 days. Car-a-holiday is never the cheapest option, even if the holiday is longer than11 days.

3. Input = 9; output = 540 × 5 + 950 = 3 650

4.



5. Bradley and his phones:

- Offer 1: "ADVANCED MOBILE! Lowest call cost! Popular handset! $20 when you sign, plus 60 cents per call!"
- Offer 2: "GENIE RENTALS has a basic charge of only $10, and calls cost $1,40 each."
- Offer 3: "HI-PRO for mobile hire! We charge only $1,00 per call! Sign up for $30."

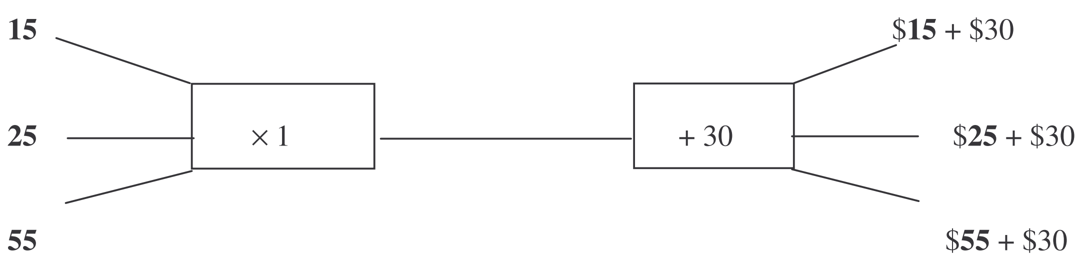
TABLES:

Number of calls:	10	20	30	40	50	60
Advanced mobile:	$26	$32	$38	$44	$50	$56
Genie rentals:	$24	$38	$52	$66	$80	$94
Hi-Pro:	$40	$50	$60	$70	$80	$90

- Flow diagrams:
- Advanced mobile:

Genie rentals:

Hi-Pro:



6. Genie Rentals is the cheapest as long as he won't want to make more than about 10 calls. Hi-Pro is never the cheapest. He is likely to get the best deal from Advanced Mobile if he wants to stay for a while.

7. It is easier to compare costs from the table. A graph would be easier still.

..ACTIVITY 1.3

- From the table, the unit cost of raisins plus packaging varies between 4,77 cents and 4,87 cents.
- As the packaging is supposed to be very cheap, the raisins will be somewhat less than R73 per kilogram.

TEST

- This unit has no test.

LEARNING UNIT 3.2

Graphical representations

Discussion

Basic graphical literacy

- The first part serves only to familiarise learners with the general appearance of a graph. Help them understand that the legends to the left and bottom of the graph contain meaningful information.
- In this section the importance of correct and adequate labelling of graphs has not been emphasized in the learner's module. This is mainly to keep the graphs legible. The teacher should point out that titles and other explanatory labels are necessary, and at appropriate times discuss the value of and need for annotation of graphs. Learners should always label their own graphs properly.
- It will be difficult, as it often is with graphs, to be completely accurate in readings taken from the graph. The main idea is that they learn where and how readings can be taken, and not to want perfectly accurate answers. It is important that they be encouraged to motivate their answers – this will lead them to try and make logical sense of the work, and not to only guess.

1.1 South Asia 1.2 East Asia 1.3 East Asia 1.4 No

1.5 Roughly speaking, the increase was about in the same ratio – each increased by about 50% of what it had been.

1.6 SA started from a very low base (almost no TV sets) and increased fast. The US started with many TV sets and could therefore not increase so much.

- In question 1.6 learners should get some input from the educator, as they might not be old enough to have the necessary experience.

2.1 (a) 50 000 – 60 000 (b) about 125 000 (c) nearly a million

2.2 more than 2.3 (see below) 2.4 About thirty years

2.5 Less than ten years 2.6 Yes – the graph goes up to the right.

- Question 2.3 – think Second World War!
- Question 2.7: The main idea is that it is impossible for the graph to keep on going upwards forever.
- Question 3 uses a graph from an area in the Western Cape – maybe it will be possible to find something close to the home range of the learners.

3.1 Between 100 m and 110 m

3.2 About 215 m

3.3 Nearly 3 000 m from Papegaaiberg

- The teacher can help a great deal to make learners more graphically literate by looking for graphs to show and discuss, and to encourage learners to do the same. An atlas usually has graphs of various kinds. Later in the module when other graphical methods are discussed, atlases can once again be used for additional examples.

Cartesian planes

- Graph paper is very expensive. Two sheets of squared paper is included at the end of this section, instead of in the learner's module. The teacher can make photocopies of them whenever necessary
- Most learners understand coordinate systems well after a bit of practice. The hardest thing to grasp can be that the integers refer to where the *lines* are, and not to the space in between. This is essential to knowing how to deal with fractions of a unit. It is effort well-repaid to make sure they get this point mastered. Point out that it works like a ruler.

1. $4 \times 36 = 144$
2. R4H2 ; L5H4 ; L4S1 ; R2S2 (Please check these answers with the learner's module)
3. Answer not included – left as an exercise for the teacher.
4. The letters are less useful – but this is the opportunity to bring in zero (for the chairs in the passages) and negative numbers for the seats to the left and to the front.

There is a great deal of terminology coming in at this stage – the more the educator uses the correct terms, the more familiar the learners will become with them.

1. A (–5 ; 6) B (–4 ; –2) C (5 ; –5) D (2 ; 3)
   E (6 ; 0) F (0 ; 8) G (–6 ; –6)
2. Something looking like a dog should emerge.

Tables and graphs

- When working with tables, it is important to take note of the order and pattern of the top row when trying to determine a pattern for the bottom row.

1.1 (The formula is $5x + 12$) $a = 57$; $b = 72$; $c = 13$

1.2 (1 ; 17) (2 ; 22) (3 ; 27) (4 ; 32) (5 ; 37) (6 ; 42) (7 ; 47) (9 ; 57) (12 ; 72) (13 ; 77)

2. This situation illustrates a stepped graph

2.1 1,5 hours is part of two hours and 2,5 hours is part of 3 hours.

2.2 Plot only dots, and don't join them.

2.4 R245

HOMEWORK

Here is a table of the values to be plotted. Important: This is also a stepped graph.

Hours	0,5	1	1,5	2	2,5	3	3,5	4	4,5	5	5,5	6	6,5	7	7,5	8
A	125	210	295	380	465	550	635	720	805	890	975	1060	1145	1230	1315	1400
B	145	230	315	400	485	570	655	740	825	910	995	1080	1165	1250	1335	1420
C	175	175	325	325	475	475	625	625	775	775	925	925	1075	1075	1225	1225
D	200	200	400	400	600	600	800	800	1000	1000	1200	1200	1400	1400	1600	1600

If it is necessary to assess this piece of homework, then the teacher can make a suitable assessment instrument for this exercise.

Equations and graphs

- From the first exercise on the six equations, the most important teaching points are: the steepness of the slopes (both positive and negative) of the graphs; the y-intercept, and the fact that these can be deduced very easily from the equation in the standard form. The learners should be led to deduce that one needs to know only two points on a straight-line graph to be able to draw the graph.
- As these six graphs are repeatedly used, the educator has to ensure that the learners' work is correct for the subsequent exercises.
- To read the gradient from a right-angled triangle, choose usable corners to draw the two sides from; also the larger the triangle, the more accurate the values.

Graphs from equations

1.1 $y = -2x + 3$; $m = -2$ and $c = 3$

1.2 $y = 2x + 3$; $m = 2$ and $c = 3$

1.3 $y = \frac{1}{2}x$; $m = \frac{1}{2}$ and $c = 0$

1.4 $y = 4$; $m = 0$ and $c = 4$

- The gradient is read off from a graph in this section; the learners need to get an intuitive feel for the gradient from looking at it on a graph. Later we calculate it from two given points.

3.1 to 3.4 The memo is left to the teachers ingenuity.

4.1 $(0 ; 1)\ (\frac{4}{3} ; 0)$ 4.2 $(0 ; -2\frac{1}{2})\ (7\frac{1}{2} ; 0)$ 4.3 $(0 ; 0)\ (0 ; 0)$

4.4 $(0 ; -\frac{5}{3})\ (\frac{5}{4} ; 0)$ 4.5 $(0 ; -4)\ (\frac{4}{3} ; 0)$ 4.6 $(0 ; \frac{1}{2})\ (-\frac{1}{2} ; 0)$

Equations from graphs

2.1	$m = -1$; $c = 1$	$y = -x + 1$		2.2	$m = -1{,}5$; $c = -1{,}5$	$y = -1\frac{1}{2}x - 1\frac{1}{2}$
2.3	$m = \frac{5}{6}$; $c = -0{,}4$	$y = \frac{5}{6}x - 0{,}4$		2.4	$m = 2$; $c = -1$	$y = 2x - 1$
2.5	$m = -1$; $c = 0$	$y = -x$		2.6	$m = -\frac{2}{3}$; $c = 0$	$y = -\frac{2}{3}x$
2.7	$m = \frac{1}{3}$; $c = 0$	$y = \frac{1}{3}x$		2.8	$m = \frac{2}{3}$; $c = 0$	$y = \frac{2}{3}x$

3.	A: $y = 3$	B: $y = -\frac{1}{2}x$	C: $y = \frac{1}{2}x + 2$	D: $x = -1$
	E: $y = -3$	F: $x = 2$	G: $y = x$	H: $y = x - 2$
	I: $y = -\frac{1}{4}x + \frac{1}{2}$	J: $y = 0$	K: $y = \frac{1}{2}x - 2$	L: $y = -\frac{1}{2}x + 4$

4. The lines are parallel. At this point, depending on the class, the educator may want to introduce the facts that for parallel lines, $m_1 = m_2$, and for perpendicular lines, $m_1 \times m_2 = -1$.

Gradients between two points

2.1 $m = \frac{6-4}{2-4} = \frac{2}{-2} = -1$

2.2 $m = \frac{2-(-1)}{1-(-2)} = \frac{2+1}{1+2} = \frac{3}{3} = 1$

2.3 $m = \frac{5-0}{1-0} = \frac{5}{1} = 5$

2.4 $m = \frac{4-4}{-1-5} = \frac{0}{-6} = 0$

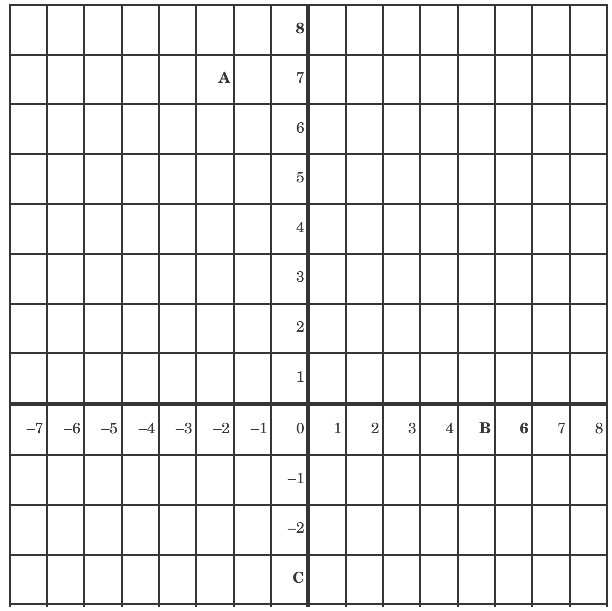
2.5 $m = \frac{0-(-3)}{7-7} = \frac{3}{0}$ which is undefined.

- Learners often confuse the meanings of the zero numerator and the zero denominator. It is wise to emphasize that a 0 denominator must be dealt with first. This means that the case of $\frac{0}{0}$ automatically is undefined (as opposed to zero).

- If time allows, ask the learners to sketch the lines above by connecting the two given points and to confirm that their answers are reasonable.

1.1 (2 ; 3)	1.2 (–3 ; –3)	1.3 (–1 ; –3)	1.4 (4 ; 0)	1.5 (4 ; 0)
2.1 (2 ; 3)	2.2 (–3 ; –3)	2.3 (–1 ; –3)	2.4 (4 ; 0)	2.5 (4 ; 0)

Test



1. Use the set of axes alongside for the following questions.

1.1 Write down the coordinates of A, B and C in the correct form.

1.2 Place dots and the letters D, E and F in the positions: D (–3 ; –5), E (0 ; –5) and F (1 ; 6).

1.3 Draw the line that goes through points A and B, very accurately.

1.4 Write down the gradient and the *y*-intercept of the line in 1.3.

1.5 Write down the defining equation of the line in 1.3.

1.6 Sketch and label the line $y = 2x – 2$.

2. Write the following equations in the standard form, and then give the values of *m* and *c* in each case.

2.1 $2y + 6x = 2$ 2.2 $12 + 6y = 3x$ 2.3 $4x + 3 = 2y + 3$

3. Calculate the gradients of each of the lines going through these sets of points.

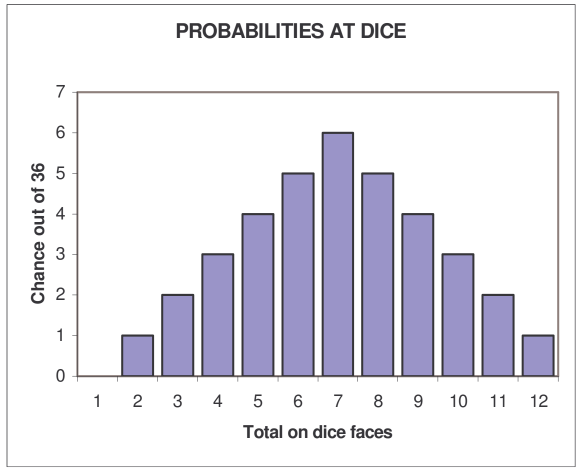
3.1 (0 ; –2) and (4 ; 0) 3.2 (3 ; 2) and (2 ; 3) 3.3 (0 ; 5) and (–2 ; 5)

4. The bar graph shows in how many ways (out of 36) you can get a certain number when you throw two ordinary dice. For example, there is no way (zero) you can get a 1 from two dice. In how many ways can you get

4.1 a four?

4.2 a seven?

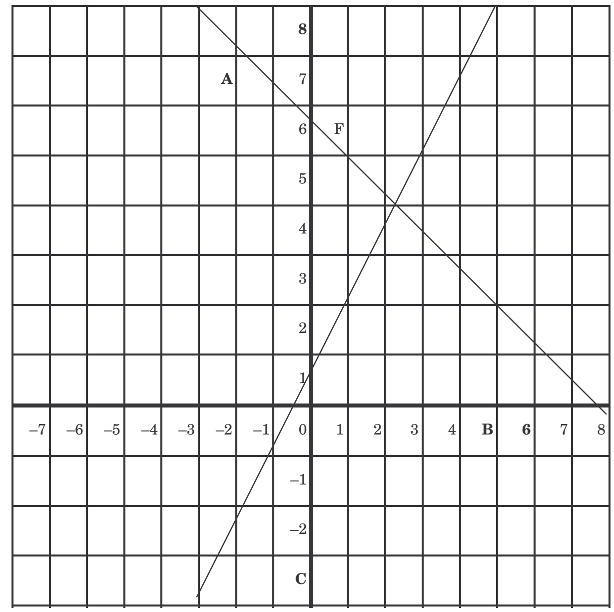
4.3 an even number?



ASSESSMENT SCALE FOR TEST

OUTCOMES	1	2	3	4
Reads points and gives coordinates	Cannot take readings	Mixes *x*- and *y*-coordinates up	One error only	All readings perfect
Places points according to coordinates	No points correct	Does not understand all coordinates	Some confusion	All points exactly correct
Finds gradient from line on graph	Not able to do it	Can't construct usable triangle	Forgets about sign	Sign and magnitude correct
Finds equation of straight line graph	No attempt made	Recognises y-intercept only	Can't form correct equation	Equation perfect
Sketches straight line graph	Cannot sketch it at all	Finds *y*-intercept only	Labels wrong/absent	Line correct and labelled
Writes equation in standard form	Knows nothing about standard form	Unsuccessful attempt	Minor error only	Final form correct
Finds gradient from two given points	No attempt	Wrong formula used	Minor error	Correct value
Reads bar graph	No attempt	One answer correct	Two answers correct	All answers correct

Memorandum of test



1.1 A (–2 ; 7) B (5 ; 0) C (0 ; –3)

1.2 on graph

1.3 on graph

1.4 $m = -1$; (0 ; 5)

1.5 $y = -x + 5$

1.6 on graph (not labelled)

2.1 $y = -3x + 1$ $m = -3$ $c = 1$

2.2 $y = \frac{1}{2}x - 2$ $m = \frac{1}{2}$ $c = -2$

2.3 $y = 2x$ $m = 2$ $c = 0$

3.1 $m = \frac{1}{2}$

3.2 $m = -1$

3.3 $m = 0$

4.1 three ways

4.2 six ways

4.3 eighteen ways

LEARNING UNIT 3.3

Equations

DISCUSSION

- The link between "word sums" and algebraic solution of equations must be emphasised. Translating words into algebra is not always easy, but the learners can be consoled with the idea that it does get easier with practice. It might be worthwhile giving them an exercise in making up word sums from bare equations.
- When it comes to exponential equations, draw a distinction between, for example, the *expression* $\sqrt{16}$ and the *equation* $x^2 = 16$. $\sqrt{16}$ is a positive number, namely 4. $x^2 = 16$ is a quadratic equation with (as the learners will discover) two solutions, one –4 and one +4.

...ACTIVITY 3.1

Say Amy has x marbles; therefore Leon has $3x$ marbles.

$3x = 48$ $\therefore x = 16$

Amy has 16 marbles

1. Let Mrs Jacobs have Rx; Mr. Jacobs has Rx + 55,30

   $x + 55{,}30 = 295{,}45$

   $x = 295{,}45 - 55{,}30$

   $x = 240{,}15$

   Mrs Jacobs has R240,15 in her purse

2. Say I thought of a

   $7 \times a \div 3 = 49$ $\therefore 7a = 49 \times 3$ $\therefore 7a = 147$ $\therefore a = 147 \div 7$ $\therefore a = 21$

   The number I thought of was 21

3. Let y be the value you multiply dollars by to get rands (the exchange rate)

   $5{,}75 \times y = 41{,}69$ $\therefore y = 41{,}69 \div 5{,}75$ $\therefore y = 7{,}25$

   The rand/dollar exchange rate is 7,25

...ACTIVITY 3.2

2. Let y be Mrs Williams' wage

   $y - 535 = 900 \div 2$ $\therefore y = 450 + 535$ $\therefore y = 985$

   Mrs Williams' wage is R985. She doesn't have to worry.

4. Say their house is b kilometres from school.

   $(b + b) + (b + 2b) = 1{,}5 \quad \therefore 5b = 1.5 \quad \therefore b = 1{,}5 \div 5 \quad \therefore b = 0{,}3$

   Their house is 300m from the school.

5. (a) $x = 7$ (b) $x = 5{,}5$ (c) $x = 30$ (d) $x = 42$

6. (a) $x = 10$ (b) $x = 3{,}5$ (c) $x = 10$ (d) $x = 24$

7. (a) $x = –5$ (b) $x = \frac{7}{3}$ (c) $x = 45$ (d) $x = 1$

..ACTIVITY 3.5

1. *Algebraically*: *LK*: $\mathbf{y} = (\mathbf{2}) = 2$ *RK*: **2** *LK* = *RK*; yes.

2. *Algebraically*: *LK*: $\mathbf{y} = (\mathbf{0}) = 0$ *RK*: $2(\mathbf{0}) – 1 = 0–1 = –1$ *LK* ≠ *RK*; no.

   Algebraically: *LK*: $\mathbf{y} = (\mathbf{1}) = 1$ *RK*: $2(\mathbf{1}) – 1 = 2–1 = 1$ *LK* = *RK*; yes.

   Algebraically: *LK*: $\mathbf{y} = (\mathbf{2}) = 2$ *RK*: $2(\mathbf{1½}) – 1 = 3–1 = 2$ *LK* = *RK*; yes.

PROBLEMS

1. Substitute 6 for a in $2a – 3b = 0 \therefore 2(6) – 3b = 0 \quad \therefore 12 = 3b \quad \therefore b = 4$

   $(a ; b) = (6 ; 4)$
2. $(x ; y) = (6 ; –1)$
3. No, the lines intersect at (–3 ; 4)
4. No, the equation is false: *LK* = –2 en *RK* = 2.

..ACTIVITY 3.6

4. –2
5. a) $x = \pm 8$ b) $x = \pm 6$ c) no real solution d) $x = \pm 7$

   e) $x = \pm 3{,}5$ f) $x = \pm 4$ g) $x = \pm 2{,}3$
6. a) $a = +4$ b) $a = –1$ c) $a = \pm\sqrt{8}$ d) $a = \pm 3$

Test

1. Find the values of the variable which make each of the following equations true.

1.1 $5x = 75$

1.2 $a - 4 = 3$

1.3 $15 - y = 16 - y$

1.4 $4x + 29 = 29$

1.5 $4(a - 3) = 2(2a - 6)$

1.6 $3x^2 - 27 = 0$

2. Find out whether 3 is a solution for any of these equations, but <u>not</u> by solving the equation.

2.1 $3(x - 2) = x$ 2.2 $x^3 = -27$ 2.3 $5(x + 3) - 2(x + 3) = 3x + 9$

3. Solve the following problems by using algebraic equations.

3.1 The difference between a number and double the number is 7. What is the number?

3.2 If Alison were 2 years younger, she would be half her brother's age. Their ages differ by 5. How old is Alison?

4. Check your own answer to question 3.2

5.1 Solve algebraically for x and y: $3a + ½b = 0$ en $a = -2$

5.2 Where do the lines $y = 2x - 4$ en $y = 2$ cross? Find the answer algebraically.

5.3 Does the point (1 ; 1) lie on both line $y = 1$ and line $y = -x + 1$? Do algebraically.

5.4 Do the lines $y = 3$ and $y = x$ intersect? Find the answer algebraically.

Assessment scale for test

OUTCOMES	1	2	3	4
Divides to remove coefficient	Coefficient not recognised	Coefficient vanishes	Subtracts or adds coefficient	Correct division
Moves terms properly	Wrong addition / subtraction	Haphazard moving of terms	Only simple cases correct	All correct
Simplifies brackets	Ignores brackets	Cannot multiply	Sign errors only	Correct
Special cases understood	Ignores possibility	Gives wrong answer to correct work	$x = 0$ puzzles	Correct answers
Proving correctness of solution	No attempt	No *LHS* / *RHS* system	Some errors	Valid work and answers
Deals with word problems	No or worthless try	Attemps some work	Gets correct answers by trial and error	Uses effective procedure
Substitution of values	Wrong	Many errors	Minor errors	Always correct
Gets correct answer to exponential equations	Cannot deal with powers	Only easy problems correct	Negative / positive cases confusing	All answers correct
Simultaneous equations	No atttempt	Tries to solve separately	Mostly correct	Correct, with answer in coordinate form

Memorandum of test

1.1 $x = 15$ 1.2 $a = 7$ 1.3 no real solution

1.4 $x = 0$ 1.5 any real number 1.6 $x = \pm 9$

2.1 ***LK*** = ***RK****;* yes 2.2 ***LK*** ≠ ***RK***; no 2.3 ***LK*** = ***RK***; yes

3.1 $2x - x = 7$ OR $x - 2x = 7$; both 7 and –7 are acceptable answers

3.2 $2(x - 2) = x + 5$ She is 9 years old

5.1 $(a\ ;\ b) = (-2\ ;\ 12)$ 5.2 $(x\ ;\ y) = (3\ ;\ 2)$ 5.3 no 5.4 yes

LEARNING UNIT 3.4

Statistics

Discussion

- The learners module is very complete and mostly self-explanatory. A great deal of the work focuses on the skills needed to deal with the organisation and analysis of information.
- The teachers who work with more than one grade 9 group have the opportunity to do comparative statistics in addition to what the module deals with. This is always of great interest to the learners and also of great value as an application of general statistics.

Statistics provide answers

Where possible, the open-ended questions in this section can be researched by the learners. They might have access to sources of information that they can share with the rest of the group.

Data collection

Experiments need to be planned very carefully. The one with the cooldrink could be very successful and interesting. Do find out beforehand whether there is controversy in the class about whether two cola-type of drinks really taste the same, or whether they can be told apart.

Another easy experiment is as follows:

- One by one the learners are brought into a room. They are blindfolded and a clip or peg is put over the nose so that they cannot smell what they eat. They then get a small piece of apple or raw potato to chew (both can be swallowed – the potato will not do any harm). Afterwards, but before their nose is liberated, only one of the following questions is asked of the taster: "Was that a piece of apple?" or "Was that a piece of raw potato?". About equally many must be asked the potato question as the apple question. It is best that the person doing the blindfolding and feeding don't know which is given, if possible.
- The fact is that raw potato and apple taste much the same if you cannot smell them. This is an experiment that illustrates the large role that the nose plays in our enjoyment of the taste of food.
- The thought-experiment of the question asked about the TV news, will be referred to again later. The discussion at this stage (about the procedure) will make the later work more relevant.

<u>The three averages</u>

It will be helpful if the teacher can prepare for the work with the heights of the learners. It does not take very long if everything is in place, and if the procedure is well thought out.

The measures of dispersion

If there is time the learners can produce bar graphs of the frequencies of the deviations of the two classes.

Comparing test marks for two groups

Even with the three averages, and the range and mean deviation it is difficult to separate the levels of attainment in the two groups. We return to these data (yes it is plural) later in the work on statistics.

Graphs

There is nothing very difficult in the graphs – all the points are discussed in the learner's module. The only point that needs emphasising is that it is a great temptation to use straight lines to connect points that have been plotted. Think carefully about what the data represent, and whether the coordinates of the points on the drawn line have any meaning whatsoever

Information from graphs

The graph on the car purchase comparison is an example of where it would have been wrong, instead of drawing a bar graph, to plot the points and join them with lines.

Direct relationships are illustrated by sentences with: " the more . . . the more . . ." as in "The harder I study, the better my marks get". Inverse relationships are of the "the more . . . the less . . ." type, as in "The more careful the government is with its spending, the less tax we have to pay".

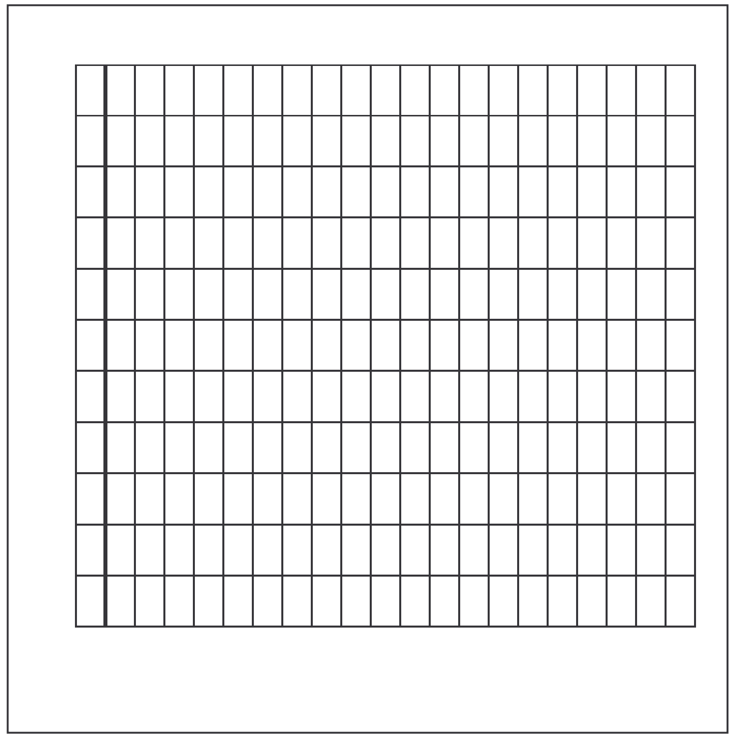
The stem-and-leaf graph shows (finally) the difference between the two groups' marks.

Ask a geography educator whether she can find a recent population demographic pyramid of the country. If you can find a few older ones to illustrate the big changes in the demographics of South Africa, you can make the learners sit up and appreciate statistics.

The following test requires learners to draw a pie chart, for which they will need protractors and compasses. Please warn them to bring these along for the test.

Test

1. In a questionnaire completed by 45 clients of a video hire shop, the following responses were obtained about the number of videos of each category the clients had taken out in a month's time. These figures are totals for all the clients.



Action: 153

Animation: 46

Comedy: 106

Documentary: 19

Drama: 33

Foreign language: 5

Kiddies: 210

Love story:74

Martial arts: 88

Murder: 52

Thriller: 94

War: 20

1.1 How many videos were taken out altogether during the month?

1.2 What is the mean number of videos taken out by one client in the month?

1.3 Use the squared paper and draw a bar graph of the number of videos in each category taken out. Be sure to label the graph properly.

2. The learners in the class with dogs weighed their dogs, and this is the frequency table of the results

DOG	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y
Kilo's	19	9	7	7	23	19	5	17	9	15	7	5	9	7	19	3	17	11	13	7	7	21	21	13	3

2.1 How many dogs do the learners in this class have altogether?

2.2 Calculate the median, mode and arithmetic mean of the weights.

2.3 What is the range of the weights?

3. Learners were asked about their favourite supper dish, and these were the five top choices of 120 learners. Make a properly labelled pie chart of the frequencies.

Fried chicken and chips	30
Pizza and salad	48
Stew and vegetables	12
Vegetable lasagna and rice	12
Mince on bread	18

ASSESSMENT SCALE FOR TEST

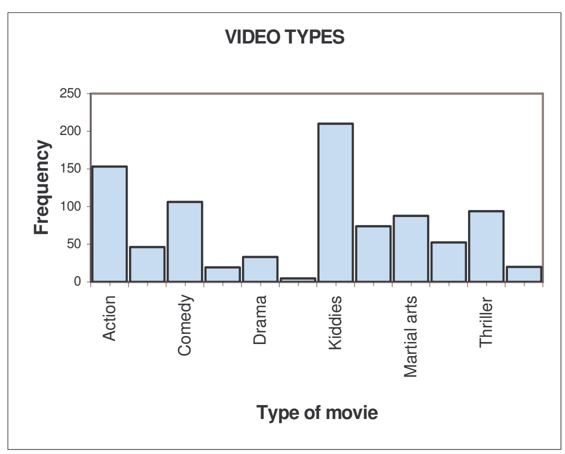
OUTCOMES	1	2	3	4
Calculates totals correctly	Does not know where to start	Adds some values	Inaccurate answer	Correct value
Calculates arithmetic mean	Does not know method	Cannot use method	Inaccurate work	Answer exact and correct
Draws bar graph	Completely wrong	Some acceptable bars	Labels left out	Correct and labelled graph
Reads table	Does not understand meaning of table	Attempts calculation	Answer wrong	Correct answer
Finds mode	No attempt	Makes attempt	Picked wrong value	Correct answer
Calculates median	Has no idea what it means	Uses correct method	Minor error only	Correct value
Calculates range	No attempt	Wrong formula used	Minor error	Correct answer
Draws pie chart	No attempt	Reasonable attempt	All correct except slice angles	Chart size and labels correct

Test - memorandum

1.1 900 videos

1.2 20 videos

1.3 (not completely labelled)



2.1 25 dogs

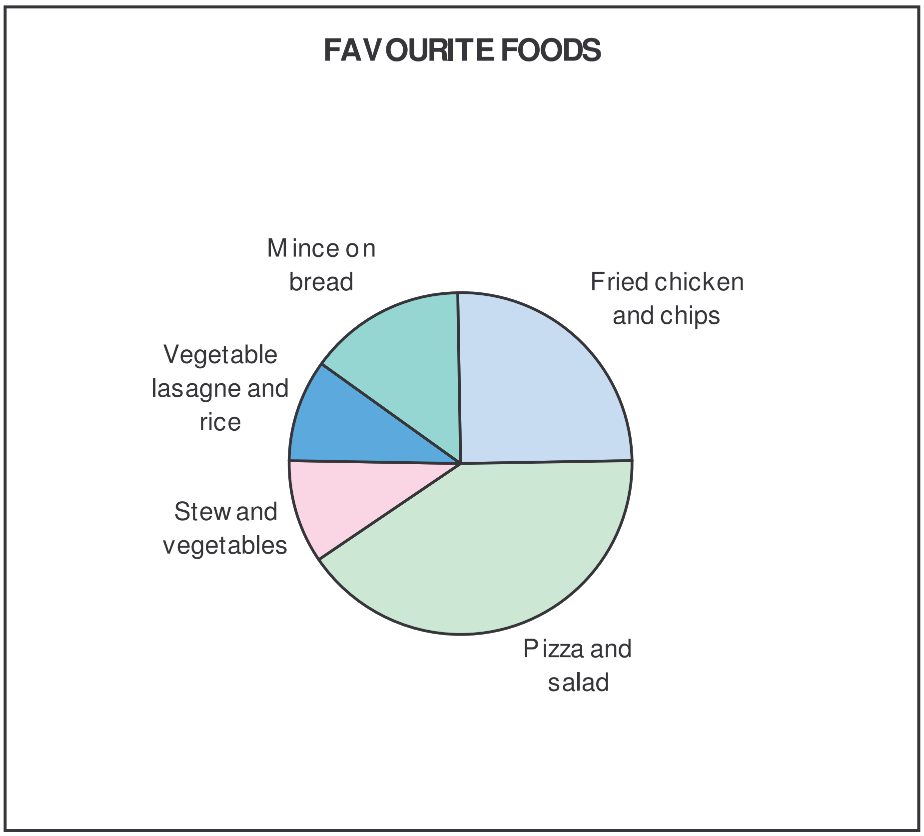
2.2 Median: 9 kg

Mode: 7 kg

Arithmetic mean:

293 ÷ 25 = 11,72 kg

2.3 23 – 3 = 20 kg



LEARNING UNIT 3.5

Probability

DISCUSSION

- The learner's module is very complete, with many examples.
- The teacher can spend time doing actual experiments (tossing a coin or throwing dice or drawing cards from a deck) and allowing the learners to practise their tallying skills for a frequency table.
- One gets dice with four faces, eight faces, and even more. These make very interesting experimental material.
- It is easy to make a cloth bag to put marbles in for some of the experiments.

Probabilities

Some comments only – the learners will (with guidance) have fun with the statements.

1.4 Relevant again later on.

1.5 A statement that is very hard to judge.

1.6 Encourage learners to figure out that this can't possibly be true.

1.8 True – as most class sizes are larger that 24, this means that more than half of the classes must have at least one pair of learners with the same birthday – let learners do some research.

2.1 Only 1 in 6

2.2 They will have to find some smarties and experiment!

2.4 True (unless the die is biased – and this does happen)

There are more aspects of risk that can be discussed – feel free to explore the subject with the learners if there is time.

Test

There is no test for this unit.

- This guide has to include two A4 sheets: one squared paper and one set of axes.
- Two paper copies of each are supplied – not electronically.

MODULE 4

Content of module

This module is the smallest of the four modules. This is due to the time needed to complete the Common Task for Assessment. Learning Unit 4.1 is important for further study in Mathematics, and should be completed thoroughly. LU 4.2 and LU 4.3 have scope for extension if time allows.

Assessment

Assessment has been kept to a minimum in this module, mainly because the CTA will demand assessment time, but also to give the educator the choice and opportunity to tailor any extra assessment needed to the requirements of her situation.

LEARNING UNIT 4.1

Quadrilaterals

Discussion

Vocabulary

There is a deal of new vocabulary in this LU. The learners can be expected to have a varied previous exposure to some of these concepts. Even words that appear to be in common use sometimes have a more restricted use in the mathematical context.

Approach

The module takes a hands-on approach to getting to know the basic quadrilaterals. Encourage the learners to experiment until they understand the characteristics. They should be encouraged to develop for themselves a portfolio of geometric basics to be used in further years.

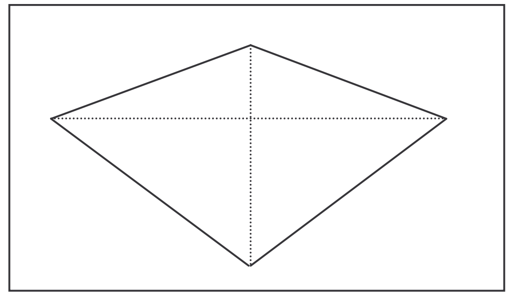
Many learners find spatial work challenging – they might need extra help to measure lengths and particularly angles correctly and accurately. This is no reason, though, to ignore these aspects of the work, but rather to pay more attention to it – after, all these are important life skills. Make sure you have a few usable rulers and protractors in the classroom.

Make as many copies of the figure sheets as the learners need – they must be encouraged to come to grips with problem-solving in geometry. Some selected answers are given here.

1.2 The dotted line in the figure of the rhombus is NOT a line of symmetry – this can be established very quickly by folding on the line.

1.4 There are surprising few differences between the rectangle and the square. When the results of these investigations are tabulated, it will be clear from the table.

1.5 Be careful that learners don't assume that trapeziums have to be symmetrical. From the trapezium, it is also useful to impress upon them that parallel lines do not need to be the same length – it is a surprisingly common fallacy.



1.6 The dotted line in the kite is actually a diagonal – unusual as it lies outside the figure. It is of course not a line of symmetry; in fact it corresponds to the "short" diagonal of the kite next to it.

2. Yes, a rhombus IS a parallelogram.

3. A very interesting question – the obvious answer is to take one of the non-parallel sides and make it parallel to the opposite side. But in doing this, one necessarily changes the length of one of the parallel sides! So, one has to change TWO things!

4. Revisit the theorem that says that co–interior angles between parallel lines are supplementary. If the class has not yet seen it, they might like to be shown that a quadrilateral is made up of two triangles, proving that the sum of the interior angles of a quadrilateral is 360°.

Apart from the characteristics of diagonals of various quadrilaterals, they are of importance in that they divide quadrilaterals into triangles, for which we have already a body of knowledge ready to be applied.

Ask learners to complete the table in pencil – they are likely to be wrong or unsure about some facts and they must be left with a correct and useful tool when they have done this exercise.

The comparison exercise will yield not only one set of correct answers – the quadrilaterals are versatile in their application. The real use of this exercise is for learners to *justify* their answers by providing reasons, and so to convince a listener. The educator's role is to keep on saying "Why do you say that?", and to teach the group members not to accept statements from each other without explanation and motivation. This skill (of being able to justify the statements one makes) is of wide and vital general application.

Other books might have different sequences of presenting the definitions of the quadrilaterals; this is perfectly acceptable. At this point it might be useful to have the learners assemble the cut-out quadrilaterals in some sort of coherent "family tree", showing the resemblances.

Measurement of areas

Although not strictly a part of quadrilaterals, the work on triangles is central to much of what the learners need to know for quadrilaterals. It also forms a useful connection between known and the unknown. Another piece of prior knowledge to be revisited is the Theorem of Pythagoras.

One of the most common problems encountered with area calculations is that the base and height are mismatched. It will affect other calculations later, so it is worth taking a little trouble now to make sure everyone has mastered the principle. The exercise of measuring the lengths and confirming that each matched pair gives the proper answer (and maybe illustrating that mismatched pairs give wrong answers) should serve to fix the concept permanently.

The learners should not need to memorise the deduction of the area formulas, but rather to understand the *process* used to arrive at the answer, as they often need those skills to solve geometrical problems.

The given problems are very simple, in that all that is needed in most cases is to substitute the values into the formula, and be careful and accurate in determining the answers.

1. 169 cm^2 (always make sure learners use the proper units)
2. 84,5 cm^2 (not necessary to calculate the side length)
3. 32,4 cm^2 4 60 cm^2
4. 36 cm^2 6 16,56 cm^2
5. 17,5 cm^2
6. 66 cm^2 (the diagonals are perpendicular; use Pythagoras)
7. 153,75 cm^2 10 212,5 cm^2

Application

The problems vary from very easy to reasonable hard. Depending on the requirements of the class, the educator may include harder work, and even problems that require the systematic proving of statements from given sketches. Even in these problems, most of them have an extra question which requires a logical process to arrive at the answer.

1.1 a = 3 cm b = 4,243 cm c = 9 cm^2 The diagonals *bisect* the 90° internal angles.
1.2 a = 5 cm b = 24 cm^2 The diagonals of a rhombus are perpendicular to each other.
1.3 a = 12 cm b = 60 cm^2 Rectangles have equal diagonals.
1.4 a = 65° b = 115° c = 27 cm^2 A rectangle is a parallelogram!

2.1 6 cm
2.2 x = 152 cm^2 Ko-binnehoeke by ewewydige lyne is supplementêr.
2.3 x = 27 cm
2.4 x = 9 cm

3.1 x = 45° All angles of a square are equal; here on is x and another is 3x.
3.2 x = 40° Adjacent angles have to be supplementary and 70° + 110° = 180°.
3.3 x = 80° Only one pair of sides marked parallel.
3.4 x = 80° The triangle is isosceles.

Test

There is no test for this unit.

LEARNING UNIT 4.2
Perspective Drawing

DISCUSSION

Background

Apart from persons who need to have technical drawings made as part of their jobs, most people are unaware of the importance of projections. Those learners who take technical drawing as a subject, will feel very confident about this work, and their expertise will be helpful to the other learners.

The explanations in the learner's module are fairly complete. Remind those learners who have forgotten about the concept of *scale*, linking it with scale factors in *similarity*. It is in work like this (and cartography) that its usefulness become apparent.

Isometric drawings are very easy to make if one uses isometric paper (like that included in the learner's module), otherwise it requires sophisticated use of draughtman's equipment. It might be a good idea to provide some rectangular object (like a shoebox) for the learners to draw.

It is not practical to teach grade 9 learners the tricky mathematical techniques for making a perspective, even an easy one-point perspective, drawing. The trick with making it on a window works well enough to give them an idea of how it should look, and gives them the personal involvement. If there is an architect or draughtsman available who can lend or give a perspective drawing to the class, that would be a bonus.

For those learners who have artistic inclinations, it might be instructive to look up the artists who started using perspective in the early fifteenth century. They deliberately chose their material to illustrate the power of this new technique.

Test

There is no test for this unit.

LEARNING UNIT 4.1
Transformations

DISCUSSION

Vocabulary

The rather technical vocabulary and notations for this unit needs to be used and stressed so that learners can become comfortable with it. Answers to the questions follow.

Translation

First diagram.

The five shapes are rectangle, triangle, oval, parallelogram and pentagon.

The pentagon has been translated five units down, and one unit left.

A (1 ; 5) → A′ (6 ; 5) B (1 ; 1) → B′(6 ; 1) C (4 ; 1) → C′(9 ; 1)

The parallelogram is 4 units away on a bearing of 180°.

Second diagram.

The shapes are rectangle, hexagon, rhombus, square and trapezium.

The learners can be asked to evaluate each others' work.

Reflection

Third diagram.

Reflection is an important transformation. It is connected with symmetry; an important property in geometry. It is also the only transformation that involves *flipping* or *turning over* the shape. Bringing a little pocket mirror with a straight edge to school to illustrate the concept of *mirror image* might be a good idea.

The parallelogram has been reflected about a vertical line *between* the two mirror images.

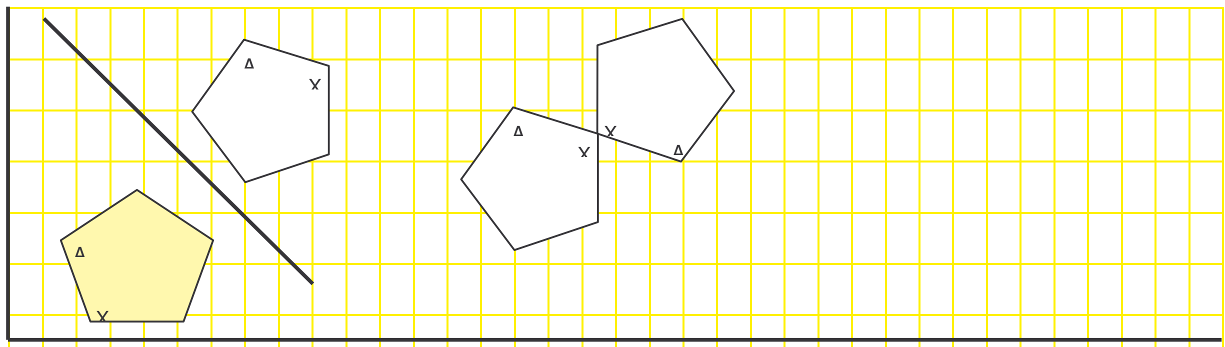
The circle is a special case as it has infinitely many lines of symmetry. This might lead to a good discussion of symmetry.

There is nothing difficult about the fourth diagram – it is simply for practice. The fifth diagram, also easy, will illustrate how symmetry can be produced by reflection.

Rotation

If time allows, allow learners to practise rotation with cut–out shapes and a pin. This will be particularly useful for those who still have problems with angular measurements.
The square was rotated through 180°.
The parallelogram was rotated through 225° clockwise. (Or 135° anti–clockwise.)
The parallelogram is the only one who will present difficulties in coordinate mapping due to the fractional units, but the rest is straightforward.

The square was translated (a) 6 units on a bearing of 090° or (b) 6 units to the right.
The square was reflected around the vertical line with defining equation $x = 18$.



This gives the answer to the rotation exercise. It is interesting to note the changes in the position of the vertex marked A in the pentagon.

Tessellations

The section on tessellations is given only for the enjoyment of it, and to illustrate the beauty of the variety that can be produced. There is a great deal of material available on the subject. Learners can be urged to explore what the local library has to offer.

The designs have been left incomplete deliberately. The learners can be invited to colour their completed designs and put them on the notice board for everybody's pleasure.

Test

There is no test for this unit.